

A Concurrent Logical Framework

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(Joint work with Frank Pfenning, David Walker, and Kevin Watkins)



CLF

- ▶ Where it comes from
 - ▶ Logical Frameworks
 - ▶ The LF approach
- ▶ What it is
 - ▶ True concurrency
 - ▶ Monadic encapsulation
 - ▶ A canonical approach
- ▶ What next?



All about Logical Frameworks

Represent and reason about object systems

- ▲ Languages, logics, ...
 - ▲ Often semi-formalized as deductive systems
 - ▲ Reasoning often informal
- ▲ Benefits
 - ▲ Formal specification of object system
 - ▲ Automate verification of reasoning arguments
 - ▲ Feed back into other tools
 - ▲ Theorem provers, PCC, ...



The LF Way

Identify fundamental mechanisms and build them into the framework (soundly!)

➤ done (right) once and for all instead of each time

- ⌈
 - ♣ Modular constructions: $[\Sigma\text{-Algebras}]$
 - ♣ $app\ f\ a$
 - ♣ Variable binding, α -renaming, substitution $[LF]$
 - ♣ $\lambda x. x+1$
 - ♣ Disposable, updateable cell $[LLF]$
 - ♣ $\lambda^{s'}. f^s$
 - ♣ **True concurrency** $[CLF]$



It's all about *Adequacy*

Informal

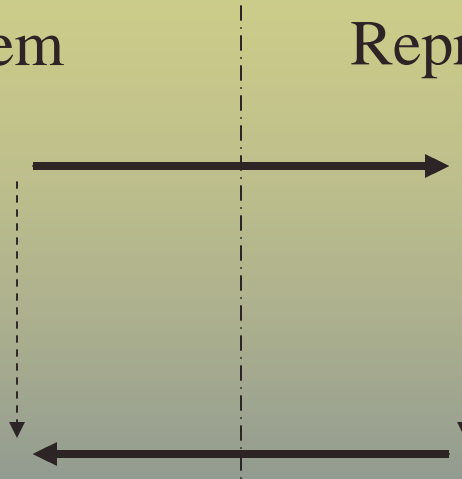
Object system

Representation

Task

- complex
- long
- tedious

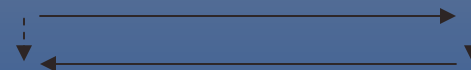
Automated



- ✦ Adequacy: correctness of the transcription
- ✦ LF: make adequacy as simple as possible



rather than



(Gödel numbers)

Representation Targets

Mottos, mottos, mottos ...

▲ LF: *judgments-as-types / proofs-as-objects*

$$\underbrace{3+5=8}_{\text{Judgment}} \Rightarrow \underbrace{N}_{\text{object}} : \underbrace{\text{ev } (+ \ 3 \ 5) \ 8}_{\text{type}}$$

(a statement we want to make)

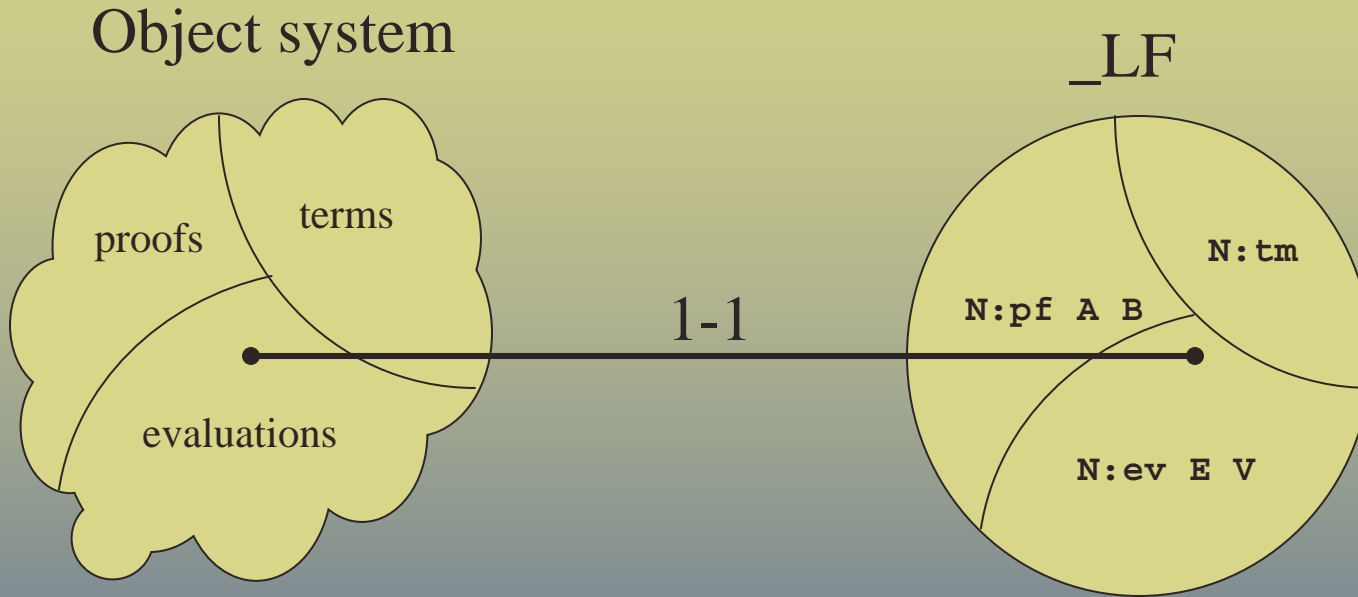
▲ LLF: *state-as-linear-hypotheses / imperative-computations-as-linear-functions*

▲ CLF: *concurrent-computations-as-monadic-expressions / ...*

▲ nextLF: *blablablablabla-as-blablablablablablabla / blablablablablablablabl-as-blablablablablablablablablablabla*



Make it Canonical, Sam



Each object of interest has exactly 1 representation

▶ Canonical objects:

- ▶ η -long, β -normal $_LF$ term
- ▶ Decidable, computable

But what is LLF?

Types

(“asynchronous” constructors of ILL)

Types: $A ::= a \mid \Pi x:A. B \mid A \multimap B \mid A \& B \mid T$

Terms

Terms: $N ::= x \mid \lambda x:A. N \mid N_1 N_2$
 $\lambda^x x:A. N \mid N_1 \wedge N_2 \mid$
 $\langle N_1, N_2 \rangle \mid \text{fst } N \mid \text{snd } N \mid$
 $\langle \rangle$

Main judgment

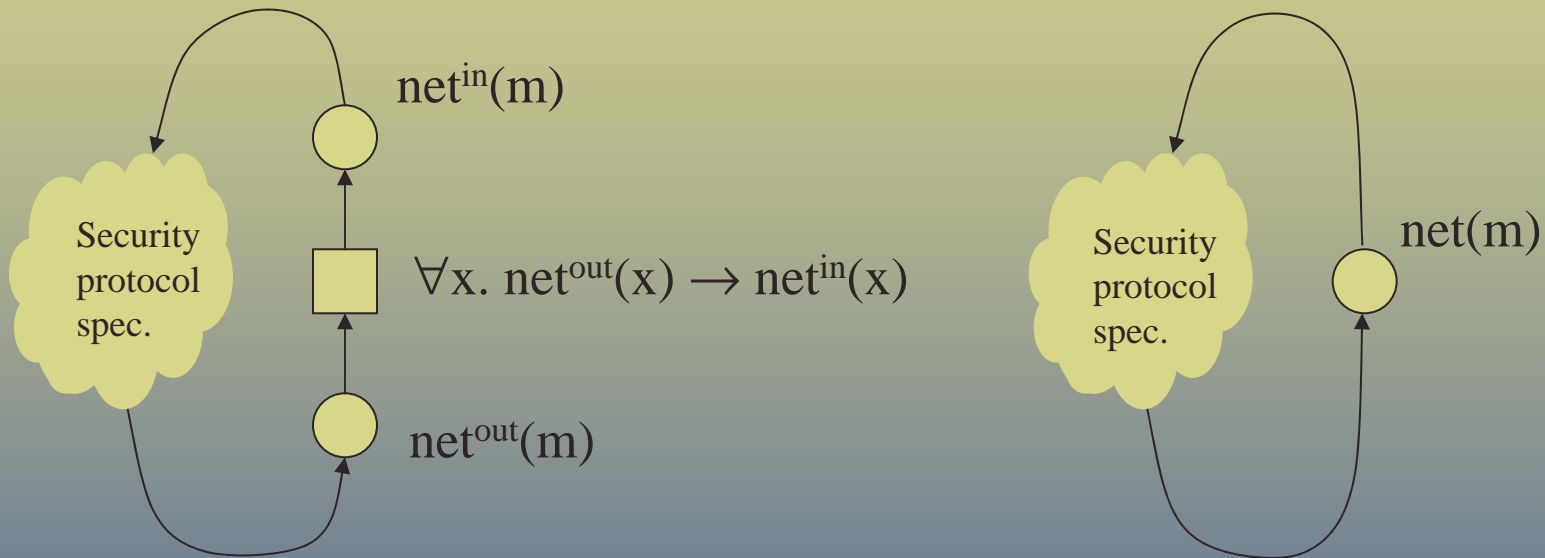
Main judgment: $\Gamma ; \Delta \vdash N : A$



CLF



An Example



Many instances can be executing concurrently

LLF Encoding

`net : step o- netout m`
`o- (netin m -o step) .`

✦ LLF forces continuation-passing style

✦ Consider 2 independent applications:

✦ $\lambda n_1^i. \text{net} \hat{\ } n_1^o \hat{\ } (\lambda n_2^i. \text{net} \hat{\ } n_2^o \hat{\ } C)$

✦ $\lambda n_2^i. \text{net} \hat{\ } n_2^o \hat{\ } (\lambda n_1^i. \text{net} \hat{\ } n_1^o \hat{\ } C)$

Should be indistinguishable (*true concurrency*)

✦ Equate them at the meta-level

`same-trace T1 T2 o- ...`

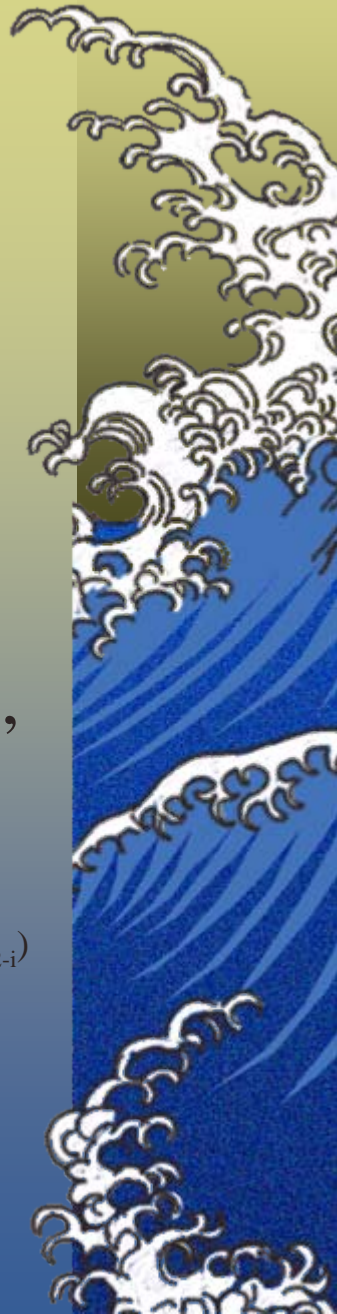
Never-ending even for small system!



Encoding in Linear logic

$$\forall m. \text{net}^{\text{out}} m \multimap \text{net}^{\text{in}} m$$

- ▶ Much simpler
- ▶ In general, requires “synchronous” operators
 - ▶ \otimes and $\mathbf{1}$
- ▶ Concurrency given by “commuting conversions”
$$\begin{aligned} & \text{let } x_1 \otimes y_1 = N_1 \text{ in } (\text{let } x_2 \otimes y_2 = N_2 \text{ in } M) \\ &= \text{let } x_2 \otimes y_2 = N_2 \text{ in } (\text{let } x_1 \otimes y_1 = N_1 \text{ in } M) \end{aligned} \quad \text{if } x_i, y_i \notin \text{FV}(R_{2-i})$$
- ▶ ... looks like what we want ...



However ...

- ▶ Commuting conversions are too wild
 - ▶ Allow permutations we don't care for
- ▶ Synchronous types destroy uniqueness of canonical forms
 - ▶ `nat:type. z:nat. s:nat->nat. c:1.`
 - ▶ Natural numbers: `z`, `s z`, `s (s z)`, ...
 - ▶ What about `let 1 = c in z`? What if `c` is linear?
- ▶ No good! ☹



Monadic Encapsulation

Separate synchronous and asynchronous types

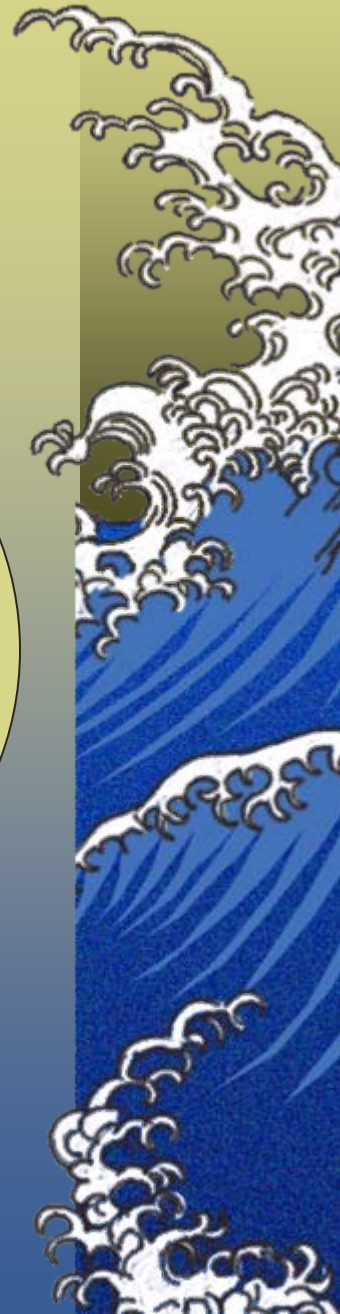
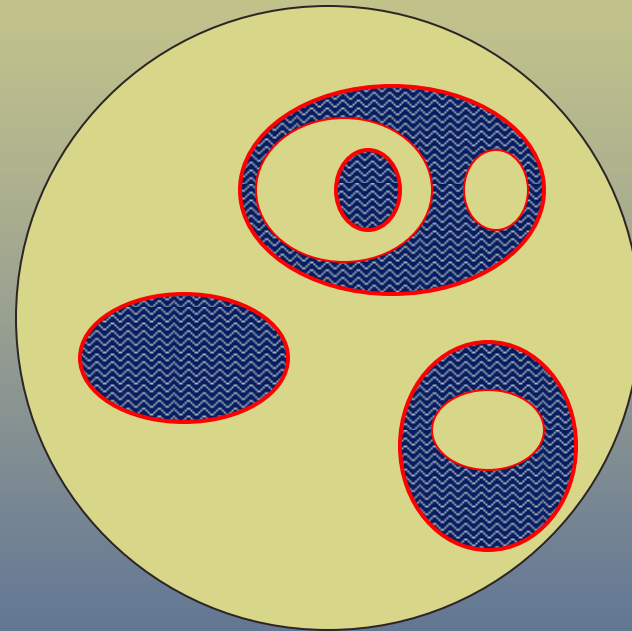
✦ *Outside* the monad

- ✦ LLF types (asynchronous)
- ✦ η -long, β -normal forms

✦ *Inside* the monad

- ✦ Synchronous types
- ✦ Commuting conversions
 - ✦ *Concurrency equation*
- ✦ η -long, β -normal forms

✦ Monad is a sandbox for synchronous behavior



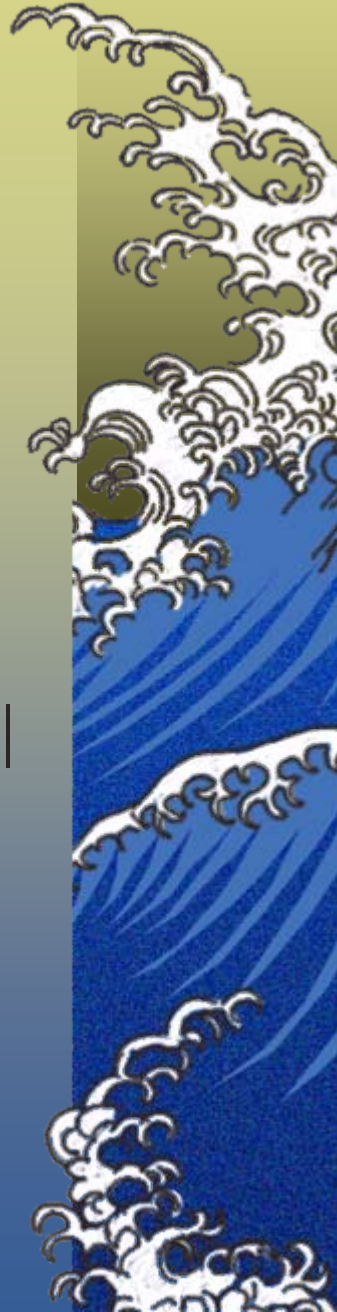
CLF

Types

- ▶ $A ::= a \mid \Pi x:A. B \mid A \multimap B \mid A \& B \mid T \mid \{S\}$
- ▶ $S ::= A \mid !A \mid S_1 \otimes S_2 \mid 1 \mid \exists x:A. S$

Terms

- ▶ $N ::= x \mid \lambda x:A. N \mid N_1 N_2 \mid \lambda^x x:A. N \mid N_1^{\wedge} N_2 \mid \langle N_1, N_2 \rangle \mid \text{fst } N \mid \text{snd } N \mid \langle \rangle \mid \{E\}$
- ▶ $E ::= M \mid \text{let } \{p\} = N \text{ in } E$
- ▶ $M ::= N \mid !N \mid M_1 \otimes M_2 \mid 1 \mid [N, M]$
- ▶ $p ::= x \mid !x \mid p_1 \otimes p_2 \mid 1 \mid [x, p]$



Example in CLF

`net : netin m -o { netout m } .`

▲ Relating the 2 specifications

▲ 2 sets of CLF declarations

▲ Meta-level definition of trace transformation

`simplify-net {Ti/o} {T}`

▲ Trivial mapping

▲ Permutations handled automatically

▲ No need to take action

▲ Critical for more complex examples



The Canonical Approach

_LF meta-theory:

- ▲ Decidability of type-checking
 - ▲ Existence of unique canonical forms
 - ▲ Substitution theorem, ...

A progression of techniques

- ▲ LF: start with equality modulo β , η over all terms
 - ▲ ~10 years to prove [several Ph.D. theses, book]
- ▲ LLF: start with equality modulo β over η -long terms
 - ▲ ~6 months to prove [thesis]
- ▲ CLF: work only with η -long, β -normal terms
 - ▲ ~2 weeks to prove [method is the thesis]
 - ▲ Applicable with minimal effort to other languages



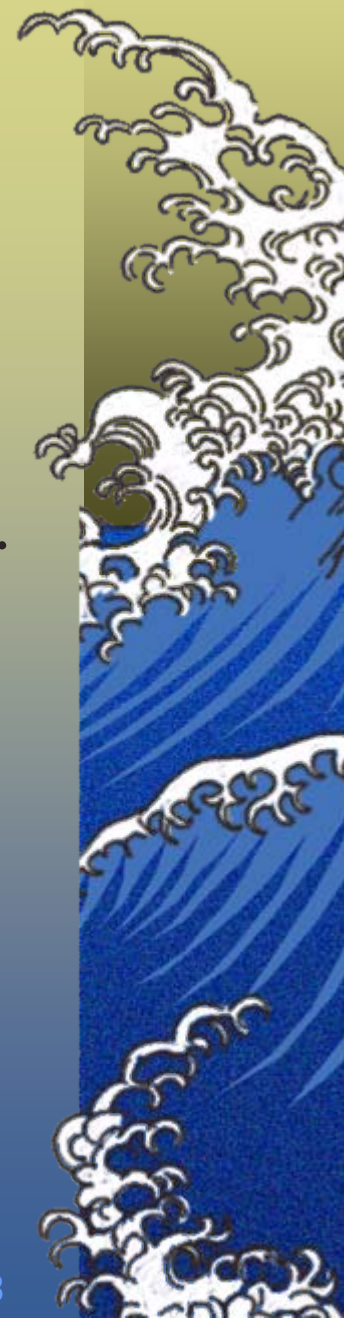
Examples and Applications

- ▶ π -calculus
 - ▶ Synchronous
 - ▶ Asynchronous
- ▶ Concurrent ML
- ▶ Petri nets
 - ▶ Execution-sequence semantics
 - ▶ Trace semantics
- ▶ MSR security protocol specification language
- ▶ No implementation ... yet ...



Further Reading

- ▶ Watkins, Cervesato, Pfenning, Walker:
A Concurrent Logical Framework: the Propositional Case, Oct. 2002
- ▶ CPWW: *A Concurrent Logical Framework*, Jan. 2002
- ▶ Forthcoming technical reports
 - ▶ *A Concurrent Logical Framework I: Judgments and Properties*
 - ▶ *A Concurrent Logical Framework II: Examples and Applications*
- ▶ NOT the paper in the proceedings



What Next?



Future Work

- ▲ Further development
 - ▲ Appropriate operational semantics
 - ▲ Irrelevant types
 - ▲ Multiple monads, ...
- ▲ Further experience
 - ▲ More concurrent systems
 - ▲ Process algebras
 - ▲ Security protocols, ...
 - ▲ Reasoning
 - ▲ Trace-base reasoning
 - ▲ Process equivalences, ...



Conclusions

CLF

- ✦ A logical framework that internalizes **true concurrency**
- ✦ **Monadic encapsulation** tames commuting conversions
- ✦ **Canonical approach** to meta-theory
- ✦ Good number of **examples**
- ✦ *This is just the beginning ... plenty more to do!*

