

A Linear Logical Framework

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Overview

A **Logical Framework** is a formalism designed to represent and reason about deductive systems

Aim:

- identify the principles underlying logics and programming languages [Pfenning'92; Michaylov,Pfenning'91; Shankar'94; Pfenning'95]

Intended applications:

- design of new and better logics and programming languages
- program verification and certification [Necula'97]

Limitations:

- ineffective with imperative formalisms [Pfenning'94]

State

So far, **no** simple, general and effective treatment of the recurring notion of **state**

- store of an imperative programming language
- database
- communication among concurrent processes, ...

A recent approach: **Linear Logic** [Girard'87]

- adequate for **representing** state and imperative computation [Chirimar'95; Hodas, Miller'94; Wadler'90]
- ineffective for **reasoning** about them

Thesis Contribution

- Design of a formalism, *LLF*, that combines
 - the meta-reasoning power of traditional logical frameworks
 - the possibility of linear logic of handling state
- First linear type theory in literature
- Conservative over *LF* [Harper,Honsell,Plotkin'93]
- Used to represent
 - imperative programming languages
 - substructural logics
 - games, ...and to reason about them

Logical Frameworks

Formalisms specially designed to provide **effective meta-representations** of **formal systems**

formal system

programming languages, logics, ...

meta-representation

represent language constructs, model their semantics, encode properties and their proofs

effectiveness

immediacy and executability

$$\text{Logical framework} = \text{meta-language} + \text{representation methodology}$$

Prior Achievements

- Logic
 - intuitionistic, classical, higher-order [Harper,Honsell,Plotkin'93]
 - modal [Avron,Honsell,Mason'89; Pfenning,Wong'95; Pfenning,Davies'96]
 - linear [Pfenning'95]
- Cut elimination [Pfenning'95]
- Logical interpretations [Pfenning,Rohwedder]
- Program extraction [Anderson'93]
- Categorical grammars and Lambek calculus [Penn'95]
- Church-Rosser theorem [Pfenning'92]
- Category theory [Gehrke'95]
- Theorem Proving [Pfenning'92]
- Logic programming [Pfenning'92]

Prior Achievements (Cont'd)

- *Mini-ML*
 - type preservation [Pfenning,Michaylov'91]
 - compiler correctness [Pfenning,Hannan'92]
 - compiler optimization [Hannan]
 - polymorphism [Pfenning'88; Harper'90]
 - *CPS* conversion, *callcc* [Pfenning,Danvy'95]
 - exceptions [Necula]
 - subtyping [van Stone]
 - refinement types [Pfenning'93]
 - partial evaluation [Hatcliff'95; Davies'96]
- Lazy functional programming
 - λ -lifting [Leone]
 - lazy evaluation [Okasaki]
 - monads [Gehrke'95]

Meta-Language

- Logics
 - Horn clauses (*Prolog*)
 - Higher-order hereditary Harrop formulas ($\lambda Prolog$ [Miller,Nadathur'88], *Isabelle* [Paulson'93])
 - Classical linear logic (*Forum* [Miller'94])
- Type theories
 - λ^Π (*LF* [Harper,Honsell,Plotkin'93])
 - Calculus of Constructions (*Coq* [Dowek&al'93], *Lego* [Pollack'94])
 - Martin-Löf's type theories (*ALF* [Nordström'93], *NuPrl* [Constable&al'86])
 - $\lambda^{\Pi \multimap \circ \& \top}$ (*LLF* [Cervesato'96])

Representation Methodology

Judgments-as-Types / Derivations-as-Objects

- Each object **judgment** is represented as a **base type**
- The **context** of an object judgment is encoded in the **context** of the meta-language
- Object-level **inference rules** are represented as **constants** that map derivations of their premisses to a derivation of their conclusion
- **Derivations** of an object judgment are represented as **canonical terms** of the corresponding base type

Representation of the Context

$$\begin{array}{c}
 \text{⌈} \quad \mathcal{T} \quad \text{⌋} \\
 \text{⌈} \quad \Omega \vdash e : \tau \quad \text{⌋} \\
 \hline
 M
 \end{array}$$

- **Term-based representation**

$$\cdot \vdash_{\Sigma} M : \text{has_type} \text{⌈}\Omega\text{⌋} \text{⌈}e\text{⌋} \text{⌈}\tau\text{⌋}$$

We must encode *explicitly*

- context operations (lookup, insertion, ...)
- context-related properties (weakening, exchange, ...)

Representation of the Context (Cont'd)

$$\frac{\boxed{x_i : \tau_i, \dots} \vdash \mathcal{T} \vdash}{\Omega \vdash e : \tau} = M$$

- **Exploitation of the meta-language context**

$$\ulcorner \Omega \urcorner \vdash_{\Sigma} M : \text{has_type } \ulcorner e \urcorner \ulcorner \tau \urcorner$$

where for each $x_i : \tau_i$ in Ω ,

$$\ulcorner x_i : \tau_i \urcorner = x_i : \text{exp}, t_i : \text{has_type } x_i \ulcorner \tau_i \urcorner$$

- context operations reduce to meta-level primitives
- meta-theoretic properties are inherited from the meta-language

λ^{II} , the Meta-Language of LF

- **Syntax**

Kinds $K ::= \text{type} \mid \Pi x:A. K$

Type families $P ::= a \mid P M$

Types $A ::= P \mid \Pi x:A. B$

Objects $M ::= x \mid c \mid \lambda x:A. M \mid M N$

- **Semantics**

$\Gamma \vdash_{\Sigma} M : A$

“ M has type A
in Γ and Σ ”

Context
 $x:A, \dots$

Signature
 $a:K, \dots, c:A, \dots$

λ^{Π} , the Meta-Language of LF (Cont'd)

$$\frac{\Gamma, x:A \vdash_{\Sigma} M : B}{\Gamma \vdash_{\Sigma} \lambda x:A. M : \Pi x:A. B} \text{ lam}$$

$$\frac{\Gamma \vdash_{\Sigma} M : \Pi x:A. B \quad \Gamma \vdash_{\Sigma} N : A}{\Gamma \vdash_{\Sigma} M N : [N/x]B} \text{ app}$$

- **Main properties**

- is strongly normalizing
- admits unique canonical forms
- type checking is decidable
- can be implemented as a logic programming language (*Elf* [Pfenning'94])

The Problem

$$\begin{array}{c}
 \text{c}_i = v_i, \dots \quad \lceil \quad \quad \quad \rceil \\
 \mathcal{E} \\
 S \triangleright K \vdash e \hookrightarrow a \quad = \quad M
 \end{array}$$

- **Term-based representation**

$$\cdot \vdash_{\Sigma} M : \text{eval} \lceil S \rceil \lceil K \rceil \lceil e \rceil \lceil a \rceil$$

... as before

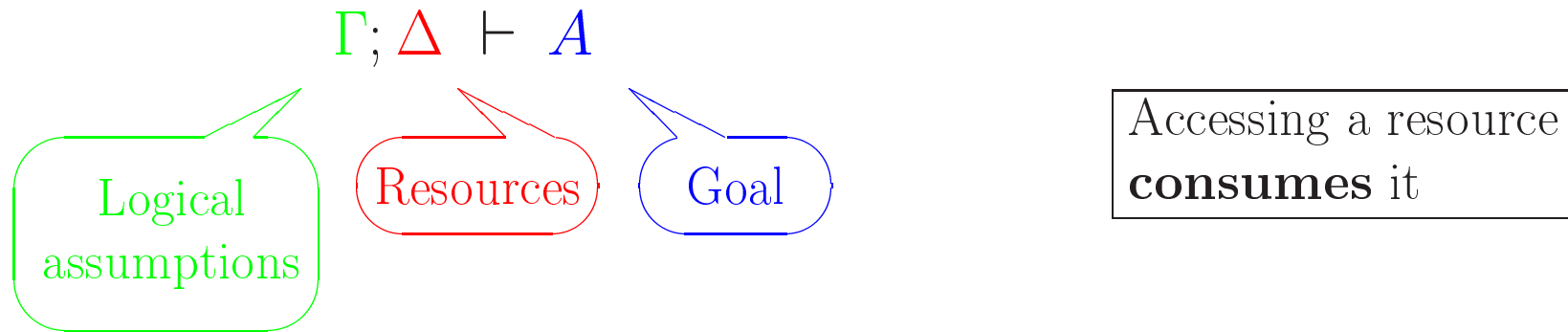
- **Context-based representation**

$$\lceil S \rceil \vdash_{\Sigma} M : \text{eval} \lceil K \rceil \lceil e \rceil \lceil a \rceil$$

This does not work!

- S is subject to *destructive operations* (e.g. assignment)
- current logical frameworks do not allow removing assumptions from the context

Linear Logic in Brief



Main resource operators

- $A \otimes B$ = “ A and B simultaneously”
- $A \& B$ = “ A and B alternatively”
- \top = “resource sink”
- $A \multimap B$ = “ B assuming A as a resource”
- $A \rightarrow B$ = “ B assuming A as a logical hypothesis”

A Simple Situation

$\$$ = “I have one dollar”

C = “I buy a coke”

F = “I buy French fries”

$\$ \rightarrow C$ = “With one dollar, I can buy a coke”

$\$ \rightarrow F$ = “With one dollar, I can buy French fries”

$\$ \rightarrow C, \$ \rightarrow F, \$ \vdash C \wedge F$

“With **one** dollar, I can buy
both a coke **and** French fries” !!

$$\frac{\frac{\frac{\$ \rightarrow C, \$ \rightarrow F, \$ \vdash \$ \rightarrow C}{\$ \rightarrow C, \$ \rightarrow F, \$ \vdash \$}}{\$ \rightarrow C, \$ \rightarrow F, \$ \vdash C}}{\frac{\frac{\Gamma \vdash \$ \rightarrow F \quad \Gamma \vdash \$}{\$ \rightarrow C, \$ \rightarrow F, \$ \vdash F}}{\underbrace{\$ \rightarrow C, \$ \rightarrow F, \$ \vdash C \wedge F}_{\Gamma}}}$$

Propositions vs. Resources

$\$ \rightarrow C$ and $\$ \rightarrow F$ are **propositions** (*logical assumptions*)

- either *true* or *false*
- accessible as many times as needed

$\$$ is a **resource**

- either *available* or *consumed*
- once consumed, it cannot be used again

Note: the derivation is uncontroversial if we have only propositions

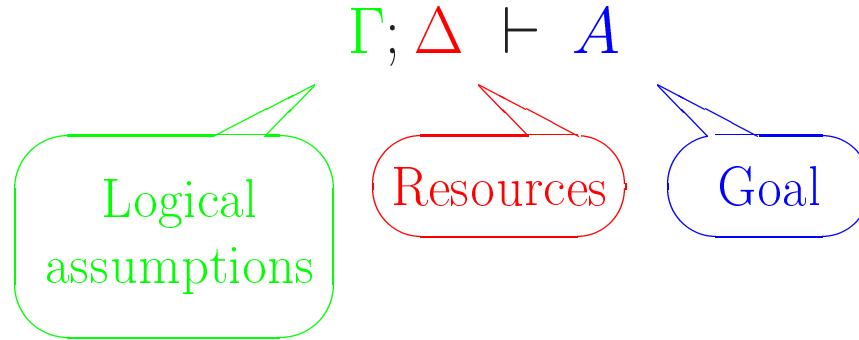
ss = “the sun shines”

sg = “I wear sunglasses”

ic = “I crave ice-cream”

$$ss \rightarrow sg, ss \rightarrow ic, ss \vdash sg \wedge ic$$

Linear Logic



Resource operators

- $\wedge \implies \otimes$ $A \otimes B = \text{“}A \text{ and } B \text{ simultaneously”}$
- $\rightarrow \implies \multimap$ $A \multimap B = \text{“}B \text{ assuming } A \text{ as a resource”}$

$$\frac{
 \frac{
 \frac{\Gamma; \cdot \vdash \$ \multimap C}{\Gamma; \$ \vdash C} \quad
 \frac{\Gamma; \$ \vdash \$}{\Gamma; \$ \vdash \$}
 }{
 \Gamma; \$ \vdash C
 } \quad
 \frac{
 \frac{\Gamma; \cdot \vdash \$ \multimap F}{\Gamma; \$ \vdash F} \quad
 \frac{\Gamma; \$ \vdash \$}{\Gamma; \$ \vdash \$}
 }{
 \Gamma; \$ \vdash F
 }
 }{
 \underbrace{\$ \multimap C, \$ \multimap F}_{\Gamma}; \$, \$ \vdash C \otimes F
 }$$

A Step Back

$$\$ \rightarrow C, \$ \rightarrow F, \$ \vdash C \wedge F$$

can also be interpreted as

“With one dollar, I can buy a coke and french fries, but not at the same time”

More resource operators

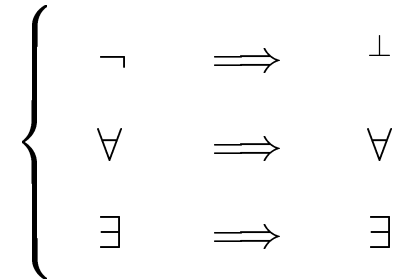
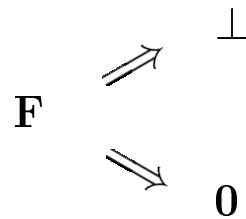
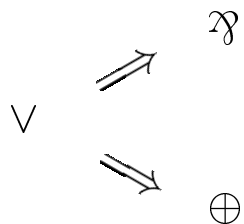
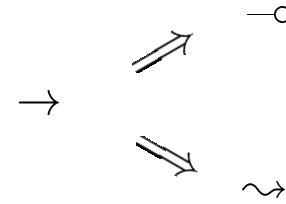
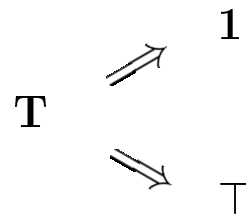
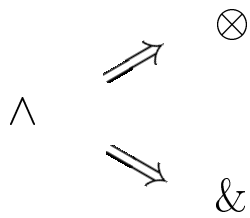
- $\wedge \implies \&$ $A \& B = \text{“}A \text{ and } B \text{ alternatively”}$

$$\frac{\frac{\overline{\Gamma; \cdot \vdash \$ \multimap C} \quad \overline{\Gamma; \$ \vdash \$}}{\Gamma; \$ \vdash C} \quad \frac{\overline{\Gamma; \cdot \vdash \$ \multimap F} \quad \overline{\Gamma; \$ \vdash \$}}{\Gamma; \$ \vdash F}}{\underbrace{\Gamma \vdash \$ \multimap C, \$ \multimap F}_{\Gamma}; \$ \vdash C \& F}$$

Linear Operators

Context splitting \Rightarrow multiplicatives

Context sharing \Rightarrow additives



Some Inference Rules

$$\frac{}{\Gamma, A; \cdot \vdash A} \text{int}$$

$$\frac{}{\Gamma; A \vdash A} \text{lin}$$

$$\frac{\Gamma, A; \Delta \vdash B}{\Gamma; \Delta \vdash A \rightarrow B} \rightarrow \text{I}$$

$$\frac{\Gamma; \Delta \vdash A \rightarrow B \quad \Gamma; \cdot \vdash A}{\Gamma; \Delta \vdash B} \rightarrow \text{E}$$

$$\frac{\Gamma; \Delta, A \vdash B}{\Gamma; \Delta \vdash A \multimap B} \multimap \text{I}$$

$$\frac{\Gamma; \Delta_1 \vdash A \multimap B \quad \Gamma; \Delta_2 \vdash A}{\Gamma; \Delta_1, \Delta_2 \vdash B} \multimap \text{E}$$

$$\frac{\Gamma; \Delta \vdash A \quad \Gamma; \Delta \vdash B}{\Gamma; \Delta \vdash A \& B} \& \text{I}$$

$$\frac{\Gamma; \Delta \vdash A \& B}{\Gamma; \Delta \vdash A} \& \text{E}_1$$

$$\frac{\Gamma; \Delta \vdash A \& B}{\Gamma; \Delta \vdash B} \& \text{E}_2$$

$$\frac{}{\Gamma; \Delta \vdash \top} \top \text{I}$$

Exponentials

Observe that \wedge corresponds to both \otimes and $\&$ when the resource context is **empty**

The same holds for all connectives **except** \rightarrow

$$\frac{\Gamma, A; \Delta \vdash B}{\Gamma; \Delta \vdash A \rightarrow B} \rightarrow \mathbf{I}$$

$$\frac{\Gamma; \Delta \vdash A \rightarrow B \quad \Gamma; \cdot \vdash A}{\Gamma; \Delta \vdash B} \rightarrow \mathbf{E}$$

Can we get rid of \rightarrow ? We do not want to, but we can:

Interpret logical assumptions as *inexhaustible resources*

$!A$ = “as many copies of A as you wish”

$$\Gamma, A; \Delta \vdash C \iff \Gamma; \Delta, !A \vdash C$$

$$A \rightarrow B \iff (!A) \multimap B$$

Observations

- Linear logic is a **conservative extension** of traditional logic:

The natural translation of judgments maintains:

- derivability
 - **derivations**
-
- Direct representation of resources

- **Meta-language:** $\lambda^{\Pi \multimap \& \top}$, a type theory based on Π , \multimap , $\&$ and \top
- **Representation methodology:** judgments-as-types, but provides direct encoding of state in the linear context
- **Range of applicability:** declarative and imperative formalisms

$\lambda^{\Pi \multimap \& \top}$, the Meta-Language of *LLF*

• Syntax

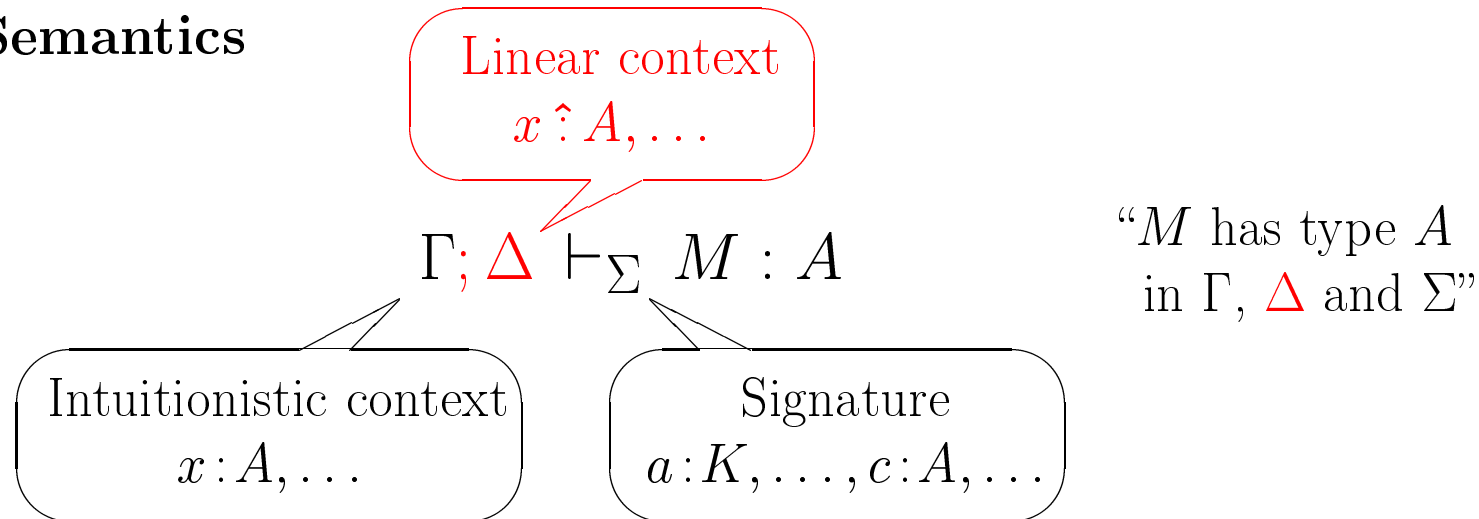
Kinds $K ::= \mathbf{type} \mid \Pi x:A. K$

Type families $P ::= a \mid P M$

Types $A ::= P \mid \Pi x:A. B$
 $\mid A \multimap B \mid A \& B \mid \top$

Objects $M ::= x \mid c \mid \lambda x:A. M \mid M N$
 $\mid \hat{\lambda}x:A. M \mid M \hat{\cdot} N \mid \langle M, N \rangle \mid \mathbf{fst} M \mid \mathbf{snd} M \mid \langle \rangle$

• Semantics



$\lambda^{\Pi \multimap \& \top}$, Some Inference Rules

$$\frac{}{\Gamma, x:A; \cdot \vdash_{\Sigma} x:A} \text{ivar}$$

$$\frac{}{\Gamma; x \hat{A} \vdash_{\Sigma} x:A} \text{lvar}$$

$$\frac{\Gamma, x:A; \Delta \vdash_{\Sigma} M:B}{\Gamma; \Delta \vdash_{\Sigma} \lambda x:A. M : \Pi x:A. B} \text{ilam}$$

$$\frac{\Gamma; \Delta \vdash_{\Sigma} M : \Pi x:A. B \quad \Gamma; \cdot \vdash_{\Sigma} N:A}{\Gamma; \Delta \vdash_{\Sigma} M N : [N/x]B} \text{iapp}$$

$$\frac{\Gamma; \Delta, x \hat{A} \vdash_{\Sigma} M:B}{\Gamma; \Delta \vdash_{\Sigma} \hat{\lambda} x:A. M : A \multimap B} \text{llam}$$

$$\frac{\Gamma; \Delta_1 \vdash_{\Sigma} M : A \multimap B \quad \Gamma; \Delta_2 \vdash_{\Sigma} N:A}{\Gamma; \Delta_1, \Delta_2 \vdash_{\Sigma} M \hat{N} : B} \text{iapp}$$

$$\frac{\Gamma; \Delta \vdash_{\Sigma} M:A \quad \Gamma; \Delta \vdash_{\Sigma} N:B}{\Gamma; \Delta \vdash_{\Sigma} \langle M, N \rangle : A \& B} \text{pair}$$

$$\frac{\Gamma; \Delta \vdash_{\Sigma} M : A \& B}{\Gamma; \Delta \vdash_{\Sigma} \text{fst } M : A} \text{fst} \quad \frac{\Gamma; \Delta \vdash_{\Sigma} M : A \& B}{\Gamma; \Delta \vdash_{\Sigma} \text{snd } M : B} \text{fst}$$

$$\frac{}{\Gamma; \Delta \vdash_{\Sigma} \langle \rangle : \top} \text{unit}$$

$\lambda^{\Pi=\circ\&\top}$: Main Properties

Lemma (*Church-Rosser property*)

If $M_1 \equiv M_2$, then there is N such that $M_1 \longrightarrow^* N$ and $M_2 \longrightarrow^* N$

Lemma (*strong normalization*)

If $\Gamma; \Delta \vdash_{\Sigma} M : A$ is derivable, then M is strongly normalizing

Theorem (*canonical forms*)

If $\Gamma; \Delta \vdash_{\Sigma} M : A$, then there exist a unique term N in canonical form such that $M \longrightarrow^* N$ and $\Gamma; \Delta \vdash_{\Sigma} N : A$

Immediacy in *LLF*

Direct correlation between an object system and its encoding

LLF gives direct support to recurrent representation patterns

- binding constructs via λ -abstraction
- derivations as proof-terms
- state manipulation via linear constructs

Computational Properties of *LLF*

- Allows automatic proof verification

Theorem (*decidability of type checking*)

It can be recursively decided whether there exist a derivation for the judgment

$$\Gamma; \Delta \vdash_{\Sigma} M : A$$

- Supports proof search

Theorem (*abstract logic programming language*)

$\lambda^{\Pi \multimap \& \top}$ is an *abstract logic programming language*

LLF, Summary

- combines the meta-reasoning power of logical frameworks with the ability of handling state of linear logic
- is a conservative extension of the logical framework LF

Theorem (*conservativity over LF*)

If Γ , M and A do not mention linear constructs, $\Gamma; \cdot \vdash_{\Sigma} M : A$ is derivable in LLF
iff $\Gamma \vdash_{\Sigma} M : A$ is derivable in LF

- can be implemented as a linear logic programming language
- has been used for the representation of
 - imperative programming languages
 - non-traditional logics
 - languages with non-standard binders
 - puzzles and solitaires
 - planning
 - imperative graph search

Case Study: *MLR*

MLR is a fragment of *ML* with

- references
- value polymorphism
- recursion

Types $\tau ::= \dots \mid \mathbf{1} \mid \tau_1 \rightarrow \tau_2 \mid \tau \text{ ref}$

Expressions $e ::= x$
| $\langle \rangle$
| $\mathbf{lam} \ x.e$
| $e_1 \ e_2$
| \dots
| c
| $\text{ref } e$
| $!e$
| $e_1 := e_2$

Store $S ::= \cdot \mid S, c = v$

Expressions

$\text{exp} : \text{type}.$
 $\text{cell} : \text{type}.$

 $\text{unit} : \text{exp}.$
 $\text{lam} : (\text{exp} \rightarrow \text{exp}) \rightarrow \text{exp}.$
 $\text{app} : \text{exp} \rightarrow \text{exp} \rightarrow \text{exp}.$
 \dots
 $\text{loc} : \text{cell} \rightarrow \text{exp}.$
 $\text{ref} : \text{exp} \rightarrow \text{exp}.$
 $\text{deref} : \text{exp} \rightarrow \text{exp}.$
 $\text{assign} : \text{exp} \rightarrow \text{exp} \rightarrow \text{exp}.$

MLR: Typing

$$\Omega \vdash e : \tau$$

“e has type τ in Ω ”

Context

$x_i : \tau_i, \dots, c_j : \sigma_j, \dots$

Expression

Type

Representation:

$$\ulcorner \Omega \urcorner \vdash_{\Sigma} \ulcorner \mathcal{T} \urcorner : \text{exp_type} \ulcorner e \urcorner \ulcorner \tau \urcorner$$

$x_i : \text{exp}, \quad t_i : \text{exp_type} \quad x_i \ulcorner \tau_i \urcorner, \quad \dots$
 $c_j : \text{cell}, \quad l_j : \text{cell_type} \quad c_j \ulcorner \sigma_j \urcorner, \quad \dots$

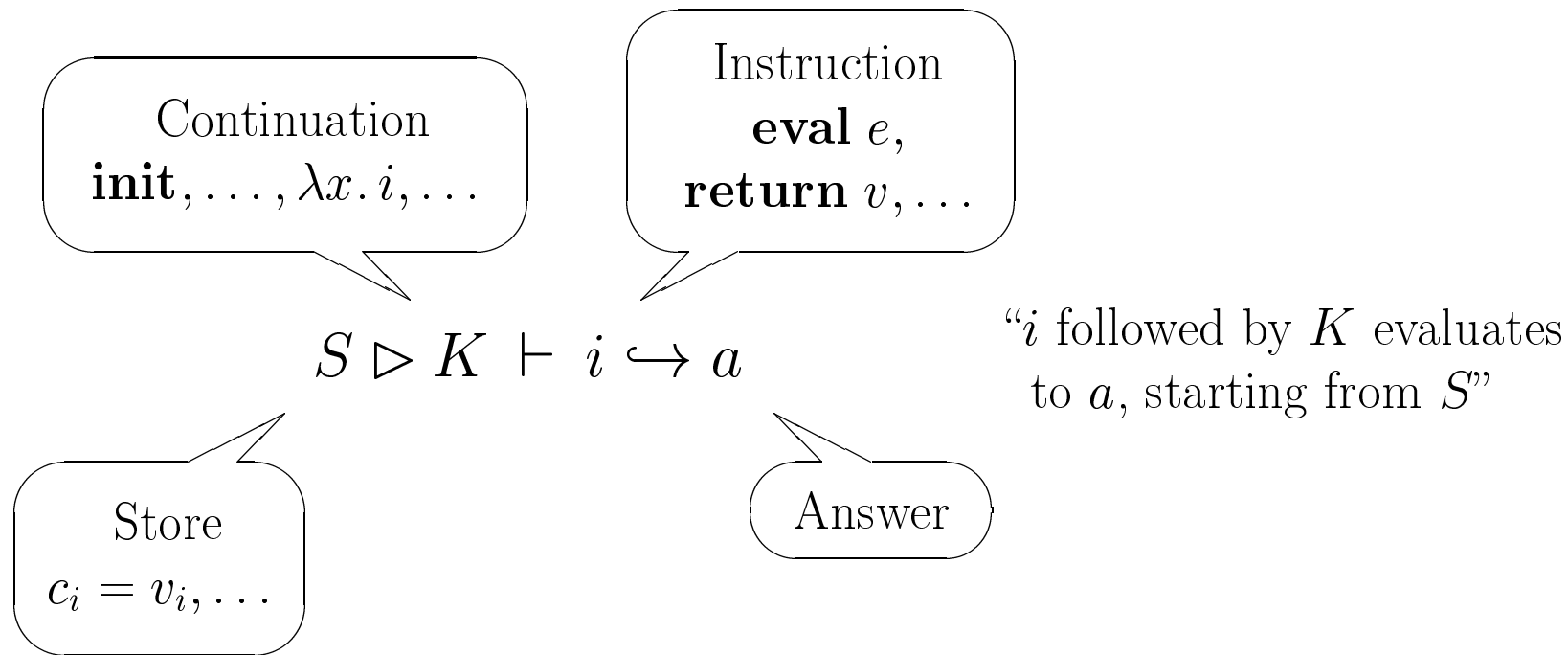
$$\frac{\Omega \vdash e_1 : \tau \text{ \textbf{ref} } \quad \Omega \vdash e_2 : \tau}{\Omega \vdash e_1 := e_2 : \mathbf{1}} \text{et_assign}$$

```
et_assign : exp_type E1 (rf T)
           -> exp_type E2 T
           -> exp_type (assign E1 E2) 1.
```

$$\frac{\Omega \vdash e : \tau \text{ \textbf{ref} }}{\Omega \vdash !e : \tau} \text{et_deref}$$

```
et_deref  : exp_type E (rf T)
           -> exp_type (deref E) T.
```

MLR: Evaluation



Representation:

$$\lceil S \rceil \vdash_{\Sigma} \lceil \mathcal{E} \rceil : \mathbf{eval} \lceil K \rceil \lceil i \rceil \lceil a \rceil$$

$c_i : \mathbf{cell}, \textcolor{red}{h_i} \hat{\mathbf{contains}} c_i \lceil v_i \rceil, \dots$

MLR: Some Imperative Rules

$$\frac{S', c = v, S'' \triangleright K \vdash \mathbf{return} \langle \rangle \hookrightarrow a}{S', c = v', S'' \triangleright K \vdash c := v \hookrightarrow a} \text{ev_assign}$$

```
ev_assign :    (contains C V    -o eval K (return unit) A)
               -o (contains C V' -o eval K (assign2 (loc C) V) A).
```

$$\frac{S', c = v, S'' \triangleright K \vdash \mathbf{return} v \hookrightarrow a}{S', c = v, S'' \triangleright K \vdash !c \hookrightarrow a} \text{ev_deref}$$

```
ev_deref  :  read C V
              & eval K (return V) A
              -o eval K (ref1 (loc C)) A.

rd :  contains C V
      -o <T>
      -o read C V.
```

MLR: Adequacy

Adequacy theorem (*Evaluation*)

Given a store $S = (c_1 = v_1, \dots, c_n = v_n)$, a continuation K , an instruction i and an answer a , all closed, there is a compositional bijection between derivations \mathcal{E} of

$$S \triangleright K \vdash i \hookrightarrow a$$

and canonical *LLF* objects M such that

$$\ulcorner S \urcorner \vdash_{\Sigma} M : \mathbf{eval} \ulcorner i \urcorner \ulcorner a \urcorner$$

is derivable, where

$$\ulcorner S \urcorner = \left[\begin{array}{c} c_1 : \mathbf{cell}, \textcolor{red}{h_1 \hat{=} \mathbf{contains} \, c_1 \ulcorner v_1 \urcorner} \\ \dots \\ c_n : \mathbf{cell}, \textcolor{red}{h_n \hat{=} \mathbf{contains} \, c_n \ulcorner v_n \urcorner} \end{array} \right]$$

MLR: Type Preservation

- Functional core: implemented in *LF* [Michaylov,Pfenning'91]
- References [Tofte'90; Harper'94]: implemented in *LLF* [Cervesato'96]

Theorem (*type preservation*)

If $S \triangleright K \vdash i \hookrightarrow a$, with $\Omega \vdash i : \tau$, $\Omega \vdash K : \tau \Rightarrow \sigma$ and $\Omega \vdash S : \Omega$, then $\Omega \vdash a : \sigma$

Proof: by induction on the evaluation derivation

The high level of abstraction of the representation permits **transcribing** this proof into an *LLF* specification capturing its computational contents

- each case yields one declaration
- the meta-reasoning is itself *linear*

Representation

```
tpev : eval K I A -> cont_type K T S -> instr_type I T -> ans_type A S -> type.
```

Future Developments: Implementation

Indispensable for tackling larger applications

- **Interpreter**

- context management [Hodas,Miller'94; Cervesato,Hodas,Pfenning'96]
- unification [Cervesato,Pfenning'96]
- term reconstruction

- **Compiler**

- WAM [Warren'83]
- embedded implication/quantification [Nadathur,Jayaraman,Kwon'95]
- types [Kwon,Nadathur,Wilson'91]
- higher-order unification
- proof-terms
- linearity

Future Developments: Applications

- Specification and verification of
 - real-world programming languages (e.g. *SML '96*, *Java*)
 - communication protocols
 - logics
- Proof-Carrying Code [Necula'97]

Use of logical frameworks technology to determine that it is **safe** to execute code provided by an **untrusted** producer

 - user extensions to the kernel of the operating system
 - mobile code in distributed/Web computing
 - foreign code extensions to a safe programming language

LLF can provide a direct handling of *resources* and a better representation of memory

(courtesy George Necula)

Future Developments: Miscellaneous

- Type theoretic extensions of LLF (e.g. dependent linear types, non-commutativity)
- Computer-assisted development environments for logics and programming languages (schema checking [Pfenning,Rohwedder'96], meta-logical frameworks [Schürmann'95])
- Educational software for logic and the theory of programming languages