Asterix calculus - classical computation in detail

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We present Asterix calculus (also denoted ${}^{*}\mathcal{X}$ calculus), built from names instead of variables. Asterix is designed to stand in computational correspondence with classical logic represented in the sequent calculus. More precisely, in the sequent system $G1$ [1], featuring explicit structural rules weakening and contraction.

It is possible to define many variants of Gentzen sequent systems. The basic Genzen systems for classical and intuitionistic logic denoted as $G1, G2$ and $G3$ are formalized in [2] and later revisited in [1]. In brief, the essential difference between $G1$ and $G3$ is the presence or absence of explicit structural rules. The distinguishing point of $G2$ is the use of the so-called mix instead of a cut rule.

In the context of the Curry-Howard paradigm, we have the following correspondence between classical logic's system $G1$ and ${}^{*}\mathcal{X}$-terms:

$$\begin{array}{rcl} \textit{Proofs} & \Leftrightarrow & \textit{Terms} \\ \textit{Propositions} & \Leftrightarrow & \textit{Types} \\ \textit{Cut ellimination} & \Leftrightarrow & \textit{Reduction} \end{array}$$

Having explicit terms for weakening and contraction at hand is an advantage strategically speaking. On the one hand we reveal the computational role of these constructors (erasure and duplication, respectively).

On the other hand, having these terms explicit, and thus a very fine grained calculus, we can identify which syntactically different terms (proofs) should be considered the same; by providing equations identifying terms up-to trivial rules-permutation.

Of course the calculus retains the desirable properties of its predecessors: type preservation, linearity preservation, strong normalisation of typed terms.

Besides Asterix ($^*\mathcal{X}$) [3, 4] there is also Obelix ($\mathcal{X}$ calculus) [5, 6]. Informally speaking, these calculi are classical analogues of intuitionistic $\lambda \mathsf{lxr}$, featuring explicit substitution, weakening and contraction [7], and $\lambda \mathsf{x}$, featuring explicit substitution [8], respectively.

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