Relating Reasoning Methodologies in Linear Logic and Process Algebra

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Linearity 2012
Tallinn, Estonia – 1 April 2012

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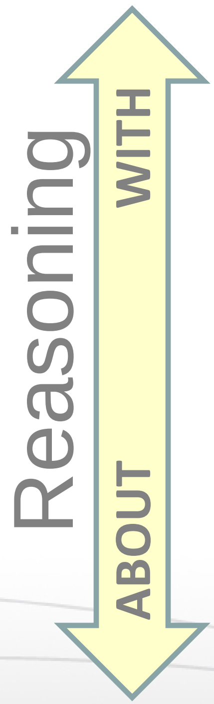
Worlds Apart


Logic

- Mechanisms
  - Derivability
  - Search
  - Cut-elimination
  - Invertibility
- Methods
  - Structural induction
  - Logical equivalence

Inductive



Process Algebra

- Mechanisms
  - Reduction
  - Structural equivalence
- Methods
  - Observational equivalence
  - Testing
  - Simulation
  - Bisimulation

Co-inductive

Related Work – PA vs. Logic

- Encodings
  - Long history back to the Chemical Abstract Machine

Reasoning with logic to reason about PA

- Reasoning
  - Miller, 1992:
    - Fragment of LL used to observe traces
  - Lincoln & Saraswat, 1991:
    - Γ |- Δ understood as process Γ passing test Δ
  - McDowell, Miller & Palamidessi, 2003:
    - LL + definitions to express simulation as derivability
  - Tiu & Miller, 2004:
    - Nominal logic to capture bisimulation

This work

Explore *one* relationship between methods to

- reason <u>about</u> logic – Inductive
- reason <u>about</u> process algebra – Co-inductive

… very initial steps

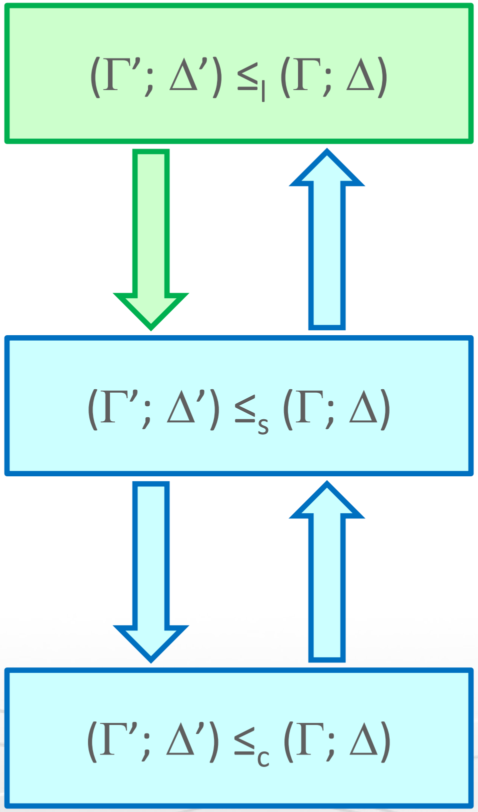
- Motivations
  - Growing interest in using logic for concurrency
  - Use co-inductive reasoning in logic
    - CLF

Outline

Logical Preorder

Simulation Preorder

Contextual Preorder



Logical Preorder $(\Gamma'; \Delta') \leq_l (\Gamma; \Delta)$

Simulation Preorder $(\Gamma'; \Delta') \leq_s (\Gamma; \Delta)$

Contextual Preorder $(\Gamma'; \Delta') \leq_c (\Gamma; \Delta)$

Our Linear Logic

$$A ::= a \mid 1 \mid A \otimes B \mid \top \mid A \,\&\, B \mid a \multimap B \mid\ !A$$

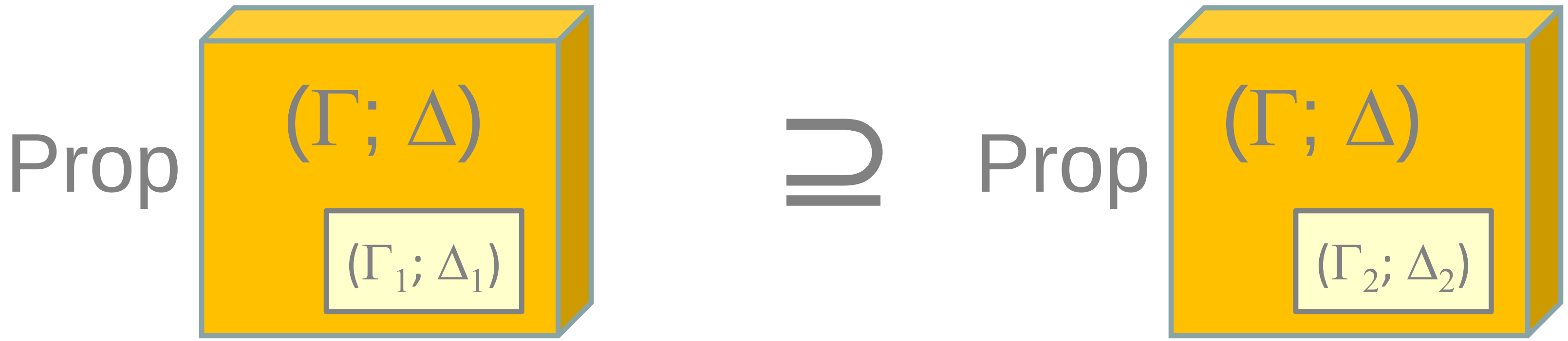
$$\Gamma; \Delta \vdash A$$

$$\frac{}{\Gamma; a \vdash a} \qquad \cdots \qquad \frac{\Gamma; \cdot \vdash A}{\Gamma; \cdot \vdash\ !A} \qquad \frac{\Gamma, A; \Delta \vdash C}{\Gamma; \Delta, !A \vdash C} \qquad \frac{\Gamma, A; \Delta, A \vdash C}{\Gamma, A; \Delta \vdash C}$$

Logical Preorder – $\leq_l$

$(\Gamma_1; \Delta_1) \leq_l (\Gamma_2; \Delta_2)$ *iff* for all $(\Gamma; \Delta)$ and C,

$(\Gamma_1,\Gamma); (\Delta_1,\Delta) \mid\!- C$ *implies* $(\Gamma_2,\Gamma); (\Delta_2,\Delta) \mid\!- C$



- $(\Gamma_1; \Delta_1) \leq_l (\Gamma_2; \Delta_2)$ *iff* $\Gamma_2; \Delta_2 \mid\!- \bigotimes !\Gamma_1 \otimes \bigotimes \Delta_1$
  - Inductive!

Logical Preorder $(\Gamma'; \Delta') \leq_l (\Gamma; \Delta)$

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Process-as-Formula

Interpretation		Transitions
• a	*send*	
• 1	*null*	• $(\Gamma; \Delta, 1) \rightarrow (\Gamma; \Delta)$
• $A \otimes B$	*fork*	• $(\Gamma; \Delta, A \otimes B) \rightarrow (\Gamma; \Delta, A, B)$
• T	*stuck*	• *(none)*
• A & B	*choice*	• $(\Gamma; \Delta, A_1 \& A_2) \rightarrow (\Gamma; \Delta, A_i)$
• $a \multimap B$	*receive*	• $(\Gamma; \Delta, a, a \multimap B) \rightarrow (\Gamma; \Delta, B)$
• !A	*replicate*	• $(\Gamma; \Delta, !A) \rightarrow (\Gamma, A; \Delta)$
		• $(\Gamma, A; \Delta) \rightarrow (\Gamma, A; \Delta, A)$

The left rules of LL

Reduction-as-Search

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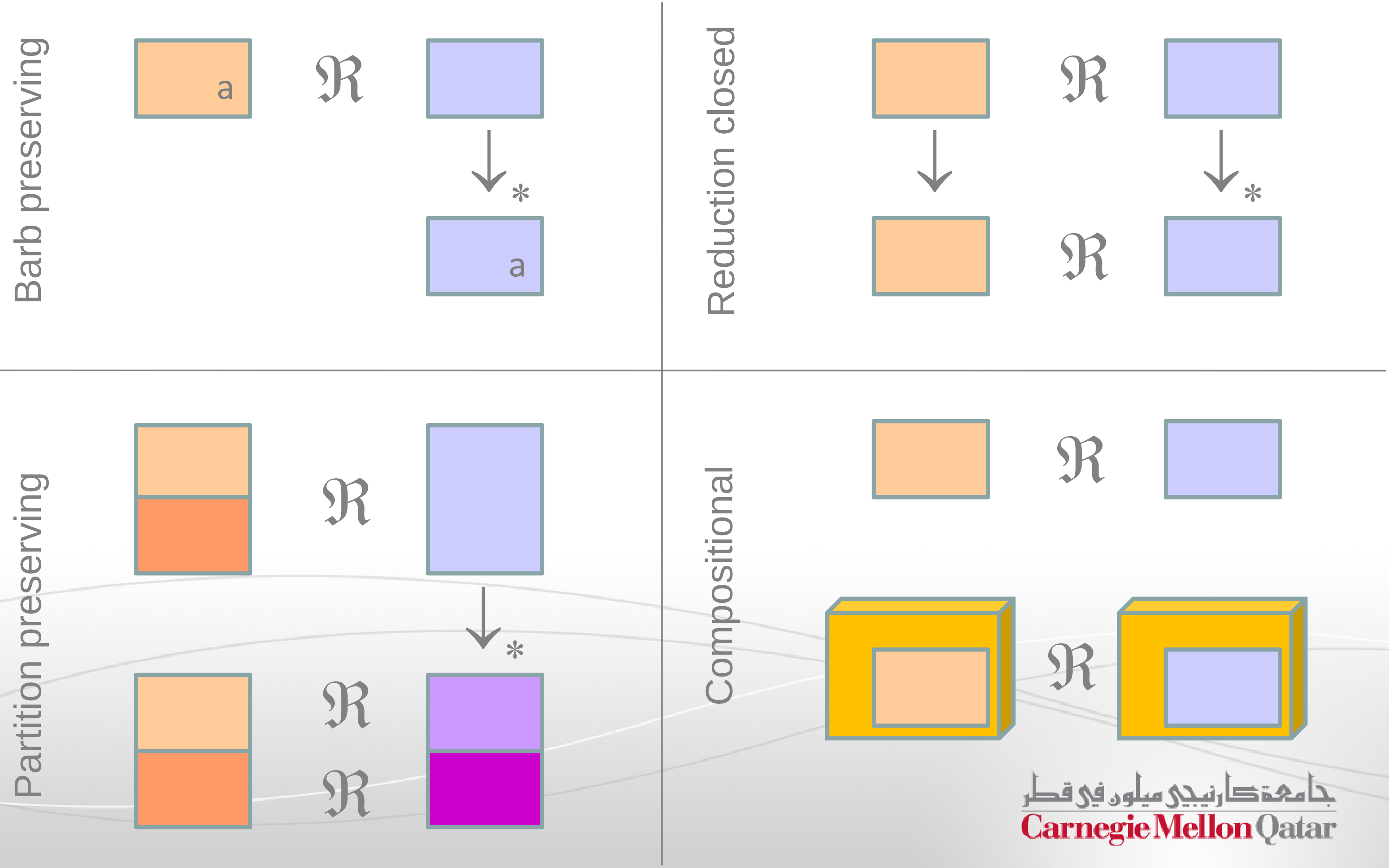
Towards a Contextual Preorder



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Contextual Preorder – $\leq_c$

The largest $\mathfrak{R}$ with these properties

- Symmetric closure is *contextual congruence*
  - AKA *reduction barbed congruence*

- Co-inductive

➢ $\leq_c$ is a preorder

Logical Preorder

$(\Gamma'; \Delta') \leq_l (\Gamma; \Delta)$

Simulation Preorder

$(\Gamma'; \Delta') \leq_s (\Gamma; \Delta)$

Contextual Preorder

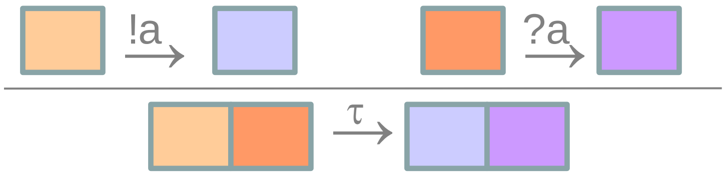
$(\Gamma'; \Delta') \leq_c (\Gamma; \Delta)$

A Labeled Transition System

$$(\Gamma; \Delta) \xrightarrow{\alpha} (\Gamma'; \Delta')$$

$$\frac{}{(\Gamma; \Delta, a) \xrightarrow{!a} (\Gamma; \Delta)}$$

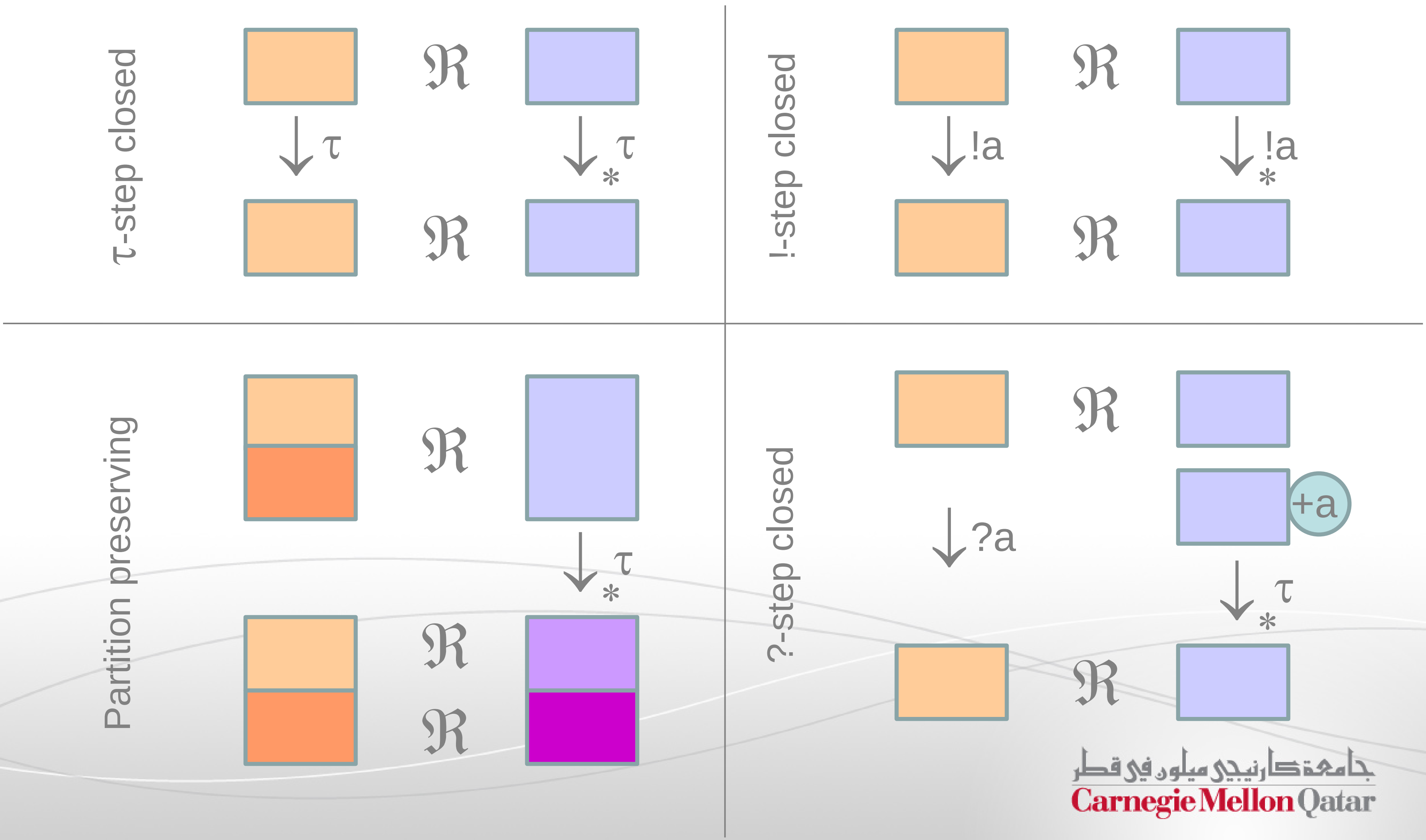
$$\frac{}{(\Gamma; \Delta, a \multimap B) \xrightarrow{?a} (\Gamma; \Delta, B)}$$



$$\frac{}{(\Gamma; \Delta, 1) \xrightarrow{\tau} (\Gamma; \Delta)} \quad \ldots$$

- Capture the other left rules

Towards a Simulation Preorder



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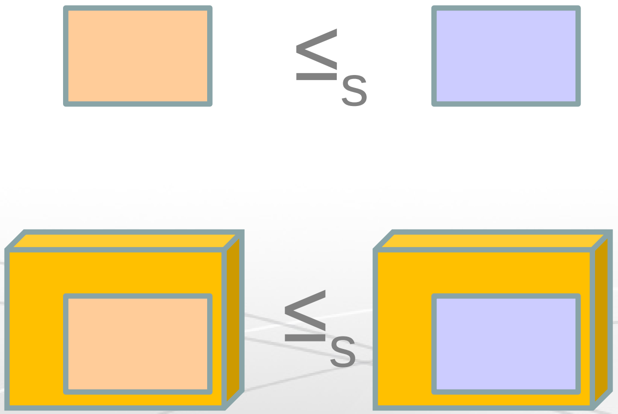
Simulation Preorder – $\leq_s$

The largest $\mathfrak{R}$ with these properties

- Co-inductive

➢ $\leq_s$ is a preorder

➢ $\leq_s$ is compositional



Logical Preorder

$(\Gamma'; \Delta') \leq_l (\Gamma; \Delta)$

Simulation Preorder

$(\Gamma'; \Delta') \leq_s (\Gamma; \Delta)$

Contextual Preorder

$(\Gamma'; \Delta') \leq_c (\Gamma; \Delta)$

$$\leq_I \Longleftrightarrow \leq_s$$

$$\leq_I \Longrightarrow \leq_s$$

- Chaining of inductive results
  - |- to $\leq_s$
  - Inductive definition of $\leq_I$
- Rather involved

$$\leq_I \Longleftarrow \leq_s$$

- Co-inductive parts
  - $\leq_s$ to |-
- Inductive parts
  - Transitive closures
  - Weak-head reduction
- Also complex

$\square$ |- A *iff* $(\cdot; A) \leq_s$ $\square$

Logical Preorder — $(\Gamma'; \Delta') \leq_l (\Gamma; \Delta)$

Simulation Preorder — $(\Gamma'; \Delta') \leq_s (\Gamma; \Delta)$

Contextual Preorder — $(\Gamma'; \Delta') \leq_c (\Gamma; \Delta)$

$$\leq_s \Longleftrightarrow \leq_c$$

$$\leq_s \Rightarrow \leq_c$$

- Relatively simple
- Mainly co-inductive
  - Compositionality
- Specific inductive parts
  - Transitive closure

$$\leq_s \Leftarrow \leq_c$$

- Direct co-inductive proof
  - Also rather simple
- Uses a few lemmas
  - Some co-inductive
  - Other inductive

What's Next

- Extend results to
  - general implication: A –o B
    - Special cases in join calculus
    - Largely beyond traditional PA
  - quantifiers: $\forall$x. A, $\exists$x. A
    - Special cases in π-calculus
- Go beyond preorder
- Implement co-inductive reasoning within CLF
  - A framework to reason about concurrent languages

Thank you!

Questions?