Automata on Infinite Objects

Seminar Presentation by
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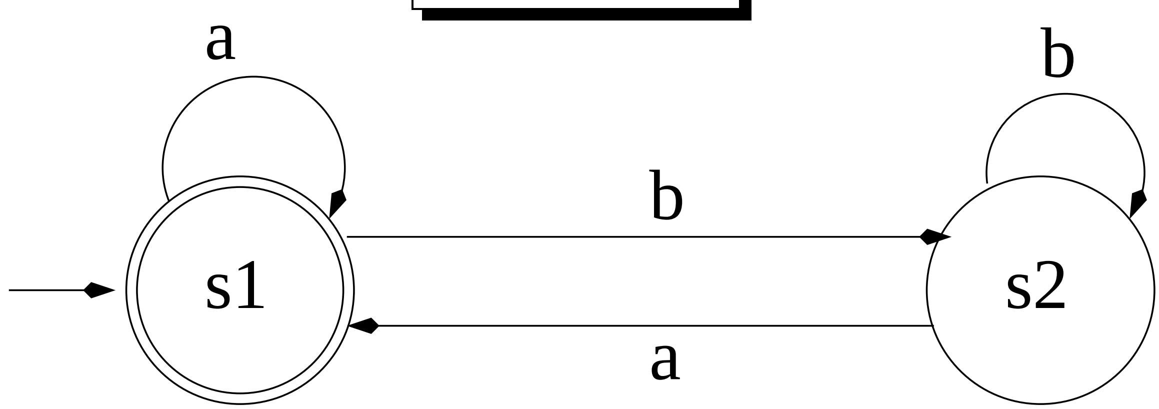
Topics

In this seminar we will look at various automata on infinte words (or ω-words). In particular:

- Büchi Automata (BA)
- Second Order Theory of One Successor (S1S)
- Muller automata (MA)
- Büchi's & McNaughtons Theorem
- DBA, NBA, DMA, NMA, ω-words

BA example



$\sum = \{a, b, c\}$

- Note for a string to be accepted, it should visit s1 infinitely often

- So the above BA accepts all strings α in which a occurs infinitely often

Büchi automata

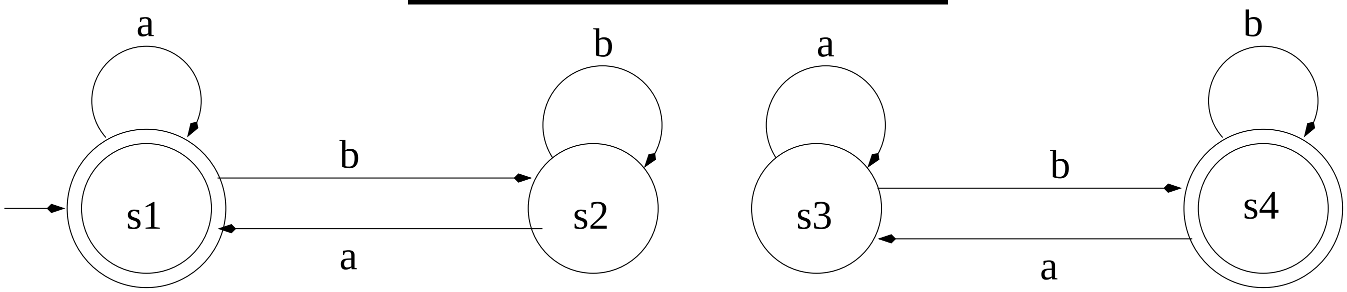
Formally, it is a tuple $\mathcal{A} = (Q, q_0, \delta, F)$, where

- Q is the finite state set
- $q_0 \in Q$ is the starting state
- δ is transition relation,$\delta \subseteq Q \times A \times Q$
- $F \subseteq Q$ is the set of final states.
- ω-word is accepted if some final state is visited infinitely many times.

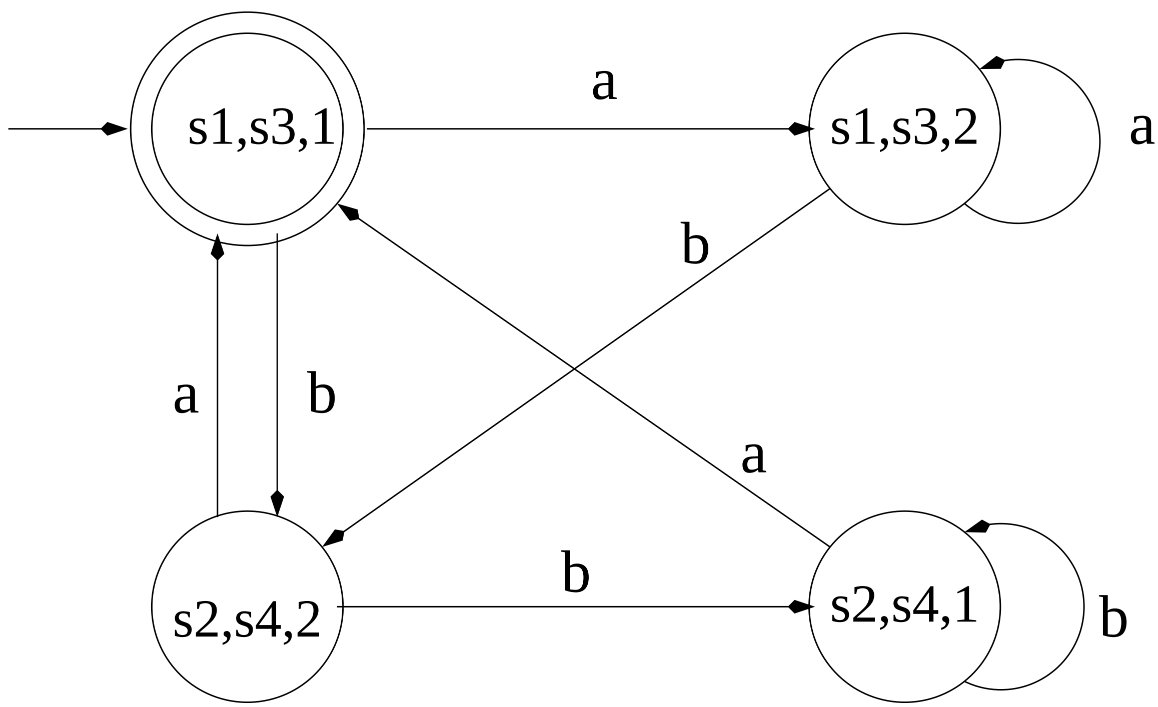
Closure Properties of Büchi Recognizable languages

- If $V \subseteq A^*$ is regular then V^w is Büchi recognizable.
- Closed under prefixing with normal finite automata.
- Closed under Union, Intersection & Complementation

Intersection example



$BA\ \mathcal{A}_1$ $BA\ \mathcal{A}_2$



BA $\mathcal{A}$ s.t $L(\mathcal{A}) = L(\mathcal{A}_1) \cap L(\mathcal{A}_2)$

- $Final\ states = \{(s1, s3, 1), (s1, s4, 1)\}$

Complementation of BA

- BA are closed under complementation

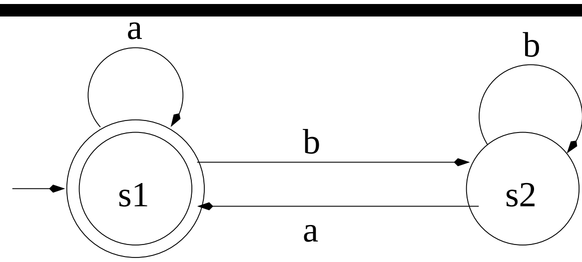
- Complementation of DBA can be done in $2 * n$ states.

- Complementation of NBA may require c^{n^2} states. (Proof involves expressing L and L^c as finite union of equivalence classes)

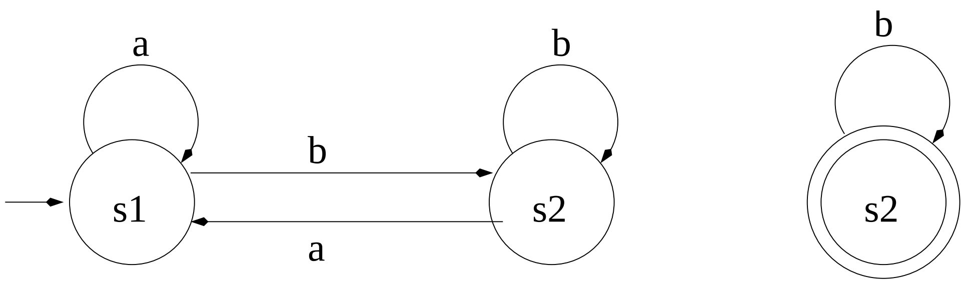
- DBA's not closed under complementation.

- $L(DBA) \subseteq L(NBA)$

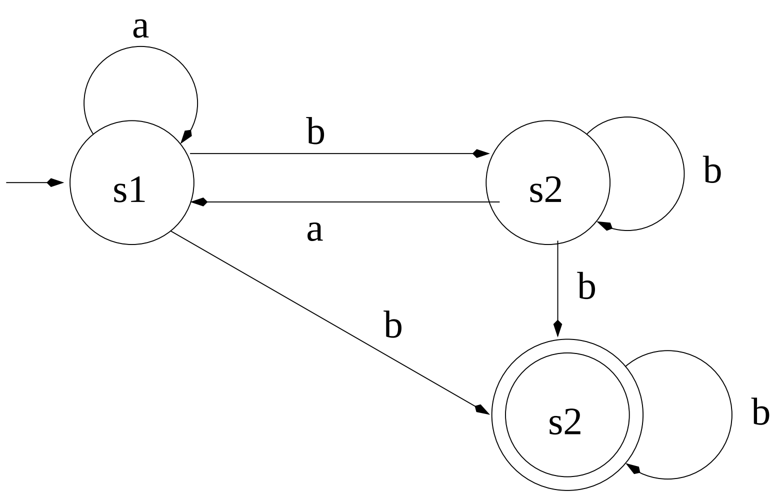
Complementation of DBA example



DBA $\mathcal{A}$ which is to be complemented



$\mathcal{A}_1$ & $\mathcal{A}_2$, two modified copies of $\mathcal{A}$



$\mathcal{A}^c$ obtained after combining $\mathcal{A}_1$ & $\mathcal{A}_2$ as above

Second Order Theory of One Successor (S1S)

Used to specify properties of w-sequences.

- A variable can be one of $x, y....$ Used for positions
- A set variable can be $X, Y, ...$
- We have a function symbol $+1$ (successor)
- $Terms$, $Atomic$ formulas, $S1A_A$ formulas

S1S formulas examples

- $\sum = \{a, b, c\}$

- Infinitely many $a's$

  $...a....a....a...........a........a......$

  $\varphi_1 : \forall x \exists y (y > x \wedge y \in Q_a)$

- Two $a's$ followed by b

  $...aab....aab....abc...........aab........bbb......$

  $\varphi_2 : \forall x \forall y (x \in Q_a \wedge y \in Q_a \wedge y = x + 1 \rightarrow$
  $\exists z (z = y + 1 \wedge z \in Q_b))$

Büchi's Theorem

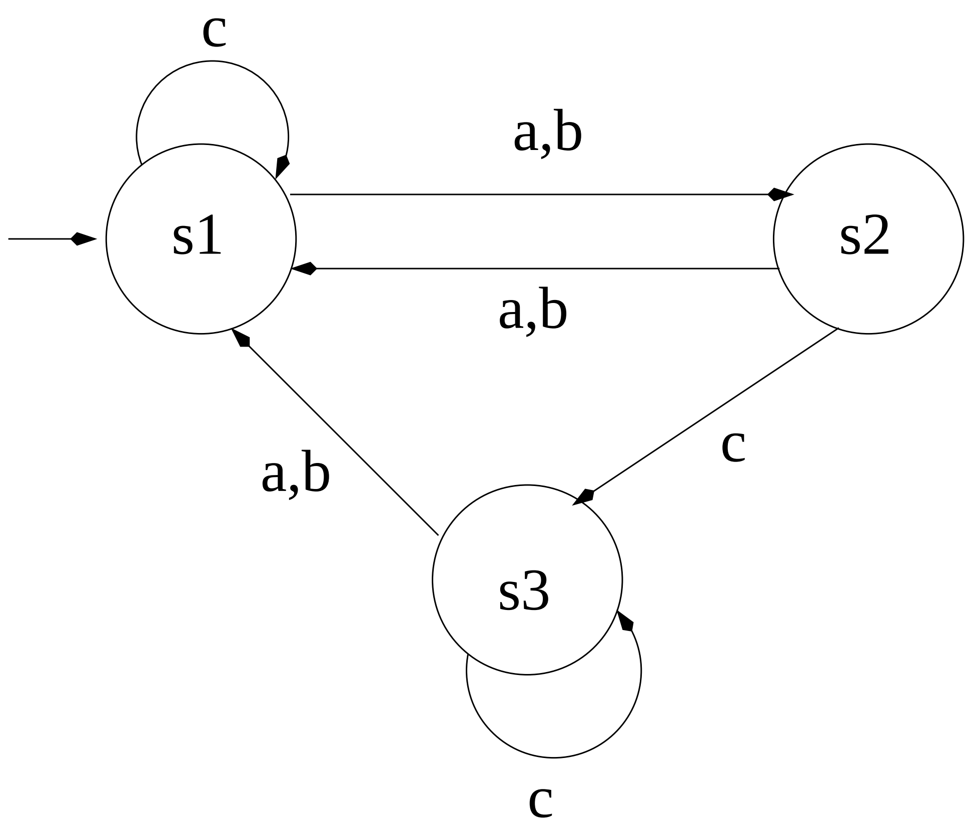
Theorem 1 *An w-language is definable in S1S iff it is regular*

- BA to S1S formula
- S1S formula to BA

Theorem 2 *Truth of sentences of SIS is decidable*

Decidability of S1S was one of the motivations for Büchi.

Muller automata example



$\mathcal{F} = \{\{s_1\}, \{s_1, s_2, s_3\}\}$

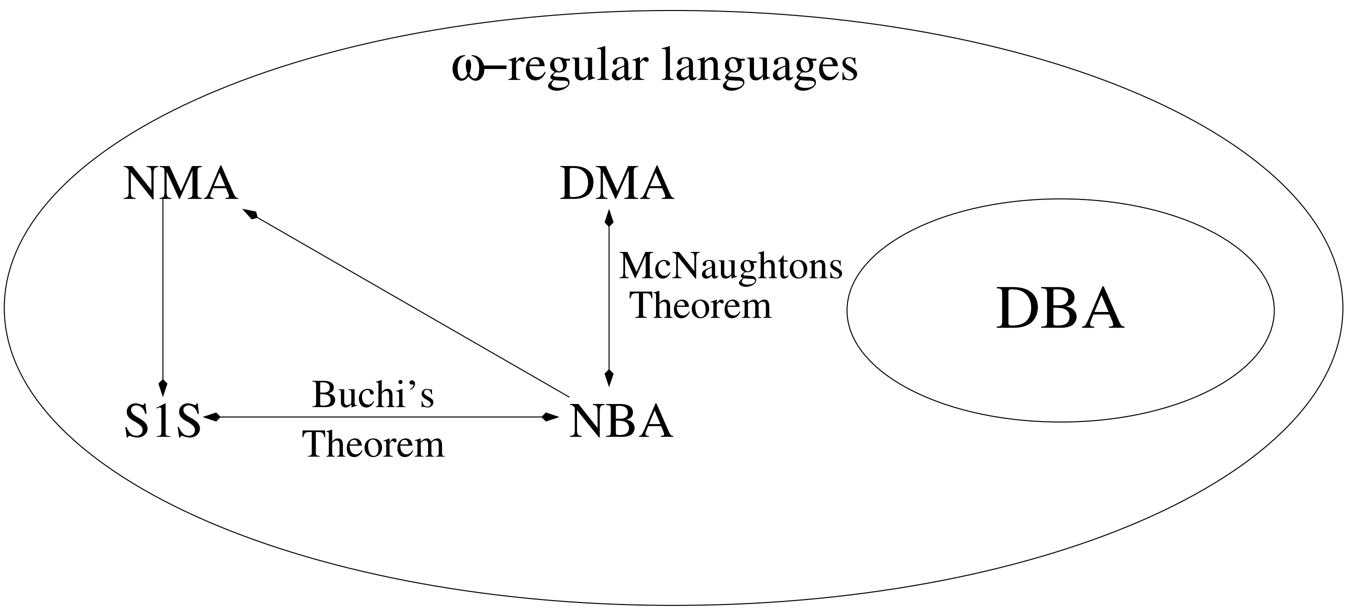
- For $\alpha = acbacb...$ the Infinity set is $\{s_1, s_2, s_3\}$. So α is Muller acceptable.

- For $\alpha = ababab...$ the Infinity set is $\{s_1, s_2\}$. So α is not Muller acceptable.

McNaughtons Theorem

Theorem 3 *An ω-language is regular (i.e, Büchi recognizable) iff it is Muller recognizable.*

Consequences



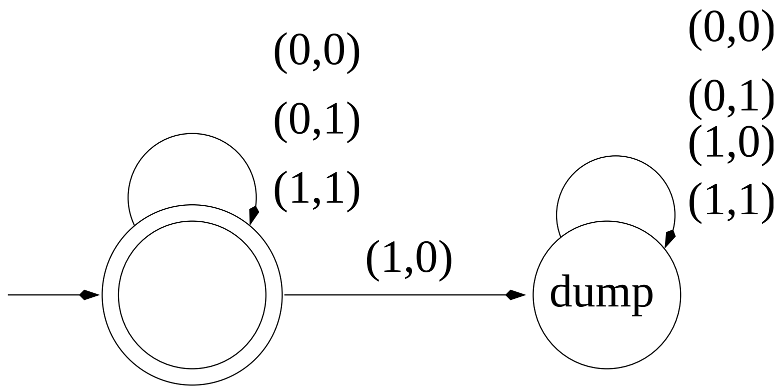
Conclusions

- Expresssive equivalence of BA, S1S, MA
- Acceptance criterion
- Union, Intersection
- Complementation of DBA Vs. NBA
- Complementation of MA

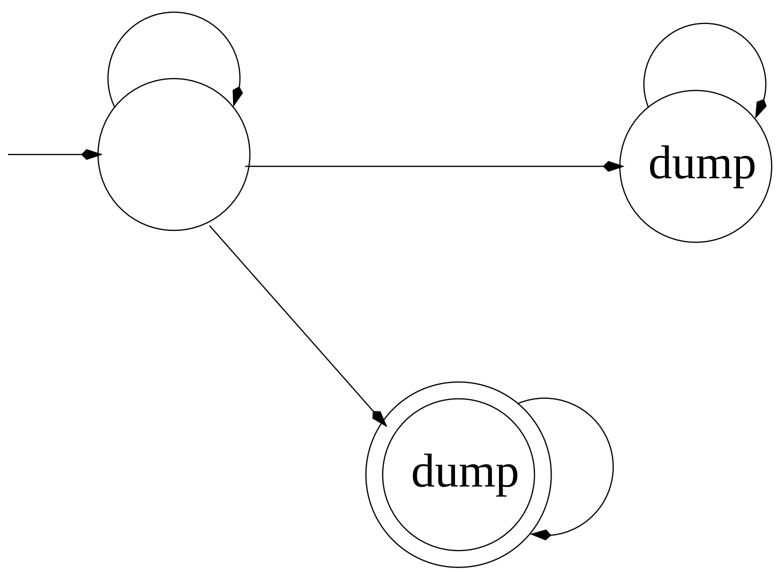
Applications

- Decidability of S1S formulas
- Behaviour of certain digital circuits
- Real Time Systems
- Programs, Specifications, Concurrent programs

Example 3



BA for $X_1 \subseteq X_2$



BA for $\varphi : \exists X_1 \exists X_2 \neg (X_1 \subseteq X_2)$

Determinism & McNaughton's Theorem

- We can show DBA's are not closed under complement.

- And hence DBA's are strictly weaker then NBA in general

- By refining the acceptance condition we can have deterministic automata which are as powerful as NBA's.

- One such deterministic version is

- **Muller Automaton**