Natural control of spring-like running: Optimised selfstabilisation

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ABSTRACT

In this study, the influence of leg rotation during the swing phase on the stability of running is addressed. Therefore, conservative spring-mass running was investigated using a return map of the apex height. A fixed angle of attack can already result in selfstabilised running as found previously. By examining the return maps of all possible angles of attack, a rotational leg control is derived adjusting a desired trajectory within one step and optimising running stability.

1 INTRODUCTION

If a steady state is reached, legged locomotion can be considered as a conservative system characterised by a constant total mechanical energy. During the flight phase, this energy is distributed to the forward and vertical direction (forward kinetic energy, corresponding to forward speed, and vertical energy, corresponding to apex height). During the contact phase, the leg mediates between these two amounts of energy. Hence, the leg angle of attack at landing can influence this energy distribution as shown previously for spring-mass running using a fixed angle of attack control (α_0 = const.) (1).

Depending on the actual situation it might be of advantage to directly control the apex height or, correspondingly, the forward speed at a given total system energy. For instance, speed could be maximised on an even ground, whereas vertical excursions are required on unpredictable (uneven) terrain. This urges for an optimised control strategy for the angle of attack α_0 to access the entire space of possible energy distributions.

Our approach to this issue is to investigate the influence of different angles of attack on the actual leg response (repulsion) during stance. In contrast to the previous presentation (1), here we ask for a rotational leg control during the swing phase, which stabilises the system at any *desired* apex height (i.e. energy distribution) for varying initial conditions. Therefore, we extend the stability analysis in terms of the return map of the apex height.

2 METHODS

The spring-mass model for fast locomotion (running) is used to derive the appropriate control strategy for a desired movement trajectory. Due to the conservative nature of the system, the return map $y_{i+1}(y_i)$ of the apex height y_{APEX} of two subsequent flight phases is applied to investigate stability.

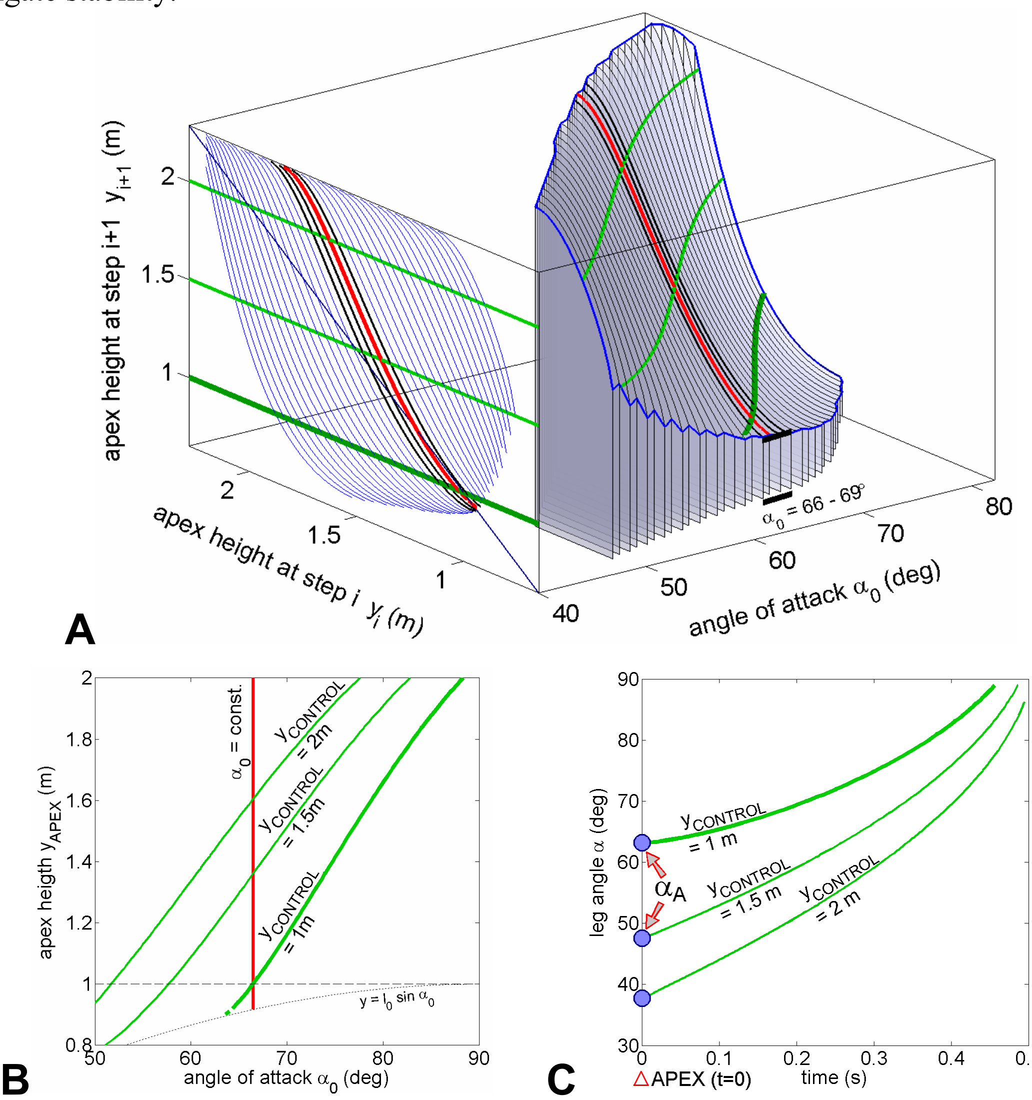

Figure 1. (A) Generalised return map $y_{i+1}(y_i, \alpha_0)$ for 5 m/s running (at 1 m apex height). Selfstabilisation can be optimised if a control fulfilling $y_{i+1}(y_i) = y_{CONTROL} =$ *const.* is applied (left plane). The required adjustment of the angle of attack α_0 with respect to the apex height y_{APEX} is shown in (B). The derived 'optimal' control strategy $\alpha(t)$ relative to the instant of apex ($t_{APEX} = 0$) using equation 1 is depicted in (C).

Starting at the apex (height y_i) and a *fixed* angle of attack α_0, the apex height y_{i+1} of the following flight phase is uniquely determined by the leg response (leg stiffness k) during stance. Periodic solutions with $y_{i+1} = y_i$ (fixed points) may be obtained for some α_0. Stable fixed points additionally require that the return map $y_{i+1}(y_i)$ intersects the diagonal ($y_{i+1} = y_i$) with a slope within (-1, 1).

To investigate the influence of *varying* leg angles of attack α_0 on the control of the apex height, we introduce a generalised return map $y_{i+1}(y_i, \alpha_0)$ collecting the return maps for all possible angles. An example is given in figure 1A for human running (mass m = 80 kg, leg length $\ell_0 = 1$ m) with a leg stiffness $k_{LEG} = 20$ kN/m in terms of a generalised surface $y_{i+1}(y_i, \alpha_0)$. For distinct angles α_0, the projection of this surface on the (y_{i+1}, y_i)-plane results in the return map $y_{i+1}(y_i)$ previously shown for $\alpha_0 = 66, 67, 68, 69°$ (figure 1A) in (1) at this conference.

3 RESULTS

To investigate the potential role of the angle of attack on the control of the apex height y_{APEX}, we ask for an 'optimal' return map. Such a return map would project all initial apex heights y_i to a desired apex height $y_{CONTROL}$ in the next flight phase, i.e. $y_{i+1}(y_i) = y_{CONTROL} =$ const. (horizontal lines with $y_{CONTROL} = 1, 1.5, 2$ m in figure 1A).

On the generalised surface $y_{i+1}(y_i, \alpha_0)$, the 'optimal' return map is represented by isolines with constant values of $y_{i+1} = y_{CONTROL}$. Projecting these isolines on the (y_i, α_o)-plane yields the required adjustment of the angle of attack α_0 depending on the preceding apex height y_i (figure 1B). For each desired apex height $y_{CONTROL}$, this results in an α(t)-dependency (figure 1C) with respect to the apex time ($t_{APEX} = 0$) using

$$t(\alpha_0) = \sqrt{\frac{2}{g}\left(y_{APEX}(\alpha_0) - \ell_0 \sin\alpha_0\right)}. \qquad \text{(equation 1)}$$

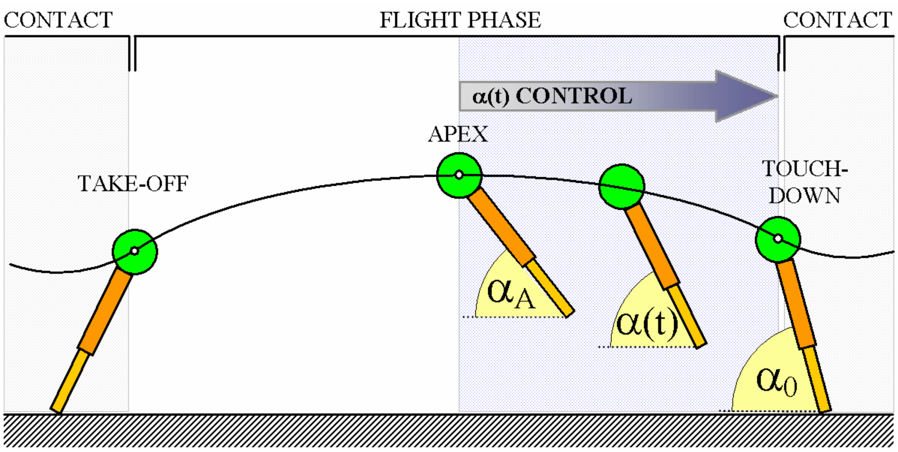

Figure 2. Selfstabilisation of spring-mass running can be improved by introducing a leg rotation during the swing phase (α(t)-control) starting with α_A at the apex.

To obtain a desired apex height $y_{CONTROL}$, there is a required leg angle α at any time $t \geq 0$ (starting at apex with $t_{APEX} = 0$, figure 1C) derived from the solution $y_{APEX}(\alpha_0)$ (figure 1B). We call this identified leg adjustment $\alpha(t)$-control (figure 2).

Comparing the $\alpha(t)$-controls for several desired apex heights $y_{CONTROL}$ (figure 1C) reveals that different $y_{CONTROL}$ require different leg angles α_A at apex. Then, applying the predicted $\alpha(t)$-adjustment forces the spring-mass system to set the corresponding apex height $y_{CONTROL}$ within one step.

4 DISCUSSION

The predicted $\alpha(t)$-control for running implies a continuous leg angle adjustment during the flight phase and allows reaching any desired apex height, i.e. energy distribution of the centre of mass trajectory, within one step regardless of the initial condition or the system history. Such flexibility can not be achieved by a control strategy keeping the leg angle during flight fixed (2).

For conservatively operating legs, the generalised surface $y_{i+1}(y_i, \alpha_0)$ can be used to predict an optimised $\alpha(t)$-strategy enforcing a desired apex height (or forward speed, respectively) within one step. Here, the stability of running with a fixed angle of attack is enhanced to a selection of a desired trajectory within one step. Such a control could be implemented by a look-up table mapping the generalised surfaces at different total system energies.

Although this method is demonstrated for spring-mass running, it can be applied to other leg behaviours with a predetermined generalised surface at a given total system energy. More general, the proposed strategy demonstrates a method to force selfstabilisation on a conservative level.

Simplifying control. Due to the almost parallel shift of the predicted $\alpha(t)$-control with different apex angles α_A corresponding to different desired apex heights $y_{CONTROL}$ (figure 1C), a simplified control can be introduced. This is illustrated in figure 3, where the optimised solution $\alpha(t)$ for $y_{CONTROL} = 1$ m is merely shifted in α by changing the apex angle α_A resulting in different controlled apex heights. The simulation of human running shows that even this simplified strategy in $\alpha(t)$ could control apex heights between 1 and 2 m. The smaller apex height (1 m) is realised within one step (at time $t \approx 14$ s), larger apex heights require just a few more steps to stabilise (at time $t \approx 2$, 6 and 18 s),

Generalisation. Assuming that a system is capable to maintain its system energy, the robustness of locomotion can be analysed on a conservative level. Using and optimising the inherent selfstabilising mechanisms, the control effort can be minimised. This holds in particular as the control strategy $\alpha(t)$ is very energy efficient (for small leg masses). Controlling a conservative system does not imply that there is no control of energy. Although the total energy is kept constant the leg adjustment determines how much energy is really used for forward locomotion (i.e. forward speed).

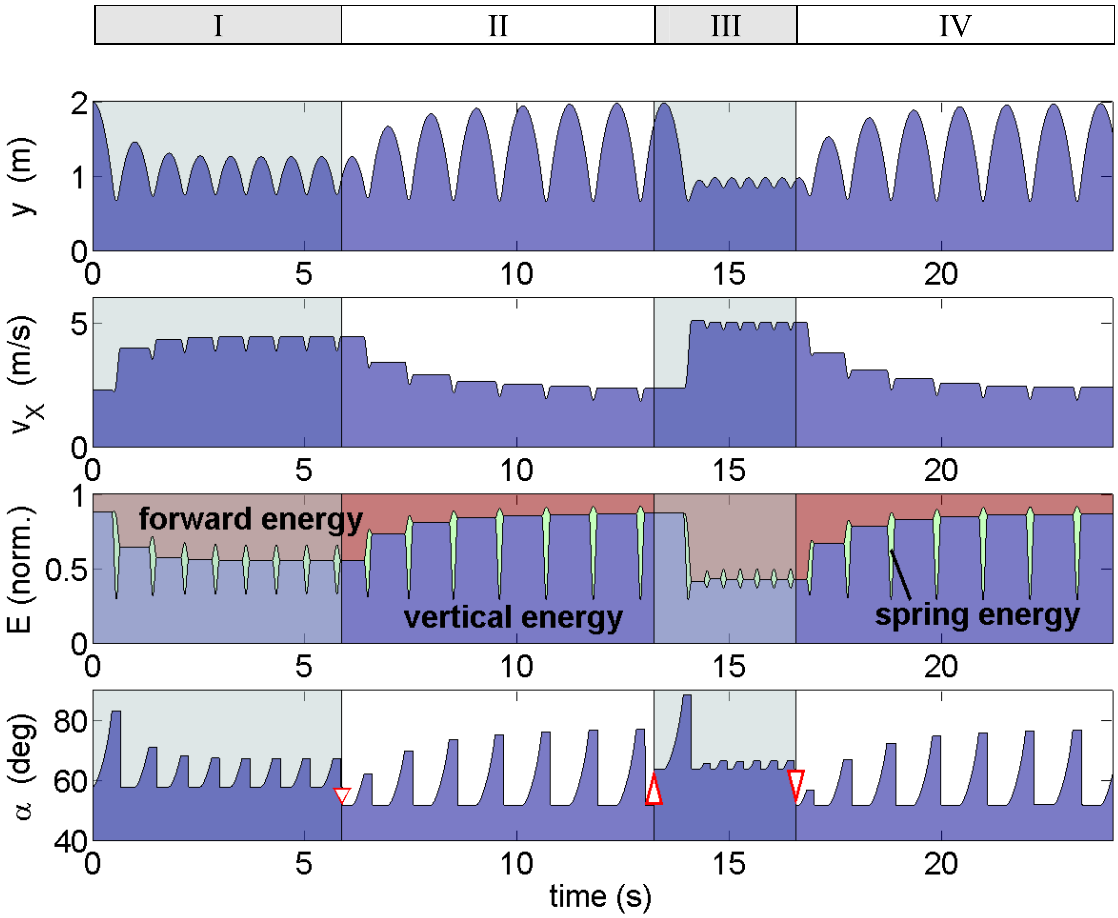

Figure 3. Control of the apex height by shifting the 'optimal' leg angle control α(t) for $y_{CONTROL}$ = 1 m in α (lowest graph). The initial leg angle at apex (I: α_A = 58°, II = IV: α_A = 52°, III: α_A = 64°) can control apex heights within 1 and 2 m (top graph). Presetting is indicated by triangles in the lowest graph. The four graphs show the time series of (i) the vertical position y of the centre of mass, (ii) the forward speed v_X, (iii) the relative energy contributions (forward kinetic energy, vertical energy, elastic energy of the spring) to the normalised total energy E, and (iv) the leg angle α.

ACKNOWLEDGEMENTS

This research was supported by an Emmy-Noether grant of the German Science Foundation (DFG) to André Seyfarth and a grant of the German Academic Exchange Service (DAAD) "Hochschulsonderprogramm III von Bund und Länder" to Hartmut Geyer.

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