10-301/601: Introduction to Machine Learning Lecture 3 – Decision Trees: Learning

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5/13/25

Front Matter

- Announcements:
  - HW1 released on 5/13 (today!), due 5/16 at 11:59 PM
    - You will submit your homework to Gradescope
      1. Submit your code to the “programming” submission slot
      2. Submit a PDF with your answers to the questions “written” submission slot
         - **You must use LaTeX to typeset your responses!**

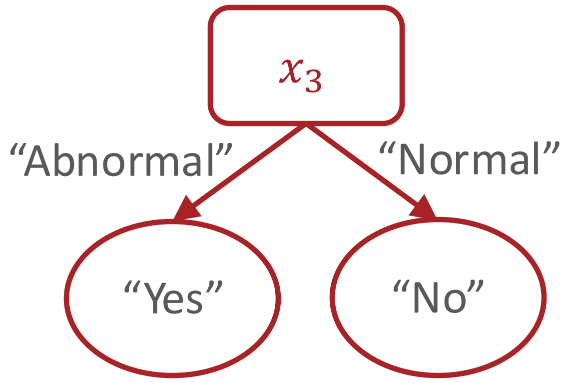
Decision Stumps: Questions

1. How can we pick which feature to split on?

2. Why stop at just one feature?
   a. How can we pick the order of the splits?

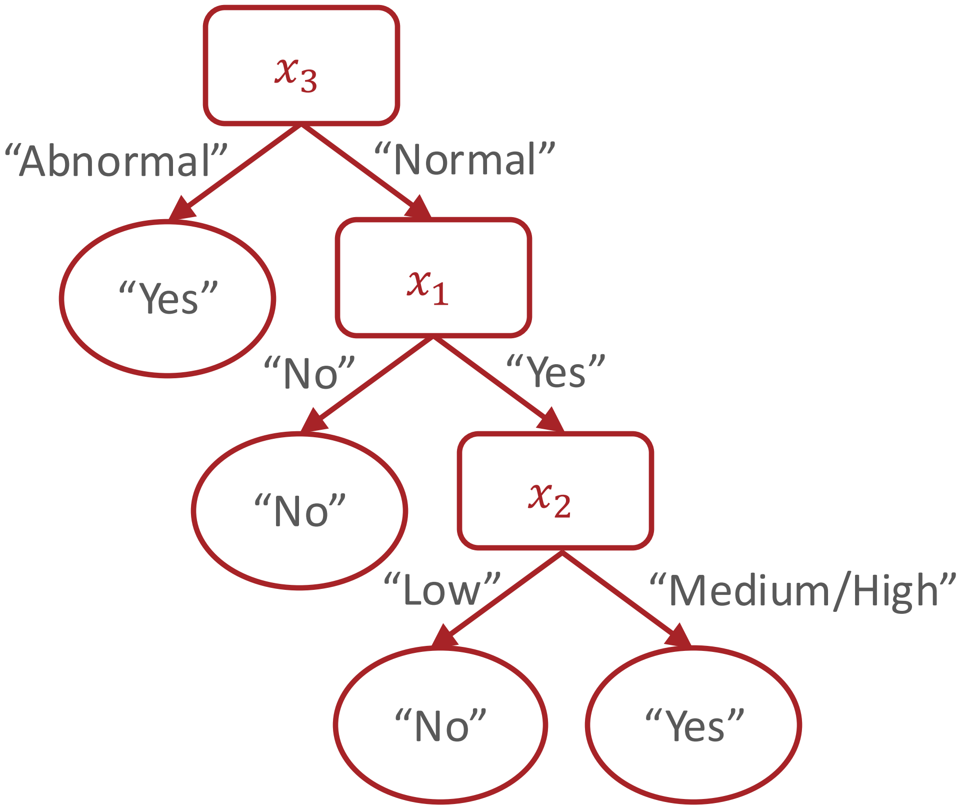
From Decision Stump …

x_1 Family History	x_2 Resting Blood Pressure	x_3 Cholesterol	y Heart Disease?
Yes	Low	Normal	No
No	Medium	Normal	No
No	Low	Abnormal	Yes
Yes	Medium	Normal	Yes
Yes	High	Abnormal	Yes



From Decision Stump to Decision Tree

x_1 Family History	x_2 Resting Blood Pressure	x_3 Cholesterol	y Heart Disease?
Yes	Low	Normal	No
No	Medium	Normal	No
No	Low	Abnormal	Yes
Yes	Medium	Normal	Yes
Yes	High	Abnormal	Yes



From Decision Stump to Decision Tree

x_1 Family History	x_2 Resting Blood Pressure	x_3 Cholesterol	y Heart Disease?
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No	Low	Abnormal	Yes
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Yes	High	Abnormal	Yes
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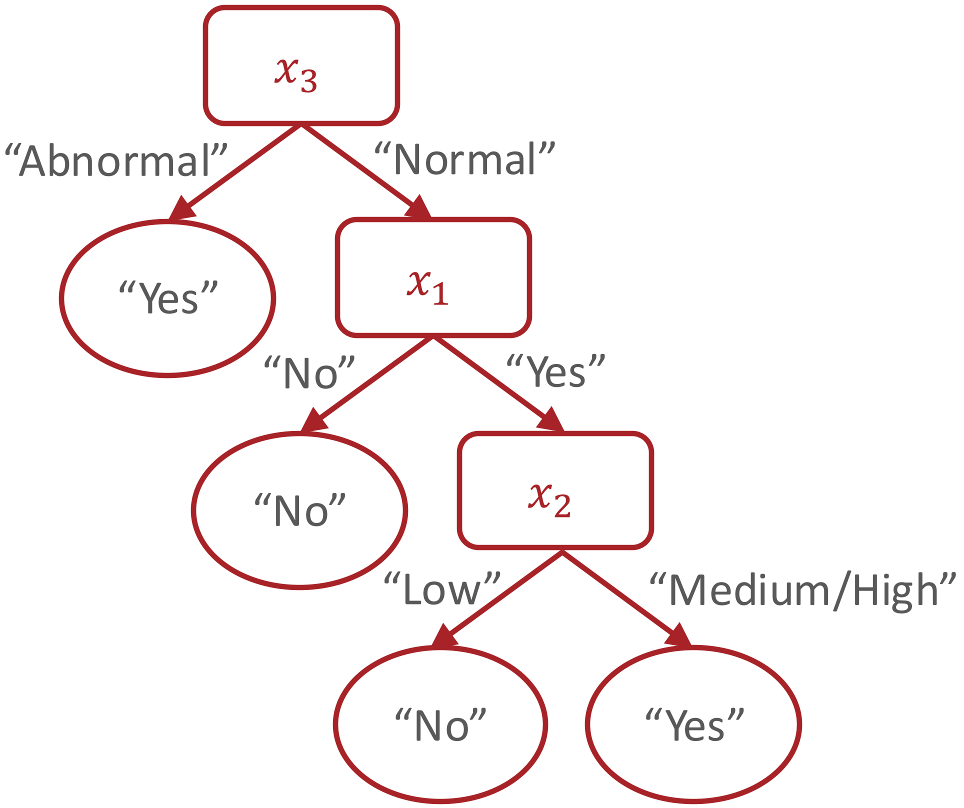

From Decision Stump to Decision Tree

x_1 Family History	x_2 Resting Blood Pressure	x_3 Cholesterol	y Heart Disease?
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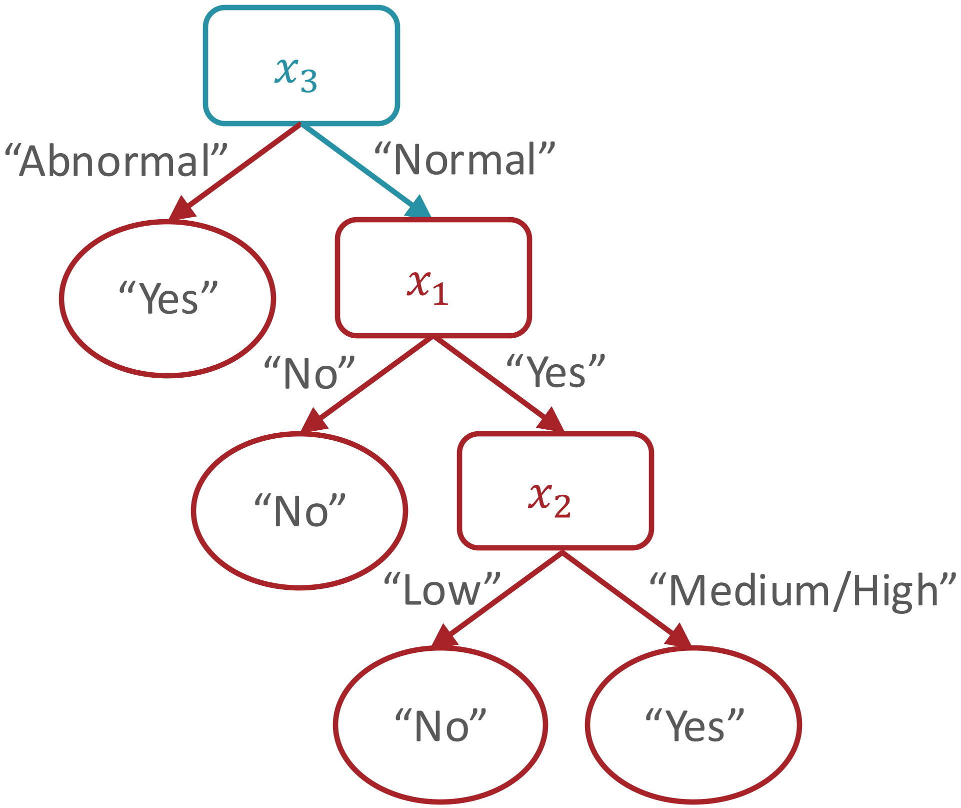
Yes	High	Abnormal	Yes
No	High	Normal	No



From Decision Stump to Decision Tree

x_1 Family History	x_2 Resting Blood Pressure	x_3 Cholesterol	y Heart Disease?
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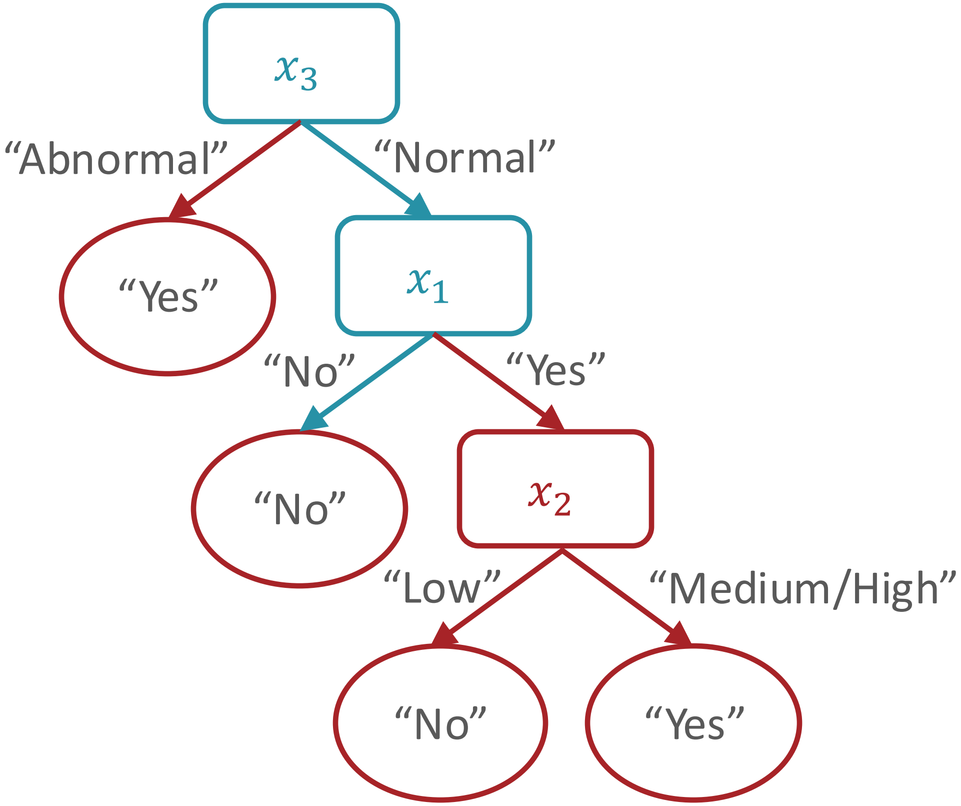

From Decision Stump to Decision Tree

x_1 Family History	x_2 Resting Blood Pressure	x_3 Cholesterol	y Heart Disease?
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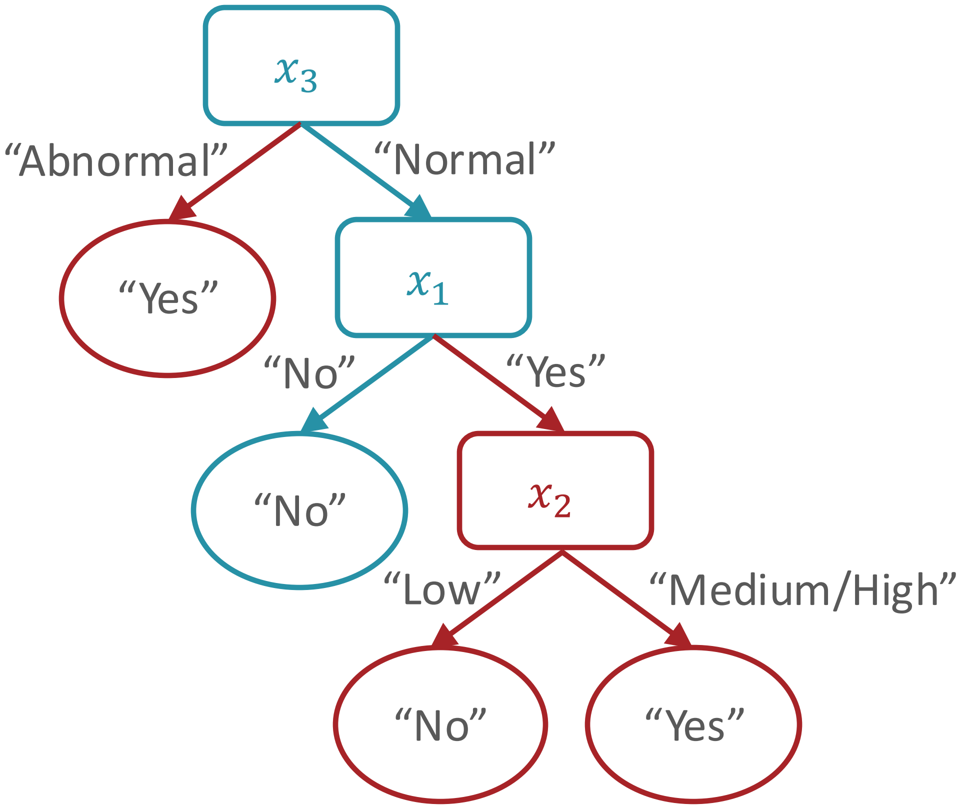
No	High	Normal	No



Decision Tree: Example

Learned from medical records of 1000 women

Negative examples are C-sections

```
[833+,167-] .83+ .17-
Fetal_Presentation = 1: [822+,116-] .88+ .12-
| Previous_Csection = 0: [767+,81-] .90+ .10-
| | Primiparous = 0: [399+,13-] .97+ .03-
| | Primiparous = 1: [368+,68-] .84+ .16-
| | | Fetal_Distress = 0: [334+,47-] .88+ .12-
| | | Fetal_Distress = 1: [34+,21-] .62+ .38-
| Previous_Csection = 1: [55+,35-] .61+ .39-
Fetal_Presentation = 2: [3+,29-] .11+ .89-
Fetal_Presentation = 3: [8+,22-] .27+ .73-
```

Figure courtesy of Tom Mitchell

Decision Tree: Pseudocode

```
def predict(x'):
  - walk from root node to a leaf node
    while(true):
       if current node is internal (non-leaf):
             check the associated attribute, x_d
             go down branch according to x'_d
       if current node is a leaf node:
             return label stored at that leaf
```

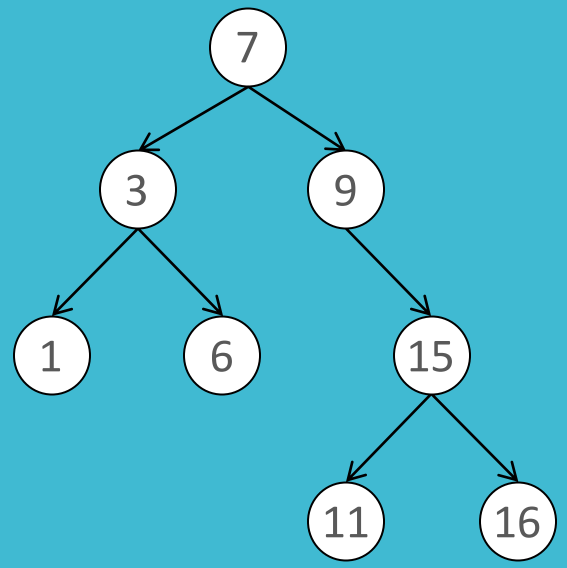
So how do we train one of these things?

def predict($\boldsymbol{x}'$):

- walk from root node to a leaf node

```
while(true):
    if current node is internal (non-leaf):
        check the associated attribute, x_d
        go down branch according to x'_d
    if current node is a leaf node:
        return label stored at that leaf
```

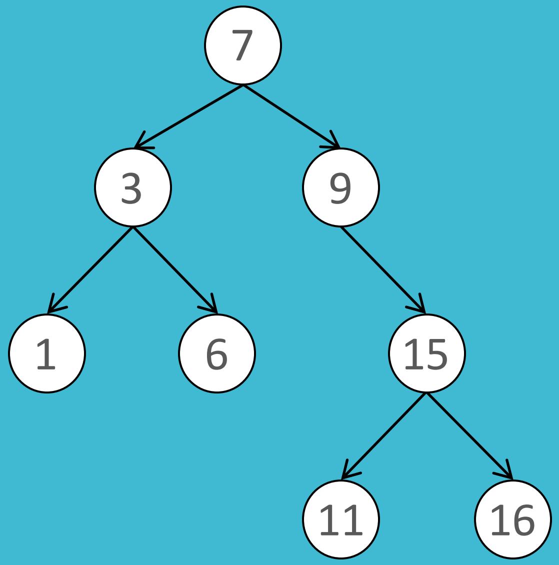
Background: Recursion



- A **binary search tree** (BST) consists of nodes, where each node:
  - has a value, v
  - up to 2 children, a left descendant and a right descendant
  - all its left descendants have values less than v and its right descendants have values greater than v
- We like BSTs because they permit search in O(log(n)) time, assuming n nodes in the tree

```
def contains_iterative(node, key):
    cur = node
    while true:
        if key < cur.value & cur.left != null:
            cur = cur.left
        else if cur.value < key & cur.right != null:
            cur = cur.right
        else:
            break
    return key == cur.value
```

Background: Recursion



- A **binary search tree** (BST) consists of nodes, where each node:
  - has a value, v
  - up to 2 children, a left descendant and a right descendant
  - all its left descendants have values less than v and its right descendants have values greater than v
- We like BSTs because they permit search in O(log(n)) time, assuming n nodes in the tree

```
def contains_recursive(node, key):
    if key < node.value & node.left != null:
        return contains(node.left, key)
    else if node.value < key & node.right != null:
        return contains(node.right, key)
    else:
        return key == node.value
```

Decision Tree: Pseudocode

```
def train(𝒟):
    store root = tree_recurse(𝒟)
def tree_recurse(𝒟′):
    q = new node()
    base case – if (SOME CONDITION):
    recursion – else:
        find best attribute to split on, x_d
        q.split = x_d
        for v in V(x_d), all possible values of x_d:
            𝒟_v = {(x^(n), y^(n)) ∈ 𝒟 | x_d^(n) = v}
            q.children(v) = tree_recurse(𝒟_v)
    return q
```

$$\mathcal{D}_v = \left\{\left(x^{(n)}, y^{(n)}\right) \in \mathcal{D} \mid x_d^{(n)} = v\right\}$$

Decision Tree: Pseudocode

```
def train(𝒟):
    store root = tree_recurse(𝒟)
def tree_recurse(𝒟′):
    q = new node()
    base case – if (𝒟′ is empty OR
        all labels in 𝒟′ are the same OR
        all features in 𝒟′ are identical OR
        some other stopping criterion):
        q.label = majority_vote(𝒟′)


    recursion – else:
    return q
```

Decision Tree: Example – How is Henry getting to work?

- Label: mode of transportation
  - $y \in \mathcal{Y} = \{\text{Bike}, \text{Drive}, \text{Bus}\}$
- Features: 4 categorial features
  - Is it raining? $x_1 \in \{\text{Rain}, \text{No Rain}\}$
  - When am I leaving (relative to rush hour)? $x_2 \in \{\text{Before}, \text{During}, \text{After}\}$
  - What am I bringing? $x_3 \in \{\text{Backpack}, \text{Lunchbox}, \text{Both}\}$
  - Am I tired? $x_4 \in \{\text{Tired}, \text{Not Tired}\}$

Data

x_1	x_2	x_3	x_4	y
Rain	Before	Both	Tired	Drive
Rain	During	Both	Not Tired	Bus
Rain	During	Both	Tired	Drive
Rain	After	Backpack	Not Tired	Bus
Rain	After	Backpack	Tired	Bus
Rain	After	Lunchbox	Tired	Drive
No Rain	Before	Backpack	Tired	Bike
No Rain	Before	Lunchbox	Not Tired	Bus
No Rain	Before	Lunchbox	Tired	Drive
No Rain	During	Backpack	Not Tired	Bus
No Rain	During	Both	Tired	Drive
No Rain	After	Backpack	Not Tired	Bike
No Rain	After	Backpack	Tired	Bike
No Rain	After	Both	Not Tired	Bus
No Rain	After	Both	Tired	Drive
No Rain	After	Lunchbox	Not Tired	Bus

Which feature would we split on first using mutual information as the splitting criterion?

x_1	x_2	x_3	x_4	y
Rain	Before	Both	Tired	Drive
Rain	During	Both	Not Tired	Bus
Rain	During	Both	Tired	Drive
Rain	After	Backpack	Not Tired	Bus
Rain	After	Backpack	Tired	Bus
Rain	After	Lunchbox	Tired	Drive
No Rain	Before	Backpack	Tired	Bike
No Rain	Before	Lunchbox	Not Tired	Bus
No Rain	Before	Lunchbox	Tired	Drive
No Rain	During	Backpack	Not Tired	Bus
No Rain	During	Both	Tired	Drive
No Rain	After	Backpack	Not Tired	Bike
No Rain	After	Backpack	Tired	Bike
No Rain	After	Both	Not Tired	Bus
No Rain	After	Both	Tired	Drive
No Rain	After	Lunchbox	Not Tired	Bus

$$H(S) = -\sum_{v \in V(S)} \frac{|S_v|}{|S|} \log_2\left(\frac{|S_v|}{|S|}\right)$$

x_1	x_2	x_3	x_4	y
Rain	Before	Both	Tired	Drive
Rain	During	Both	Not Tired	Bus
Rain	During	Both	Tired	Drive
Rain	After	Backpack	Not Tired	Bus
Rain	After	Backpack	Tired	Bus
Rain	After	Lunchbox	Tired	Drive
No Rain	Before	Backpack	Tired	Bike
No Rain	Before	Lunchbox	Not Tired	Bus
No Rain	Before	Lunchbox	Tired	Drive
No Rain	During	Backpack	Not Tired	Bus
No Rain	During	Both	Tired	Drive
No Rain	After	Backpack	Not Tired	Bike
No Rain	After	Backpack	Tired	Bike
No Rain	After	Both	Not Tired	Bus
No Rain	After	Both	Tired	Drive
No Rain	After	Lunchbox	Not Tired	Bus

$$H(S) = -\sum_{v \in V(S)} \frac{|S_v|}{|S|} \log_2\left(\frac{|S_v|}{|S|}\right)$$

$$H(Y) = -\frac{3}{16}\log_2\left(\frac{3}{16}\right)$$

$$-\frac{6}{16}\log_2\left(\frac{6}{16}\right)$$

$$= -\frac{7}{16}\log_2\left(\frac{7}{16}\right)$$

$$H(Y) \approx 1.5052$$

x_1	x_2	x_3	x_4	y
Rain	Before	Both	Tired	Drive
Rain	During	Both	Not Tired	Bus
Rain	During	Both	Tired	Drive
Rain	After	Backpack	Not Tired	Bus
Rain	After	Backpack	Tired	Bus
Rain	After	Lunchbox	Tired	Drive
No Rain	Before	Backpack	Tired	Bike
No Rain	Before	Lunchbox	Not Tired	Bus
No Rain	Before	Lunchbox	Tired	Drive
No Rain	During	Backpack	Not Tired	Bus
No Rain	During	Both	Tired	Drive
No Rain	After	Backpack	Not Tired	Bike
No Rain	After	Backpack	Tired	Bike
No Rain	After	Both	Not Tired	Bus
No Rain	After	Both	Tired	Drive
No Rain	After	Lunchbox	Not Tired	Bus

$$I(x_d;Y)$$

$$= H(Y) - \sum_{v \in V(x_d)} f_v * H\left(Y_{x_d=v}\right)$$

$$I(x_1, Y$$

$$\text{IG}(x_1, y) = -\frac{7}{16}\log_2\left(\frac{7}{16}\right)$$

x_1	x_2	x_3	x_4	y
Rain	Before	Both	Tired	Drive
Rain	During	Both	Not Tired	Bus
Rain	During	Both	Tired	Drive
Rain	After	Backpack	Not Tired	Bus
Rain	After	Backpack	Tired	Bus
Rain	After	Lunchbox	Tired	Drive
No Rain	Before	Backpack	Tired	Bike
No Rain	Before	Lunchbox	Not Tired	Bus
No Rain	Before	Lunchbox	Tired	Drive
No Rain	During	Backpack	Not Tired	Bus
No Rain	During	Both	Tired	Drive
No Rain	After	Backpack	Not Tired	Bike
No Rain	After	Backpack	Tired	Bike
No Rain	After	Both	Not Tired	Bus
No Rain	After	Both	Tired	Drive
No Rain	After	Lunchbox	Not Tired	Bus

$$I(x_d; Y) = H(Y) - \sum_{v \in V(x_d)} f_v * H(Y_{x_d=v})$$

$$I(x_1, Y) \approx 1.5052$$

$$-\frac{6}{16}\left(-\frac{1}{2}\log_2\left(\frac{1}{2}\right)\right)$$

$IG(x_1, y$ … 16 … $\left(16\right)$

x_1	x_2	x_3	x_4	y
Rain	Before	Both	Tired	Drive
Rain	During	Both	Not Tired	Bus
Rain	During	Both	Tired	Drive
Rain	After	Backpack	Not Tired	Bus
Rain	After	Backpack	Tired	Bus
Rain	After	Lunchbox	Tired	Drive
No Rain	Before	Backpack	Tired	Bike
No Rain	Before	Lunchbox	Not Tired	Bus
No Rain	Before	Lunchbox	Tired	Drive
No Rain	During	Backpack	Not Tired	Bus
No Rain	During	Both	Tired	Drive
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No Rain	After	Both	Not Tired	Bus
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$$I(x_d; Y)$$

$$= H(Y) - \sum_{v \in V(x_d)} f_v * H\left(Y_{x_d=v}\right)$$

$$I(x_1, Y) \approx 1.5052$$

$$-\frac{6}{16}(1)$$

x_1	x_2	x_3	x_4	y
Rain	Before	Both	Tired	Drive
Rain	During	Both	Not Tired	Bus
Rain	During	Both	Tired	Drive
Rain	After	Backpack	Not Tired	Bus
Rain	After	Backpack	Tired	Bus
Rain	After	Lunchbox	Tired	Drive
No Rain	Before	Backpack	Tired	Bike
No Rain	Before	Lunchbox	Not Tired	Bus
No Rain	Before	Lunchbox	Tired	Drive
No Rain	During	Backpack	Not Tired	Bus
No Rain	During	Both	Tired	Drive
No Rain	After	Backpack	Not Tired	Bike
No Rain	After	Backpack	Tired	Bike
No Rain	After	Both	Not Tired	Bus
No Rain	After	Both	Tired	Drive
No Rain	After	Lunchbox	Not Tired	Bus

$$I(x_d; Y) = H(Y) - \sum_{v \in V(x_d)} f_v * H\left(Y_{x_d=v}\right)$$

$$I(x_1, Y) \approx 1.5052$$

$$-\frac{6}{16}(1)$$

$$-\frac{10}{16}\left(-\frac{3}{10}\log_2\left(\frac{3}{10}\right)\right.$$

$$-\frac{3}{\;}\log_2\left(\frac{3}{\;}\right)$$

x_1	x_2	x_3	x_4	y
Rain	Before	Both	Tired	Drive
Rain	During	Both	Not Tired	Bus
Rain	During	Both	Tired	Drive
Rain	After	Backpack	Not Tired	Bus
Rain	After	Backpack	Tired	Bus
Rain	After	Lunchbox	Tired	Drive
No Rain	Before	Backpack	Tired	Bike
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No Rain	During	Both	Tired	Drive
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No Rain	After	Backpack	Tired	Bike
No Rain	After	Both	Not Tired	Bus
No Rain	After	Both	Tired	Drive
No Rain	After	Lunchbox	Not Tired	Bus

$$I(x_d; Y)$$

$$= H(Y) - \sum_{v \in V(x_d)} f_v * H\left(Y_{x_d = v}\right)$$

$$I(x_1, Y) \approx 1.5052$$

$$-\frac{6}{16}(1)$$

$$-\frac{10}{16}(1.5710)$$

$$\approx 0.1482$$

x_1	x_2	x_3	x_4	y
Rain	Before	Both	Tired	Drive
Rain	During	Both	Not Tired	Bus
Rain	During	Both	Tired	Drive
Rain	After	Backpack	Not Tired	Bus
Rain	After	Backpack	Tired	Bus
Rain	After	Lunchbox	Tired	Drive
No Rain	Before	Backpack	Tired	Bike
No Rain	Before	Lunchbox	Not Tired	Bus
No Rain	Before	Lunchbox	Tired	Drive
No Rain	During	Backpack	Not Tired	Bus
No Rain	During	Both	Tired	Drive
No Rain	After	Backpack	Not Tired	Bike
No Rain	After	Backpack	Tired	Bike
No Rain	After	Both	Not Tired	Bus
No Rain	After	Both	Tired	Drive
No Rain	After	Lunchbox	Not Tired	Bus

$$I(x_d;Y) = H(Y) - \sum_{v \in V(x_d)} f_v * H\left(Y_{x_d=v}\right)$$

$I(x_d,Y)$	
x_1	0.1482
x_2	0.1302
x_3	0.5358
x_4	0.5576

x_1	x_2	x_3	x_4	y
Rain	Before	Both	Tired	Drive
Rain	During	Both	Not Tired	Bus
Rain	During	Both	Tired	Drive
Rain	After	Backpack	Not Tired	Bus
Rain	After	Backpack	Tired	Bus
Rain	After	Lunchbox	Tired	Drive
No Rain	Before	Backpack	Tired	Bike
No Rain	Before	Lunchbox	Not Tired	Bus
No Rain	Before	Lunchbox	Tired	Drive
No Rain	During	Backpack	Not Tired	Bus
No Rain	During	Both	Tired	Drive
No Rain	After	Backpack	Not Tired	Bike
No Rain	After	Backpack	Tired	Bike
No Rain	After	Both	Not Tired	Bus
No Rain	After	Both	Tired	Drive
No Rain	After	Lunchbox	Not Tired	Bus

$$I(x_d;Y) = H(Y) - \sum_{v \in V(x_d)} f_v * H\left(Y_{x_d=v}\right)$$

$I(x_d, Y)$	
x_1	0.1482
x_2	0.1302
x_3	0.5358
x_4	0.5576

x_1	x_2	x_3	x_4	y
Rain	During	Both	Not Tired	Bus
Rain	After	Backpack	Not Tired	Bus
No Rain	Before	Lunchbox	Not Tired	Bus
No Rain	During	Backpack	Not Tired	Bus
No Rain	After	Backpack	Not Tired	Bike
No Rain	After	Both	Not Tired	Bus
No Rain	After	Lunchbox	Not Tired	Bus
Rain	Before	Both	Tired	Drive
Rain	During	Both	Tired	Drive
Rain	After	Backpack	Tired	Bus
Rain	After	Lunchbox	Tired	Drive
No Rain	Before	Backpack	Tired	Bike
No Rain	Before	Lunchbox	Tired	Drive
No Rain	During	Both	Tired	Drive
No Rain	After	Backpack	Tired	Bike
No Rain	After	Both	Tired	Drive

$$I(x_d;Y) = H(Y) - \sum_{v \in V(x_d)} f_v * H\left(Y_{x_d=v}\right)$$

$I(x_d, Y)$	
x_1	0.1482
x_2	0.1302
x_3	0.5358
x_4	0.5576

x_1	x_2	x_3	x_4	y
Rain	During	Both	Not Tired	Bus
Rain	After	Backpack	Not Tired	Bus
No Rain	Before	Lunchbox	Not Tired	Bus
No Rain	During	Backpack	Not Tired	Bus
No Rain	After	Backpack	Not Tired	Bike
No Rain	After	Both	Not Tired	Bus
No Rain	After	Lunchbox	Not Tired	Bus
Rain	Before	Both	Tired	Drive
Rain	During	Both	Tired	Drive
Rain	After	Backpack	Tired	Bus
Rain	After	Lunchbox	Tired	Drive
No Rain	Before	Backpack	Tired	Bike
No Rain	Before	Lunchbox	Tired	Drive
No Rain	During	Both	Tired	Drive
No Rain	After	Backpack	Tired	Bike
No Rain	After	Both	Tired	Drive

$$I(x_d;Y) = H(Y) - \sum_{v \in V(x_d)} f_v * H\left(Y_{x_d=v}\right)$$

$I(x_d, Y)$	
x_1	0.1482
x_2	0.1302
x_3	0.5358
x_4	0.5576

x_1	x_2	x_3	x_4	y
Rain	During	Both	Not Tired	Bus
Rain	After	Backpack	Not Tired	Bus
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No Rain	During	Backpack	Not Tired	Bus
No Rain	After	Backpack	Not Tired	Bike
No Rain	After	Both	Not Tired	Bus
No Rain	After	Lunchbox	Not Tired	Bus
Rain	Before	Both	Tired	Drive
Rain	During	Both	Tired	Drive
Rain	After	Backpack	Tired	Bus
Rain	After	Lunchbox	Tired	Drive
No Rain	Before	Backpack	Tired	Bike
No Rain	Before	Lunchbox	Tired	Drive
No Rain	During	Both	Tired	Drive
No Rain	After	Backpack	Tired	Bike
No Rain	After	Both	Tired	Drive

Decision Tree: Example

x_4

Not Tired

x_1	x_2	x_3	x_4	y
Rain	During	Both	Not Tired	Bus
Rain	After	Backpack	Not Tired	Bus
No Rain	Before	Lunchbox	Not Tired	Bus
No Rain	During	Backpack	Not Tired	Bus
No Rain	After	Backpack	Not Tired	Bike
No Rain	After	Both	Not Tired	Bus
No Rain	After	Lunchbox	Not Tired	Bus

Tired

x_1	x_2	x_3	x_4	y
Rain	Before	Both	Tired	Drive
Rain	During	Both	Tired	Drive
Rain	After	Backpack	Tired	Bus
Rain	After	Lunchbox	Tired	Drive
No Rain	Before	Backpack	Tired	Bike
No Rain	Before	Lunchbox	Tired	Drive
No Rain	During	Both	Tired	Drive
No Rain	After	Backpack	Tired	Bike
No Rain	After	Both	Tired	Drive

x_4

Not Tired

x_1	x_2	x_3	x_4	y
Rain	During	Both	Not Tired	Bus
Rain	After	Backpack	Not Tired	Bus
No Rain	Before	Lunchbox	Not Tired	Bus
No Rain	During	Backpack	Not Tired	Bus
No Rain	After	Backpack	Not Tired	Bike
No Rain	After	Both	Not Tired	Bus
No Rain	After	Lunchbox	Not Tired	Bus

Tired

x_1	x_2	x_3	x_4	y
Rain	Before	Both	Tired	Drive
Rain	During	Both	Tired	Drive
Rain	After	Backpack	Tired	Bus
Rain	After	Lunchbox	Tired	Drive
No Rain	Before	Backpack	Tired	Bike
No Rain	Before	Lunchbox	Tired	Drive
No Rain	During	Both	Tired	Drive
No Rain	After	Backpack	Tired	Bike
No Rain	After	Both	Tired	Drive

x_4

Not Tired

x_1	x_2	x_3	x_4	y
Rain	During	Both	Not Tired	Bus
Rain	After	Backpack	Not Tired	Bus
No Rain	Before	Lunchbox	Not Tired	Bus
No Rain	During	Backpack	Not Tired	Bus
No Rain	After	Backpack	Not Tired	Bike
No Rain	After	Both	Not Tired	Bus
No Rain	After	Lunchbox	Not Tired	Bus

Tired

x_1	x_2	x_3	x_4	y
Rain	Before	Both	Tired	Drive
Rain	During	Both	Tired	Drive
Rain	After	Backpack	Tired	Bus
Rain	After	Lunchbox	Tired	Drive
No Rain	Before	Backpack	Tired	Bike
No Rain	Before	Lunchbox	Tired	Drive
No Rain	During	Both	Tired	Drive
No Rain	After	Backpack	Tired	Bike
No Rain	After	Both	Tired	Drive

$$H\left(Y_{x_4=\text{Tired}}\right) = -\frac{6}{9}\log_2\frac{6}{9} - \frac{2}{9}\log_2\frac{2}{9} - \frac{1}{9}\log_2\frac{1}{9} \approx 1.2244$$

x_4

Not Tired

x_1	x_2	x_3	x_4	y
Rain	During	Both	Not Tired	Bus
Rain	After	Backpack	Not Tired	Bus
No Rain	Before	Lunchbox	Not Tired	Bus
No Rain	During	Backpack	Not Tired	Bus
No Rain	After	Backpack	Not Tired	Bike
No Rain	After	Both	Not Tired	Bus
No Rain	After	Lunchbox	Not Tired	Bus

Tired

x_1	x_2	x_3	x_4	y
Rain	Before	Both	Tired	Drive
Rain	During	Both	Tired	Drive
Rain	After	Backpack	Tired	Bus
Rain	After	Lunchbox	Tired	Drive
No Rain	Before	Backpack	Tired	Bike
No Rain	Before	Lunchbox	Tired	Drive
No Rain	During	Both	Tired	Drive
No Rain	After	Backpack	Tired	Bike
No Rain	After	Both	Tired	Drive

$$I\left(x_1, Y_{x_4=\text{Tired}}\right)$$
$$= H\left(Y_{x_4=\text{Tired}}\right) - \left(\frac{4}{9} H\left(Y_{x_4=\text{Tired}, x_1=\text{Rain}}\right) + \frac{5}{9} H\left(Y_{x_4=\text{Tired}, x_1=\text{No Rain}}\right)\right)$$

x_4

Not Tired | Tired

x_1	x_2	x_3	x_4	y
Rain	During	Both	Not Tired	Bus
Rain	After	Backpack	Not Tired	Bus
No Rain	Before	Lunchbox	Not Tired	Bus
No Rain	During	Backpack	Not Tired	Bus
No Rain	After	Backpack	Not Tired	Bike
No Rain	After	Both	Not Tired	Bus
No Rain	After	Lunchbox	Not Tired	Bus

x_1	x_2	x_3	x_4	y
Rain	Before	Both	Tired	Drive
Rain	During	Both	Tired	Drive
Rain	After	Backpack	Tired	Bus
Rain	After	Lunchbox	Tired	Drive
No Rain	Before	Backpack	Tired	Bike
No Rain	Before	Lunchbox	Tired	Drive
No Rain	During	Both	Tired	Drive
No Rain	After	Backpack	Tired	Bike
No Rain	After	Both	Tired	Drive

$$I\left(x_1, Y_{x_4=\text{Tired}}\right)$$

$$\approx 1.2244 - \left(\frac{4}{9}(0.8113) + \frac{5}{9}(0.9710)\right) \approx 0.3244$$

x_4

Not Tired | Tired

x_1	x_2	x_3	x_4	y
Rain	During	Both	Not Tired	Bus
Rain	After	Backpack	Not Tired	Bus
No Rain	Before	Lunchbox	Not Tired	Bus
No Rain	During	Backpack	Not Tired	Bus
No Rain	After	Backpack	Not Tired	Bike
No Rain	After	Both	Not Tired	Bus
No Rain	After	Lunchbox	Not Tired	Bus

x_1	x_2	x_3	x_4	y
Rain	Before	Both	Tired	Drive
Rain	During	Both	Tired	Drive
Rain	After	Backpack	Tired	Bus
Rain	After	Lunchbox	Tired	Drive
No Rain	Before	Backpack	Tired	Bike
No Rain	Before	Lunchbox	Tired	Drive
No Rain	During	Both	Tired	Drive
No Rain	After	Backpack	Tired	Bike
No Rain	After	Both	Tired	Drive

$$I(x_1, Y_{x_4=\text{Tired}}) \approx 0.3244$$

$$I(x_2, Y_{x_4=\text{Tired}}) \approx 0.2516$$

$$I(x_3, Y_{x_4=\text{Tired}}) \approx \mathbf{0.9183}$$

x_4

Not Tired | Tired

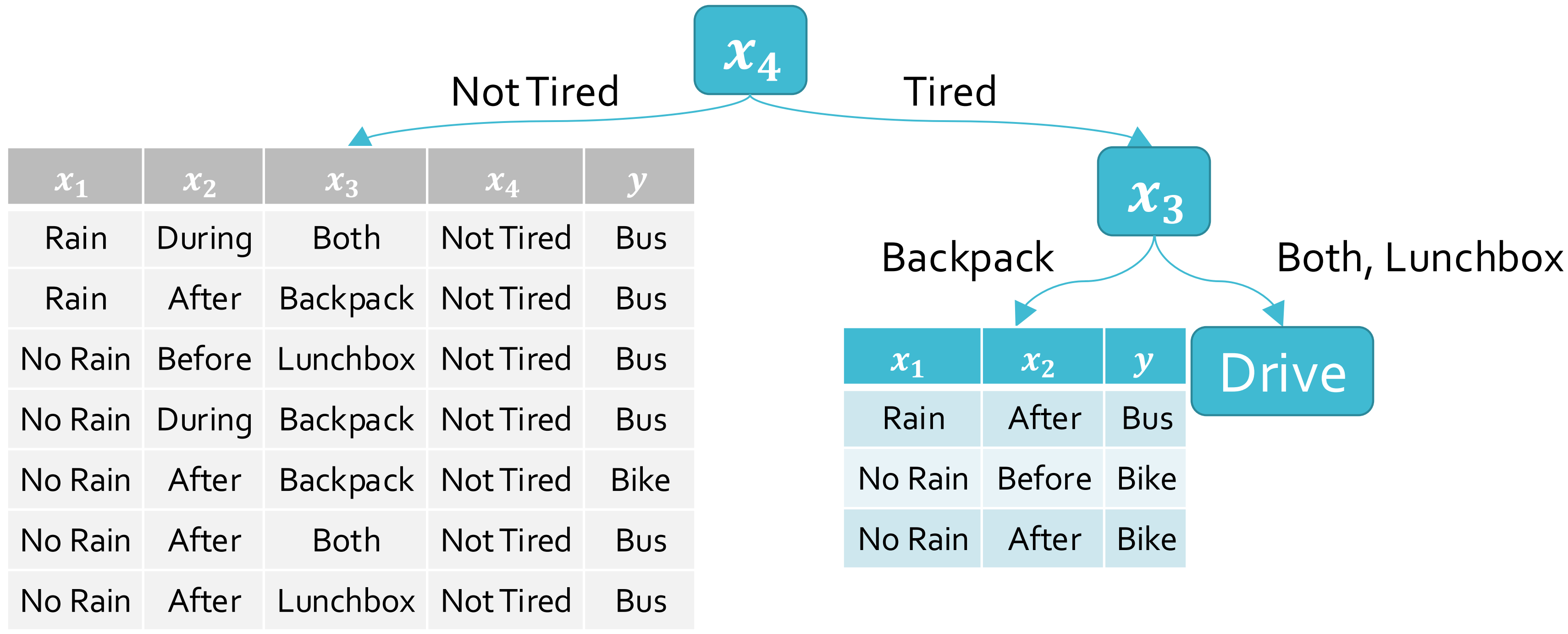
x_1	x_2	x_3	x_4	y
Rain	During	Both	Not Tired	Bus
Rain	After	Backpack	Not Tired	Bus
No Rain	Before	Lunchbox	Not Tired	Bus
No Rain	During	Backpack	Not Tired	Bus
No Rain	After	Backpack	Not Tired	Bike
No Rain	After	Both	Not Tired	Bus
No Rain	After	Lunchbox	Not Tired	Bus

x_1	x_2	x_3	x_4	y
Rain	After	Backpack	Tired	Bus
No Rain	Before	Backpack	Tired	Bike
No Rain	After	Backpack	Tired	Bike
Rain	Before	Both	Tired	Drive
Rain	During	Both	Tired	Drive
No Rain	During	Both	Tired	Drive
No Rain	After	Both	Tired	Drive
Rain	After	Lunchbox	Tired	Drive
No Rain	Before	Lunchbox	Tired	Drive

$$I\left(x_1, Y_{x_4=\text{Tired}}\right) \approx 0.3244$$

$$I\left(x_2, Y_{x_4=\text{Tired}}\right) \approx 0.2516$$

$$I\left(x_3, Y_{x_4=\text{Tired}}\right) \approx \mathbf{0.9183}$$



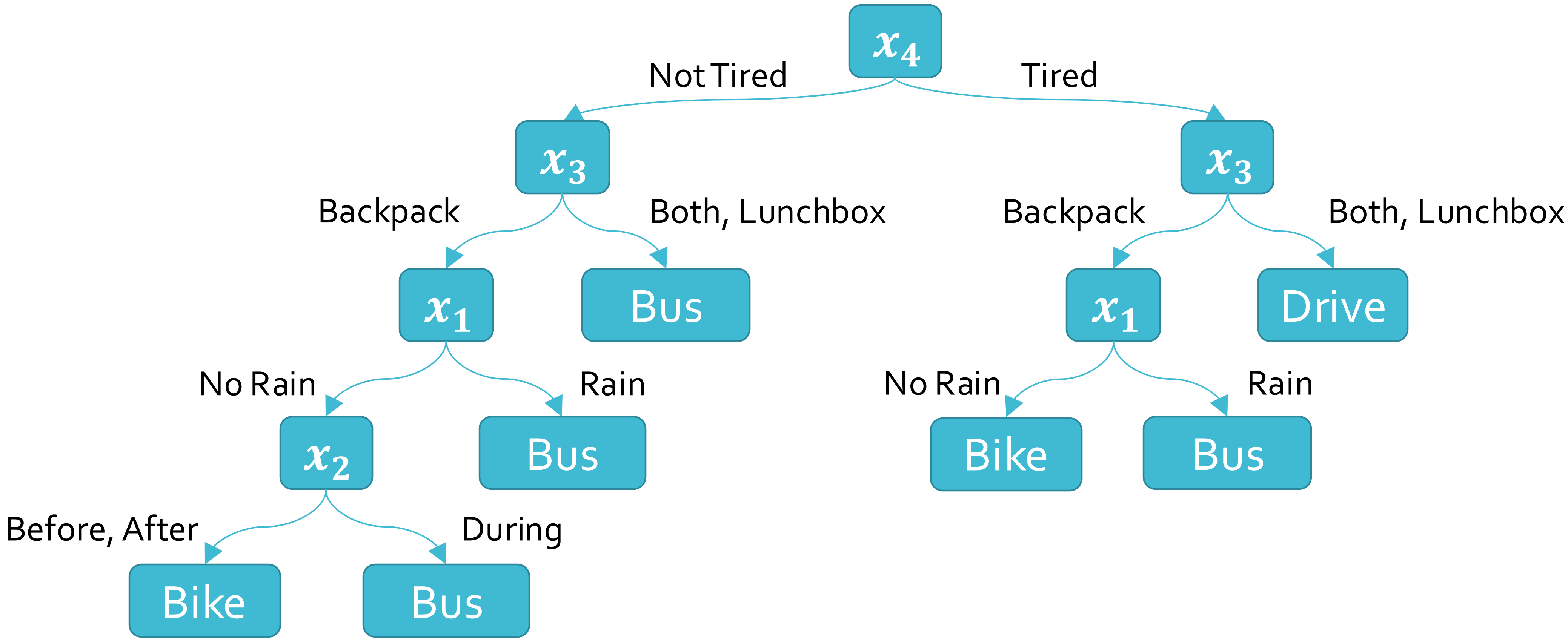
x_1	x_2	x_3	x_4	y
Rain	During	Both	Not Tired	Bus
Rain	After	Backpack	Not Tired	Bus
No Rain	Before	Lunchbox	Not Tired	Bus
No Rain	During	Backpack	Not Tired	Bus
No Rain	After	Backpack	Not Tired	Bike
No Rain	After	Both	Not Tired	Bus
No Rain	After	Lunchbox	Not Tired	Bus

x_1	x_2	y
Rain	After	Bus
No Rain	Before	Bike
No Rain	After	Bike

$$I\left(x_1, Y_{x_4=\text{Tired}}\right) \approx 0.3244$$

$$I\left(x_2, Y_{x_4=\text{Tired}}\right) \approx 0.2516$$

$$I\left(x_3, Y_{x_4=\text{Tired}}\right) \approx \mathbf{0.9183}$$


x_4
Not Tired
Tired
x_3
x_3
Backpack
Both, Lunchbox
Backpack
Both, Lunchbox
x_1
Bus
x_1
Drive
No Rain
Rain
No Rain
Rain
x_2
Bus
Bike
Bus
Before, After
During
Bike
Bus

Decision Stumps: Questions

1. How can we pick which feature to split on?

2. Why stop at just one feature?
   a. How can we pick the order of the splits?

Key Takeaways

- Decision tree prediction algorithm
- Decision tree learning algorithm via recursion