# Challenges for Principles of Parallel Programming

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Programming Technologies

# Three Central Problems

**Computation**

**A billion threads, a million nodes**

**Failure!**

➔ **Resilient computation**

**Robust computation**

**(e.g. robust automata)**

➔ **Extend core imperative calculli, lang with resilience**

➔ **(Concurrent, Distributed, Resilient, Data-centered)**

**Domain specific calculii**

**Data**

**Wide variety of data, exabytes of data**

**Establish ZooKeeper, HBase, Hadoop do not lose data, despite node failure**

**Develop consistent global state management techniques**

**Develop resilient distributed termination detection**

**X10: places, async, finish, at, atomic … resilience?**

(Agents) A,B ::= S | async A | finish A | at (p) A

**Semantics, properties** | **Static Analysis, Types** | **Synthesis** | **Compilation** | **Runtime** | **Debugging**

# CCP for Graph Algorithms: Maximal Cliques >= L

```
/* Degree Filter: Delete all vertices with degree < L */
all (x:V,y:V) =>
    if (bag(y:V=>edge(x,y)).size()<L, edge(x,y)) { delete(x,y) }

/* Edge Filter: Delete all edges (x,y) s.t. commonSize(x,y) < L */
all (x:V,y:V) =>
    if (edge(x,y), bag(z:V=>{edge(x,z),edge(z,y)}).size()<L) { delete(x,y) }

/* Phase Rule */
all (x:V,y:V)=>
   unless (delete(x,y)) if (edge(x,y))  { next edge(x,y) }
```

**Execute efficiently (and resiliently) on billions of vertices, thousands of nodes**

# Concurrent Constraint Programming

**Linear CCP (linear logic) 92, 00**

**Discrete Time (next) TCC 94**

**Universal CCP (all) 06**

**Instantaneous pre-emption (unless), Default TCC 95**

**CCP 89**

```
(Agents) A::= c; | if (c) {A} | A B | {val x:T; A}
(Config) G ::= A,…, A (multiset of agents)
G, {val x:T; A} → G,A        (x not free in G)
G, A B → G,A,B
G,c1,…,cn,if (c) A → G,A     (c1,…,cn |- c)
```

**Nested Tests, Agents (RCC) 05**

**Continuous time (HCC) 96**
**Differential equations**

**Spatial CCP 89,12**

**Epistemic CCP 12**

**Continuous space (04) (PDEs)**

# Compilation to X10

(Agents) A,B ::= e==f? g=h | {var x; A} | A A | async A | finish A | at (p) A

$$
\begin{array}{rcll}
A & \equiv & B & (B \text{ is an } \alpha\text{-renamed version of } A) \\
(A\,;\,B)\,;\,C & \equiv & A\,;\,(B\,;\,C) & \\
\mathit{async}\ A\,;\,\mathit{async}\ B & \equiv & \mathit{async}\ B\,;\,\mathit{async}\ A & \\
\mathit{new}\ x\ \mathit{new}\ y\ A & \equiv & \mathit{new}\ y\ \mathit{new}\ x\ A & \\
\mathit{new}\ x\ (A\,;\,B) & \equiv & A\,;\,(\mathit{new}\ x\ B) & (x \notin \mathit{var}(A)) \\
\mathit{new}\ x\ (A\,;\,B) & \equiv & (\mathit{new}\ x\ A)\,;\,B & (x \notin \mathit{var}(B)) \\
!A & \equiv & A\,;\,!A &
\end{array}
$$

$$
\frac{\mathit{val}(\bar{e},\sigma) = \mathit{val}(\bar{f},\sigma)}{\langle \bar{e} == \bar{f}\ ?\ \bar{g} = \bar{h}, \sigma\rangle \longrightarrow \sigma[\mathit{val}(\bar{g},\sigma) \mapsto \mathit{val}(\bar{h},\sigma)]} \quad (2)
$$

$$
\frac{\langle A, \sigma \rhd x\rangle \longrightarrow \langle A', \sigma'\rangle \mid \sigma' \quad x \notin \mathrm{dom}(\sigma)}{\langle \mathit{new}\ x\ A, \sigma\rangle \longrightarrow \langle A, \sigma'\rangle \mid \sigma'} \quad (3)
$$

$$
\frac{\langle A, \sigma\rangle \longrightarrow \langle A', \sigma'\rangle \mid \sigma'}{\begin{array}{rcl}
\langle A\,;\,B, \sigma\rangle & \longrightarrow & \langle A'\,;\,B, \sigma'\rangle \mid \langle B, \sigma'\rangle \\
\langle (\mathit{async}\ B)\,;\,A, \sigma\rangle & \longrightarrow & \langle (\mathit{async}\ B)\,;\,A', \sigma'\rangle \mid \langle \mathit{async}\ B, \sigma'\rangle \\
\langle \mathit{async}\ A, \sigma\rangle & \longrightarrow & \langle \mathit{async}\ A', \sigma'\rangle \mid \sigma' \\
\langle \mathit{finish}\ A, \sigma\rangle & \longrightarrow & \langle \mathit{finish}\ A', \sigma'\rangle \mid \sigma'
\end{array}} \quad (4)
$$

$$
\frac{A \equiv A' \quad \langle A', \sigma\rangle \longrightarrow \langle B', \sigma'\rangle \mid \sigma' \quad B' \equiv B}{\langle A, \sigma\rangle \longrightarrow \langle B, \sigma'\rangle \mid \sigma'} \quad (5)
$$

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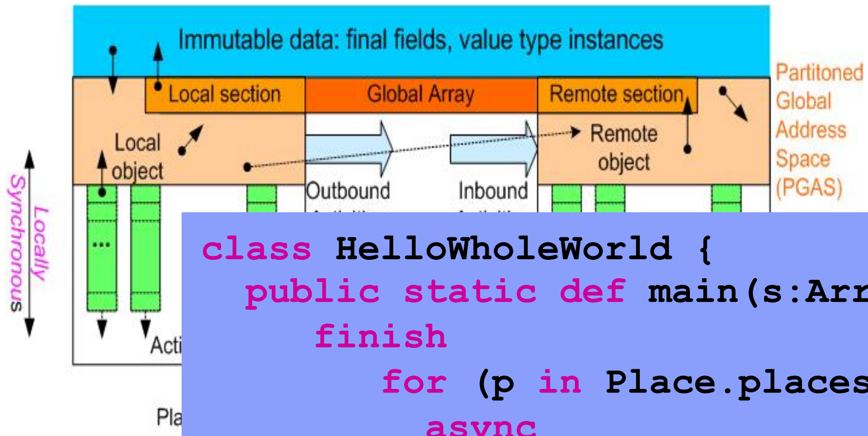
# X10 2.2: An APGAS language




```
class HelloWholeWorld {
  public static def main(s:Array[String]) {
     finish
        for (p in Place.places())
          async
            at (p)
              Console.OUT.println("(At " + p + ") "
                                  + s(0));
  }
}
```

Asychrony
- async S

- atomic S
- when (c) S

Locality
- at (P) S

Order
- finish S
- clocks

- points, regions, distributions, arrays

- Class-based single-inheritance OO
- Structs
- Closures
- True Generic types (no erasures)
- Constrained Types (OOPSLA 08)

**established**
ws best known
TS upto 3K

- Global Matrix Library shows substantial speedup over Hadoop for data analytics kernels.
- Similar performance improvement for Main Memory Map Reduce engine (M3R) over Hadoop.

**Java-like productivity, MPI-like performance**

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# But – how do we handle a billion threads?

- **X10 is (deliberately) low-level**
  - Imperative – explicit mutation, hence very “PC centric” view of computation.
  - Explicit distribution
- **How do you debug a 100,000 threads from a PC-centric point of view?**

- **Our belief**
  - Need to raise level of abstraction
  - Programming model needs to be closer to application domain
  - Implicitly concurrent
  - Statically type safe
  - Declarative
    - Support semantically-based tools, using symbolic reasoning
  - Determinate
  - Efficiently implementable!

# Research Agenda

- **Develop "broad" programming framework**
  – Declarative programs (CCP)
  – Fundamentally integrates space and time
  – Compiles to high-performance imperative programs

- **Develop tools that exploit declarative semantics**
  – Correctness at scale
  – Correct by construction
  – Partial programs, sketching
  – Declarative debugging

- **Directed at substantially raising level of programmer/ productivity**
  – (cf R, Matlab, … but at scale)
  – "domain" programmer: HPC, machine learning/BA

# Background

# Selected Bibliography


- **Saraswat, Rinard, Panangaden "Semantics of Concurrent Constraint Programming", POPL 1991**
- **Falaschi, Gabbrielli, Marriott, Palamidessi "Compositional analysis for CCP", LICS 1993**
- **Fromherz "Towards declarative debugging of CCP", 1995**
- **Saraswat, Jagadeesan, Gupta "Timed Default CCP", Journal Symbolic Comp., 1996**
- **de Boer, Gabbrielli, Marchiori, Palamidessi "Proving concurrent constraint programs correct", TOPLAS 1997**
- **Gupta, Jagadeesan, Saraswat "Computing with continuous change", Science Comp Progg. 1998.**
- **Etalli, Gabbrielli, Meo "Transformations of CCP programs", TOPLAS 2001**
- **Falaschi, Olarte, Valencia "Framework for abstract interpretation for Timed CCP", PPDP 09**
- **Gabbrielli, Palamidessi, Valencia "Concurrent and Reactive Constraint Programming", 2010**


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# Constraint systems

- **Any (intuitionistic, classical) system of partial information**
- **For Ai read as logical formulae, the basic relationship is:**
  - A1,…, An |- A
  - Read as "If each of the A1,…, An hold, then A holds"
- **|- is axiomatized through given rules.**
- **Require conjunction, existential quantification**

A,B,D ::= atomic formulae | A&B |X^A

G ::= multiset of formulae

**(Id)** A |- A (Id)

**(Cut)** G |- B G',B |- D ➔ G,G' |- D

**(Weak)** G |- A ➔ G,B |- A

**(Dup)** G, A, A |- B ➔ G,A |- B

**(Xchg)** G,A,B,G' |- D ➔ G,B,A,G' |- D

**(&-R)** G,A,B |- D ➔ G, A&B |- D

**(&-L)** G |- A G|- B ➔ G |- A&B

**(^-R)** G |- A[t/X] ➔ G |- X^A

**(^-L,*)** G,A |- D ➔ G,X^A |- D

# Constraint system: Examples

- **Gentzen**
  - G |- A iff A in G.
- **Herbrand**
  - uninterpreted first-order terms (labeled, fixed-arity trees)
- **Finite domain**
- **Propositional logic (SAT)**
- **Arithmetic constraints**
  - Naïve, linear, nonlinear
- **Interval arithmetic**

- **Orders**
- **Temporal Intervals**
- **Hash-tables**
- **Arrays**
- **Graphs**

- **Constraint systems (as systems of partial information) are ubiquitous in computer science**
  - Type systems
  - Compiler analysis
  - Symbolic computation
  - Concurrent system analysis

# Logic

Proposition: Operational Semantics is complete for constraint entailment. (Saraswat, Lincoln 1994, unpublished)

- **CCP is simply a fragment of first-order logic.**
  - Computation == Deduction
  - Unlike "Logic Programming", CCP employs "forward chaining".

- **RCC (Jagadeesan, Nadathur, Saraswat, FSTTCS 2005)**
  - Unifies and subsumes CCP and LP (forward- and backward-chaining).
  - Provides logical expression for recursive nested guards
    - i.e. "finish"
  - Localized augmentation of programs ("assume-if" reasoning, `(P=>Q)=>R)`
  - Backtracking and search

# xcc: CCP in X10

- **Basic idea**
  - Concrete language is just like X10 – classes, inheritance, interfaces, structs, functions, fields, methods, constructors, user-defined operators, type inference etc.
  - No var permitted, no need for atomic, when, finish, async, at.
    - Initially, finish, async, at may be introduced as annotations to permit efficient execution while compiler is being developed.
- **Every variable of type T is initialized with a *promise* of type T.**
  - A promise is a "logical variable" – nothing is known about it.
  - (Herbrand) Two objects are equal iff they are instances of the same class and their corresponding fields are equal.
- **Assignment (=) is re-interpreted as Tell:**
  - $e_1=e_2$ is executed as: evaluate $e_1$ to get a value $v_1$, $e_2$ to get $v_2$, and equate the two.
- **if (and ? : conditional expression evaluator) suspends until condition evaluates to true or false**
  - if = when, because of monotonicity.
- $e.m(e_1,..,e_n)$
  - e, $e_1,..,e_n$ evaluated in parallel
  - Once enough is known about e to determine the class, use dynamic lookup to determine method body
  - Body executed in parallel with arg evaluation
    - Return value is an anonymous promise constrained by return statements.

# Can computations deadlock?

- **Yes.**
  - `when(a) b` is canonical deadlocked agent.
  - Intuitively, program quiesces but can produce more when given more.
- **Deadlock is a "natural" state.**
  - Simply means the system has quiesced.
  - If you supply more information, you may get more information back.
  - E.g. almost all interesting programs would deadlock on true.

- **Semantic characterization:**
  - P does not deadlock on input a if all fixed points of P above a are stable.
    - **b >= P(a) implies b in P**
  - Observation: if P does not deadlock on d, then for any b, P(d&b)=P(d)&P(b)

**Open problem:**

Identify static type system that guarantees deadlock-freedom and permits useful idioms to be expressed.

# Declarative Debugging

- **Declarative debugging techniques can be applied to logic programs, functional programs, CCP.**
  - Ueda 98 (CCP)
  - Fromherz 93
  - Falaschi et al ICLP 07

- **Basic idea is to summarize an execution through an execution tree**
  - Node = procedure call
  - Children = calls made in the body.
  - Node associated with some data about subtree, e.g. pair of input/ output constraints.

- **Debugging**
  - Query oracle (user, specification) whether data with node is correct.
  - Identify node with incorrect data whose children have correct data …. BUG!

# Timed CCP: Basic Results

- **TCC = fragment of first-order linear temporal logic**
- **Rich algebra of defined temporal combinators (cf Esterel):**
  - always A
  - do A watching c
  - whenever c do A
  - time A on c
- **A general combinator can be defined**
  - time A on B: the clock fed to A is determined by (agent) B
- **Discrete timed synchronous programming language with the power of Esterel**
  - present is translated using defaults
- **Proof system**
- **Compilation to automata**

# Programming matter


- *Vijay Saraswat, IBM Research*
- *Radha Jagadeesan, De Paul University*


- *May 2006*

IBM Research

# Programmable matter

- **Large collection of "computing atoms" (catoms) that can**
  - Compute
  - Communicate locally (wireless)
  - Sense
  - Move
  - Adhere to each other (bond)
  - Change physical/chemical properties based on state
- **cf sensor networks**

- **Desired computations**
  - Form a particular shape
  - Sense a particular shape

How do you compute with $10^6$ computers/cubic centimeter?

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# The computational substrate

- **No shared clock.**
- **No shared gobal coordinate system.**
- **No unique ids (but random variables permitted).**
- **No shared mutable state (shared memory).**
- **Catoms randomly distributed in 3D (2D).**
- **Some small subset are "dead on arrival".**

- **Catoms can sense connections with neighboring catoms and send/receive messages.**
- **Catoms can broadcast locally.**
- **Assume boundary conditions are supplied in some fashion.**
- **Catoms are (re-)programmed by "beaming in" code.**

- **Catoms have limited power?**

Cf Amorphous computing

# The programming matter challenge

## How do you move from a global description to local actions?

- **What is the programming model for programmable matter?**
- **Global program**
  - Specifies constraints on desired interactions of system with environment.
- **Local program: Catom's view**
  - Specifies how each catom in ensemble initiates/responds to messages received from the environment.

- **Our approach: Program globally, implement locally**
  - Treat programmable matter as *matter*
  - Study how matter "computes"
    - Physics
    - Chemistry
    - Biology – developmental biology
  - Study mathematical descriptions of these processes (continuous space, time, differential eqns, stochasticity)
  - Build programming model on these descriptions
  - Compile such global programs to local catom programs: *"correct" by construction!*

## From analysis to programming

# Constraint systems

- **Any (intuitionistic, classical) system of partial information**
- **For $A_i$ read as logical formulae, the basic relationship is:**
  - $A_1, \ldots, A_n$ |- A
  - Read as "If each of the $A_1, \ldots, A_n$ hold, then A holds"
- **Require conjunction, existential quantification**

Saraswat, LICS 91

A,B,D ::= atomic formulae | A&B |X^A

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**(Dup)** G, A, A |- B ➔ G,A |- B

**(Xchg)** G,A,B,G' |- D ➔ G,B,A,G' |- D

**(&-R)** G,A,B |- D ➔ G, A&B |- D

**(&-L)** G |- A G|- B ➔ G |- A&B

**(^-R)** G |- A[t/X] ➔ G |- X^A

**(^-L,*)** G,A |- D ➔ G,X^A |- D

# Constraint system: Examples

- **Gentzen**
- **Herbrand**
  - Lists
- **Finite domain**
- **Propositional logic (SAT)**
- **Arithmetic constraints**
  - Naïve
  - Linear
  - Nonlinear
- **Interval arithmetic**
- **Orders**
- **Temporal Intervals**

- **Hash-tables**
- **Arrays**
- **Graphs**
- **Constraint systems are ubiquitous in computer science**
  - Type systems (checking, inference)
  - Static analysis
  - Symbolic computation
  - Concurrent system analysis

# Concurrent Constraint Programming

- **Use constraints for communication and control between concurrent agents operating on a shared store.**
- **Two basic operations**
  - **Tell** c: Add c to the store
  - **Ask** c **then** A: If the store is strong enough to entail c, reduce to A.

(Agents) `A ::= c`

`if (c) A`

`A,B`

`{x:T; A}`

(Config) `G ::= A,…,A`

`G,{x:T;A}` ➔ `G,A` (x not free in G)

`G, if (c) A` ➔ `G,A` (s(G) |- c)

[[A]] = set of fixed points of a closure operator

Operational semantics is complete for logical entailment of constraints.

Saraswat 89; POPL 87, POPL 90, POPL 91

# Default CCP

- `A ::= unless(c) A`
  - Run A, unless c holds at end
  - ask c \/ A
  - Leads to nondet behavior
- `unless(c) c`
  - No behavior
- unless($c_1$) $c_2$, unless($c_2$) $c_1$
  - gives $c_1$ or $c_2$
- `unless(c) d` **: gives d**
- `c, unless(c) d` **: gives c**

↑

non-monotonicity

- **[A] = set S of pairs (c,d) satisfying**
  - $S_d$ = {c | (c,d) in S} denotes a closure operator.
  - *We still have a simple denotational semantics!*
- **Operational implementation:**
  - Backtracking search
  - Compile-time determinacy analysis (not implemented)
  - Open question:
    - Efficient compile-time analysis (cf causality analysis in Esterel)
    - Use negation as failure

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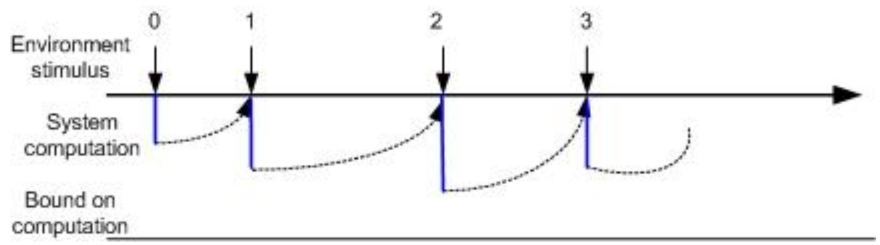
# Discrete Timed CCP (1993)




- **Synchrony principle**
  - System reacts **instantaneously** to the environment
  - Implemented by ensuring computation at each time instant is bounded.
- **Semantic idea**
  - Run a Default CCP program at each time point
  - Add a single new combinator: **A ::= hence A** (run A at every *subsequent* instant.)
  - No connection between the store at one point and the next.
  - Semantics: Sets of sequences of (pairs of) constraints

**Proof system**

- **The usual temporal combinators can be programmed:**
  - `always(A) = {A; hence A;}`
  - `do A watching c`
  - `time A on B:` the clock fed to A is determined by (agent) B
- `unless` **can be used to retract** `hence` **constraints**
  - `next(A) =`

```
{X:boolean;
  hence {
   unless(X=true) A;
   hence X=true;
  }
}
```

**Compilation to automata**

# Hybrid Systems

- **Traditional Computer Science**
  - **Discrete state, discrete change (assignment)**
  - **E.g. Turing Machine**
  - **Brittle:**
    - **Small error → major impact**
    - **Devastating with large code!**
- **Traditional Mathematics**
  - **Continuous variables (Reals), with continuous functions (e.g. sum, multiplication).**
  - **Smooth state change**
    - **Mean-value theorem**
    - **e.g. computing rocket trajectories**
  - **Robustness in the face of change**
  - **Stochastic systems (e.g. Brownian motion)**

- **Hybrid Systems combine both**
  - **Discrete control**
  - **Continuous state evolution**
  - **Intuition: Run program at every real value.**
    - **Approximate by:**
      - **Discrete change at an instant**
      - **Continuous change in an interval**
- **Primary application areas**
  - **Engineering and Control systems**
    - **Paper transport**
    - **Autonomous vehicles…**
  - **Biological Computation.**
  - **Programmable Matter**

Emerged in early 90s in the work of Nerode, Kohn, Alur, Dill, Henzinger…

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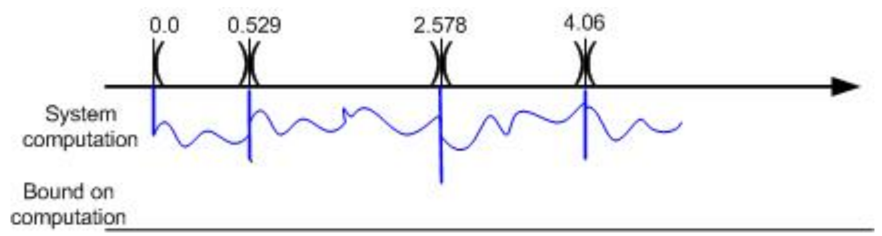
# HCC: Move to Continuous time (1995)




- ***No new combinator needed***
  - Constraints are now permitted to vary with time (e.g. `x'=y`)
- **Semantic intuition**
  - Run default CCP at each real time instant, starting with t=0.
  - Evolution of system is piecewise continuous: system evolution alternates between point phase and interval phase.
  - In each phase program determines output of that phase and program to be run in next phase.
- **Point phase**
  - Result determines initial conditions for evolution in the subsequent interval phase and `hence` constraints in effect in subsequent phases.
- **Interval phase**
  - Any constraints asked of the store recorded as transition conditions.
  - ODE's integrated to evolve time-dependent variables.
  - Phase ends when any transition condition potentially changes status.
  - (Limit) value of variables at the end of the phase can be used by the next point phase.

*Gupta, Jagadeesan, Saraswat SCP 1998*

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# Systems Biology

- **Work subsumes past work on mathematical modeling in biology:**
  - Hodgkin-Huxley model for neural firing
  - Michaelis-Menten equation for Enzyme Kinetics
  - Gillespie algorithm for Monte-Carlo simulation of stochastic systems.
  - Bifurcation analysis for Xenopus cell cycle
  - Flux balance analysis, metabolic control analysis…

- **Why Now?**
  - Exploiting genomic data
  - Scale
    - Across the internet, across space and time.
  - Integration of computational tools
  - Integration of new analysis techniques
  - Collaboration using markup-based interlingua (SBML)
  - Moore's Law!

*This is not the first time…*

# Chemical Reactions

- **Cells host thousands of chemical reactions (e.g. citric acid cycle, glycolis…)**
- **Chemical Reaction**
  - $X+Y_0 -k_0\rightarrow XY_0$
  - $XY_0 -k_{-0} \rightarrow X+Y_0$
- **Law of Mass Action**
  - Rate of reaction is proportional to product of conc of components
  - $[X]' = -k_0[X][Y] + k_{-0}[XY_0]$
  - $[Y]'=[X]'$
  - $[XY]'=k_0[X][Y]-K_{-0}[XY_0]$
- **Conservation of Mass**
- **When multiple reactions, sum mass flows across all sources and sinks to get rate of change.**
- **Same analysis useful for enzyme-catalyzed reactions**
  - Michaelis-Menten kinetics
- **May be simulated**
  - Using “deterministic” means.
  - Using stochastic means (Gillespie algorithm).

*At high concentration, species concentration can be modeled as a continuous variable.*

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# Quorum sensing (V. fischeri)

Model due to Alur et al

# Cell division: Delta-Notch signaling in X. Laevis

- **Consider cell differentiation in a population of epidermic cells.**
- **Cells arranged in a hexagonal lattice.**
- **Cells interacts concurrently with its neighbors.**
- **Delta and Notch proteins in each cell vary continuously.**
- **Cell can be in one of four states: {Delta, Notch} x {inhibited, expressed}**

- **Experimental Observations:**
  - Delta (Notch) concentrations show typical spike at a threshold level.
  - At equilibrium, cells are in only two states (D or N expressed; other inhibited).

Ghosh, Tomlin: “Lateral inhibition through Delta-Notch signaling: A piece-wise affine hybrid model”, HSCC 2001

# Delta-Notch Models

- **Model:**
  - $V_D$, $V_N$: concentration of Delta and Notch protein in the cell.
  - $U_D$, $U_N$: Delta (Notch) production capacity of cell.
  - $U_N$=sum_i (neighbors) $V_D$(i)
  - $U_D = -V_N$
  - Parameters:
    - Threshold values: HD,HN
    - Degradation rates: MD, MN
    - Production rates: RD, RN
  - Cell in 1 of 4 states: {D,N} x {Expressed (above), Inhibited (below)}
- **Stochastic variables used to set random initial state.**

**if (UN(i,j) < HN) VN' = -MN*VN,**

**if (UN(i,j)>=HN) VN' =RN-MN*VN,**

**if (UD(i,j)<HD) VD' =-MD*VD,**

**if (UD(i,j)>=HD) VD' =RD-MD*VD,**

Results: Simulation confirms observations. Tiwari/Lincoln prove that States 2 and 3 are stable.

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# Other examples

- Bouncing ball
- Thermostat controller
- Square waves
- Sine waves…

- Paper path model
- Aercam model

# Concrete HCC language

- **Arithmetic variables are interval valued.**
- **Arithmetic constraints are non-linear algebraic equations, over +, *, ^, etc.**
- **Users can add own operators as C libraries.**
- **Various combinators translated to basic combinators e.g.**

  **do A watching c → execute A, abort it when c becomes true**

  **when c do A → start A at the first instant when c holds**

  **wait N do A → start A after N time units**

  **forall C(X) do A(X) → execute a copy of A for each object X of class C**

- **Arithmetic expressions compiled to byte code**
  - Further compiled to machine code.
  - Common sub-expressions are recognized.
- **Copying garbage collector**
  - Speeds up execution
  - Allows snapshotting of state.
- **API from Java/C to use Hybrid cc as a library. System runs on Solaris, Linux, SGI and Windows NT.**

*Carlson, Gupta "Hybrid CC with Interval Constraints"*

# HCC Implementation outline

- **Constraint techniques**

**Use constraints to narrow intervals of variables, one variable at a time. Suppose f(x,y) = 0.**

**Indexicals: Rewrite as x = g(y). Set x ∈ I ∩ g(J), where x ∈ I and y ∈ J. (y can be a vector of variables.)**

**Interval splitting: If x ∈ [a, b], use binary search to find min c in [a,b] such that 0 ∈ f([c,c], J), where y ∈ J. Similary determine max such d in [a,b], and set x ∈ [c,d].**

**Newton-Raphson: Get min and max roots of f(x,J) = 0, where y ∈ J. Set x as above.**

**Simplex: Given the constraints on x, find its min and max values, and set it as above. Treat non-linear terms as separate variables.**

- **Integration techniques**

**Treat differential equations as ordinary algebraic equations on variables and their derivatives e.g. f = m * a'', x'' + d*x' + k*x = 0.**

**Various integrators are provided --- Euler, 4th order Runge Kutta, 4th order Runge Kutta with adaptive stepsize, Bulirsch-Stoer with polynomial extrapolation. Others can be added if necessary.**

**Integrators modified to integrate implicit differential equations, over interval valued variables.**

**Determine points of discrete changes (end of an interval phase) using cubic Hermite interpolation.**

*Carlson, Gupta "Hybrid CC with Interval Constraints"*

# Integration of symbolic reasoning

- **Use state of the art constraint solvers**
  - ICS from SRI
  - Shostak combination of theories (SAT, Herbrand, RCF, linear arithmetic over integers).
- **Finite state analysis of hybrid systems**
  - Generate code for HAL
- **Predicate abstraction techniques.**
- **Develop bounded model checking.**
- **Parameter search techniques.**
  - Use/Generate constraints on parameters to rule out portions of the space.
- **Integrate QR work**
  - Qualitative simulation of hybrid systems

# Spatial HCC: Move to continuous space

- **Add `A::= atOther A`**
  - Run A at all *other* points. (`atAll A = A, atOther A`)
  - Constraints may now use partial derivatives.
  - All variables now implicitly depend on space parameters (e.g. x,y,z)
- **Semantic intutions**
  - Computation now uniformly extended across space.
  - At each point, run a Default CC program.
  - Program induces its own discretization of space (into open and closed regions).

- **Programming intuition**
  - Program with vector fields, specifying how they vary across space-time.
- **Programming Matter realization**
  - Catoms represent dense computational grid.
  - Signals represented as memory cells in each catom
  - Catoms use epidemic algorithms to diffuse signals (possibly with non-zero gradients) across space.
  - Catoms use neighborhood queries to sense local minima
  - Catoms integrate PDEs by using chaotic relaxation (Chazan/Mirankar).
  - Compiler produces FSA for each catom from input program.

# Some basic programming idioms

```
// coord system
R=(0,0,0),
atAll grad(R)=(1,1,1)
// define
at(L) A :: at(R=L) A
at(I:J) A:: at(I<R&R<J) A
```

```
// vibrating 1-d string
u=0, at(R=L)u=0,
at(0<R && R<L)u=f
atAll u''t = c*c*u''x
```

Abbreviation:

**`at(boolean b) A ::`**

**`atAll if (b) A`**

**`b`** may be true at 0 or more points in space.

We will also use neighborhood queries:

**`min {e | b} (max,…)`**

**`e`** is an expression, **`b`** a **`boolean`**

**`min`** evaluated over a sphere of radius r (execution-time parameter). Also **`max`**,…

# Nagpal’s Origami Operator(1): perp


```
agent perp(boolean isP0,
   boolean isP1,
   vec R, // global coord system
   boolean line) {
  at(isP0) {
   vec(2) D0=R, atAll grad(D0)=0.0,
   at(isP1) {
      vec(2) D1=R, atAll grad(D1)=0.0,
      at(norm(D1-D0)<=eps)
        line=true
    }}}
```

```
agent perp(boolean isP0,
           boolean isP1,

           boolean line) {
  at(isP0) {
   vec(2) D0=0.0, atAll grad(D0)=1.0,
   at(isP1) {
      vec(2) D1=0.0, atAll grad(D1)=1.0,
      at(norm(D1-D0)<= eps)
             line=true
    }}}
```

**Use global coordinate system.**

**Use local coordinate systems!**

*Global coordinate systems can be banned by requiring initial agent is* `atAll A.`

IBM Research

# Nagpal’s Operator(1): perp


```
agent perp(boolean isP0,
           boolean isP1,
           boolean line) {
  at(isP0) {
    vec(2) D0=0.0, atAll grad(D0)=1.0,
    at(isP1) {
       vec(2) D1=0.0, atAll grad(D1)=1.0,
       at(norm(D1-D0) <= eps)
              line=true
     }}}
```

**Local coordinate system.**

**Propagates 2-d vectors with unit gradient.**

```
agent perp(boolean isP0,
           boolean isP1,
           boolean line) {
  at(isP0) {
    vec(1) D0=0.0,atAll grad(D0)=(1.0,0.0),
    at(isP1) {
       vec(1) D1=0.0,atAll grad(D1)=(1.0,0.0),
       at(norm(D1-D0) <= eps)
              line=true
     }}}
```

**Local *polar* coordinate system.**

**Propagates scalars with unit radial gradient, zero angular gradient.**

IBM Research

# Nagpal’s Operator(2): conn


```
agent conn(boolean isP0,
           boolean isP1,
           boolean line) {
  at(isP1) {
    vec(2) D1=0.0, atAll grad(D1)=1.0,
    at(isP0) {
       vec(2) D0=D1, atAll grad(D0)=0.0,
       at(norm(D1.unit-D0.unit)<= eps)
              line=true}}
```

```
agent conn(boolean isP0,
           boolean isP1,
           boolean line) {
  at(isP1) {
    vec(2) D1=0.0,atAll grad(D1)=(1.0,0.0),
    at(isP0) {
       vec(2) D0=0.0,atAll grad(D0)=(1.0,0.0),
       at(D0+D1-min{D0+D1})<= eps)
              line=true}}
```

**Local coordinate system.**

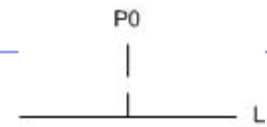
**Propagates 2-d vectors with unit gradient.**

**Local coordinate system.**

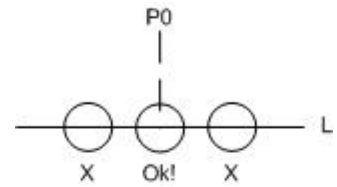
**Propagate scalars.**

**Use neighborhood minima queries.**

# Nagpal Operator (3): alt




```
agent alt(boolean isP0,
          boolean isLine,
          boolean line, boolean crossing) {
 at(isP0) {
   vec(2) D0=0.0,atAll grad(D0)=(1.0,0.0),
   at(isLine &(D0-min{isLine | D0}<= eps)) {
     crossing=true, atOther crossing=false,
     conn(isP0,crossing,line)}}
```



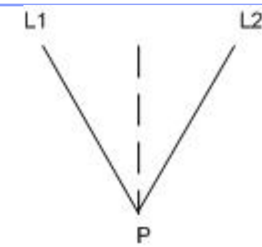

- **Find the point P1 on the line**
  - that is closest to P0
  - in its local neighborhood, considering only points on the line.
- **Draw the line from P0 to P1**

**Local coordinate system.**

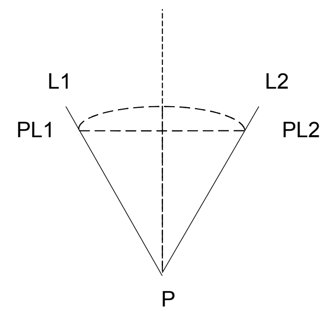
**Propagate scalars.**

**Use *conditional* neighborhood minima queries.**

# Nagpal Operator(4): Bisection




```
agent bisect(boolean isLine1,
           boolean isLine2,
           boolean line) {
  at(isLine1 & isLine2) {
    boolean isP=true,
    vec(1) P=0.0, atAll grad(P)=(1.0,0.0),
    at(isLine1&(P0-5.0)<eps) {
        boolean isPL1=true,
        at(isLine2&(P0-5.0)<eps) {
          boolean isPL2=true,atOther isPL2=false
          boolean temp,
          conn(isPL1,isPL2,temp),
          alt(isP,temp,line)}}}}
```




**Local coordinate system.**

**Propagate scalars.**

**Use other constructions.**

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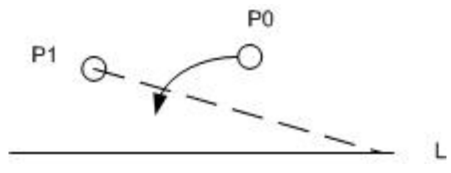
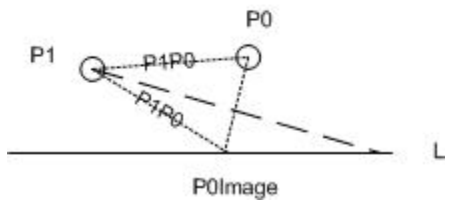
# Nagpal Operator(5): PontoL

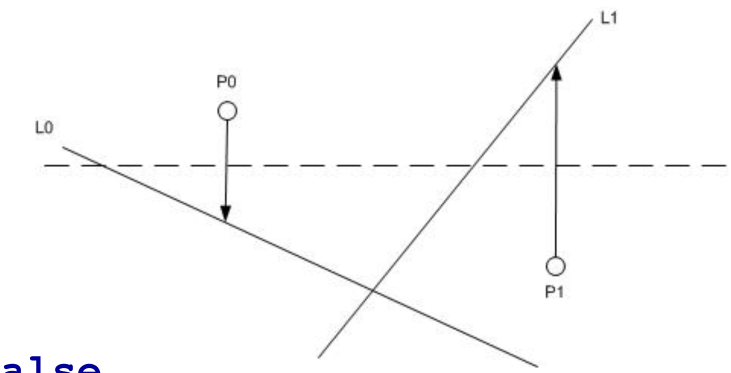


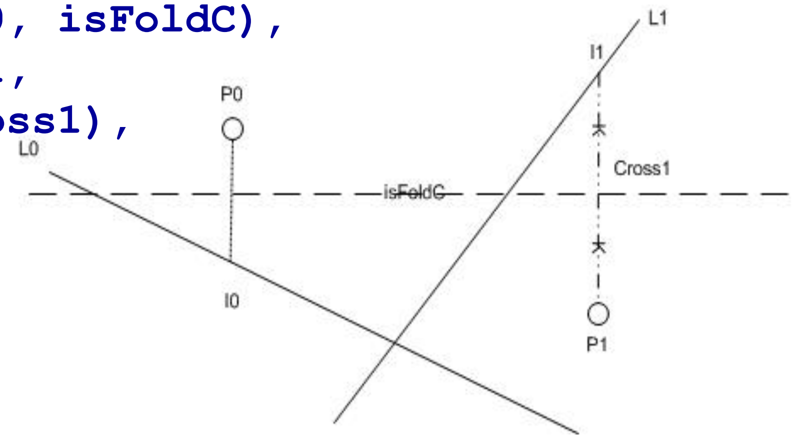

```
agent bisect(boolean isP0,
             boolean isP1,
             boolean isLine,
             boolean line) {
  at(isP0) {
    vec(1) P0=0.0, atall grad(P0)=(1.0,0.0),
    at(isP1) {
        vec(1) P1P0=P0, atAll grad(P1P0)=0.0,
        vec(1) P1=0.0, atAll grad(P1)=(1.0,0.0),
        at(isLine&(P1-P1P0)<eps) {
          boolean isP0Image=true,
          boolean temp, conn(isP0,isP0Image,temp),
          alt(isP1,temp,line)}}}}
```



# Nagpal Operator(6): P0P1ontoL0L1

```
agent lineToLines(boolean isP0,
                  boolean isP1,
                  boolean isL0,
                  boolean isL1,
                  boolean isFold) {
    at (isL0) {
        boolean isI0=true, atOther isI0=false,
        boolean isFoldC, perp(isP0, isI0, isFoldC),
        boolean isAlt1, boolean isCross1,
        alt(isP1, isFoldC, isAlt1, isCross1),
         at(isAlt1&isL1) {
             vec(1) orig=0.0,
             atAll grad(orig)=(1.0,0.0),
              at(isCross1) {
                vec(1) K = orig,
                atAll grad(cross1D)=0.0,
                at(isP1&norm(orig-2*K)<eps)
                  atAll isFold = isFoldC
          }}}
```



# Flocking

# How do u realize this on Progg Matter?

- **Work in progress!**
- **Basic intuitions**
  - Require propagation over space takes time.
  - Dilate time, dilate space.
  - Try establishing computational substrate has, at each point, same velocity of flow (in a particular direction) over time, +/- delta, *with some probability p.*
  - Therefore from each point, sufficiently widely spaced waves are guaranteed to arrive at all other points in sequence.

# Conclusion

- **We believe biological system modeling and analysis will be a very productive area for constraint programming and programming languages**

- **Handle continuous/discrete space+time**
- **Handle stochastic descriptions**
- **Handle models varying over many orders of magnitude**
- **Handle symbolic analysis**
- **Handle parallel implementations**

# HCC references


– Gupta, Jagadeesan, Saraswat "Computing with Continuous Change", Science of Computer Programming, Jan 1998, 30 (1—2), pp 3--49
– Saraswat, Jagadeesan, Gupta "Timed Default Concurrent Constraint Programming", Journal of Symbolic Computation, Nov-Dec1996, 22 (5—6), pp 475-520.
– Gupta, Jagadeesan, Saraswat "Programming in Hybrid Constraint Languages", Nov 1995, Hybrid Systems II, LNCS 999.
– Alenius, Gupta "Modeling an AERCam: A case study in modeling with concurrent constraint languages", CP'98 Workshop on Modeling and Constraints, Oct 1998.

# Controlling Cell division: The p53-Mdm2 feedback loop

- **1/ $[p53]' = [p53]_0 - [p53]*[Mdm2]*deg - d_{p53}*[p53]$**
- **2/ $[Mdm2]' = p1 + p2_{max}*(I^n)/(K^n + I^n) - d_{Mdm2}*[Mdm2]$**
  - I is some intermediary unknown mechanism; induction of [Mdm2] must be steep, n is usually > 10.
  - May be better to use a discontinuous change?
- **3/ $[I]' = a*[p53] - k_{delay}*I$**
  - *This introduces a time delay between the activation of p53 and the induction of Mdm2. There appears to be some hidden "gearing up" mechanism at work.*
- **4/ $a = c_1*sig/(1 + c_2*[Mdm2]*[p53])$**
- **5/ $sig' = -r*sig(t)$**
  - Models initial stimulus (signal) which decays rapidly, at a rate determined by repair.
- **6/ $deg = deg_{basal} - [k_{deg}*sig - thresh]$**
- **7/ $thresh' = -k_{damp}*thresh*sig(t=0)$**

*Lev Bar-Or, Maya et al "Generation of oscillations by the p53-Mdm2 feedback loop..", 2000*

# The p53-Mdm2 feedback loop

- **Biologists are interested in:**
  - Dependence of amplitude and width of first wave on different parameters
  - Dependence of waveform on delay parameter.
- **Constraint expressions on parameters that still lead to desired oscillatory waveform would be most useful!**

- **There is a more elaborate model of the kinetics of the G2 DNA damage checkpoint system.**
  - 23 species, rate equations
  - Multiple interacting cycles/ pathways/regulatory networks:
    - Signal transduction
    - MPF
    - Cdc25
    - Wee1

*Aguda "A quantitative analysis of the kinetics of the G2 DNA damage checkpoint system", 1999*

# Integration of symbolic reasoning techniques

- **Use state of the art constraint solvers**
  - ICS from SRI
  - Shostak combination of theories (SAT, Herbrand, RCF, linear arithmetic over integers).
- **Finite state analysis of hybrid systems**
  - Generate code for HAL
- **Predicate abstraction techniques.**
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