

## 15-853: Algorithms in the Real World

Computational Biology III  
- Multiple Sequence Alignment  
- Sequencing the Genome

15-853

Page 1

## Multiple Alignment

A C T \_ G T A  
A C A C G T T  
A G T G \_ T A  
C C \_ G C T A

Goal: match the “maximum” number of aligned pairs of symbols.

### Applications:

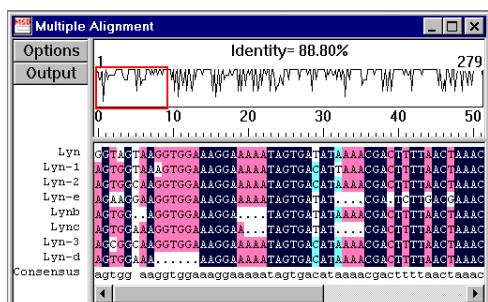
- Assembling multiple noisy reads of fragments of sequences
- Finding a canonical among members of a family and studying how the members differ

The problem is NP-hard

15-853

Page 2

## Example Output



Output from typical multiple alignment software  
DNAMAN (using [ClustalW](#))

15-853

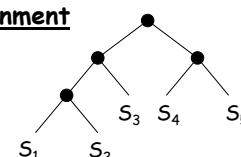
Page 3

## Scoring Multiple Alignments

1. Distance from consensus  $S_c$ :  $D = \sum_{S_i \in S} D(S_i, S_c)$

2. Pairwise distances:  $D = \sum_{S_i \in S} \sum_{S_j \in S / S_i} D(S_i, S_j)$

### Evolutionary Tree Alignment



$$D = D(S_1, S_2) + D(S_4, S_5) + D(S_{12}, S_3) + D(S_{123}, S_{45})$$

15-853

Page 4

## Approaches

**Dynamic programming:** optimal, but takes time that is exponential in  $p$

**Center Star Method:** approximation

**Clustering Methods:** also called iterative pairwise alignment. Typically an approximation.  
Many variants, many software packages

15-853

Page 5

## Using Dynamic Programming

For  $p$  sequences of length  $n$  we can fill in a  $p$ -dimensional array in  $n^p$  time and space.

For example for  $p = 3$ :

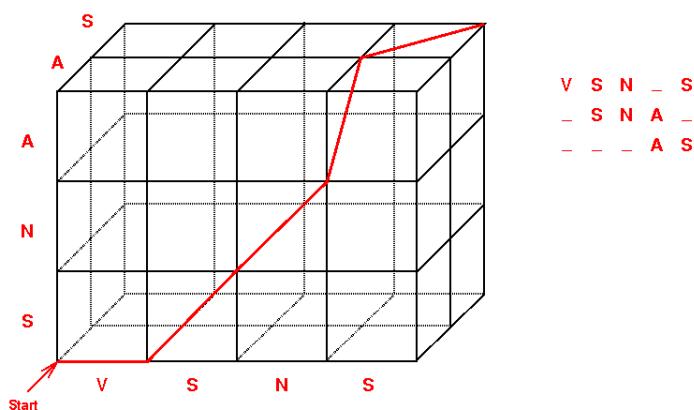
$$D_{ijk} = \min \begin{cases} D_{i-1, j-1, k-1} + d(a_i, b_j, c_k) \\ D_{i-1, j-1, k} + d(a_i, b_j, \_) \\ D_{i-1, j, k} + d(a_i, \_, c_k) \\ \dots \end{cases} \quad 7 \text{ cases}$$

where  $d(a, b, c) = d(a, b) + d(b, c) + d(a, c)$   
assuming the pairwise distance metric.  
Takes time exponential in  $p$ . Perhaps OK for  $p = 3$

15-853

Page 6

## Example



## Optimization

As in the case of pairwise alignment we can view the array as a graph and find shortest paths (or heaviest paths).

Used in a program called MSA.

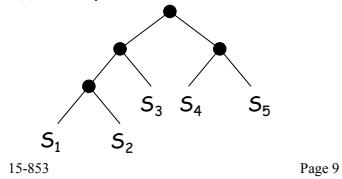
Can align 6 strings consisting of 200 bp each in a “practical” amount of time.

15-853

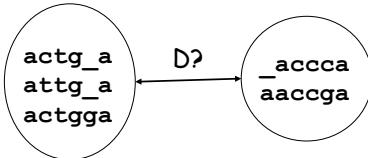
Page 8

## Using Clustering

1. Compute  $D(S_i, S_j)$  for all pairs
2. Bottom up cluster
  - I. All sequences start as their own cluster
  - II. Repeat
    - a) find the two “closest” clusters and join them into one
    - b) Find best alignment of the two clusters being joined



## Distances between Clusters



Could use difference between consensus.  
A popular technique is called the “Unweighted Pair-Group Method using arithmetic Averages” (UPGMA).  
It takes the average of all distances among the two clusters.  
Implemented in Clustal and Pileup

15-853

Page 10

## Summary of Matching

### Types of matching:

- **Global:** align two sequences A and B
- **Local:** align A with any part of B
- **Multiple:** align k sequences (NP-complete)

### Cost models

- **LCS** and **MED**
- **Scoring matrices:** Blosum, PAM
- **Gap cost:** affine, general

### Methods

- **Dynamic programming:** many optimizations
- **“Fingerprinting”:** hashing of small seqs. (approx.)
- **Clustering:** for multiple alignment (approx.)

15-853

Page 11

## Sequencing the Genome

One of the great achievements of the 21<sup>st</sup> century.

15-853

Page 12

## Tools of the Trade

### Cutting:

Arber, Nathans, and Smith, **Nobel Prize in Medicine** (1978) for “the discovery of restriction enzymes and their application to problems of molecular genetics”.

### Copying:

Mullis, **Nobel Prize in Chemistry** (1993) for “his invention of the polymerase chain reaction (PCR) method”

### Reading: (sequencing)

Gilbert and Sanger, **Nobel Prize in Chemistry** (1980) for “contributions concerning the determination of base sequences in nucleic acids”

15-853

Page 13

## Cutting

### Cutting:

- Restriction Enzymes:  
Cut at particular sites, e.g. ACTTCTAGAT
- Chemical, physical or radiation cuts  
Cut at random locations

15-853

Page 14

## Copying

### Copying:

#### **Cloning a strand of DNA**

- Cosmids: clones sequences up to 40K bps
- BAC, PAC: up to about 200K bps
- YAC (yeast artificial chromosomes): up to 1 M

#### **Copying between two specific sites**

- PCR (polymerase chain reaction): 500 bps

15-853

Page 15

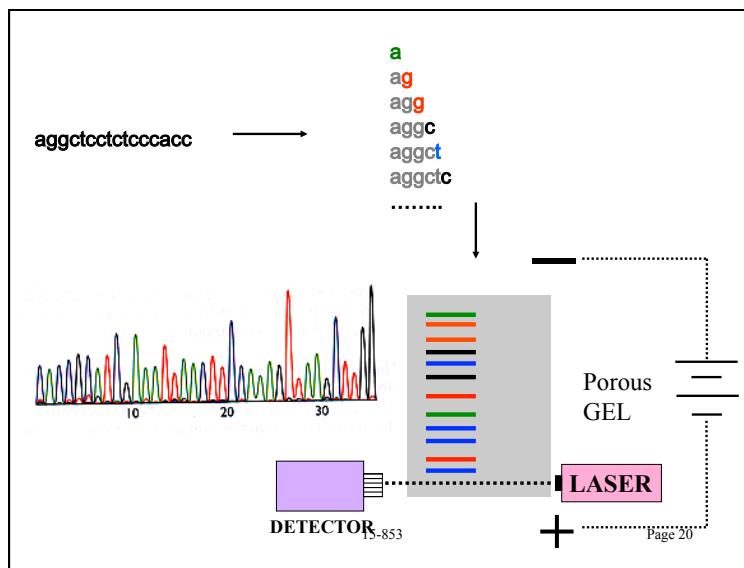
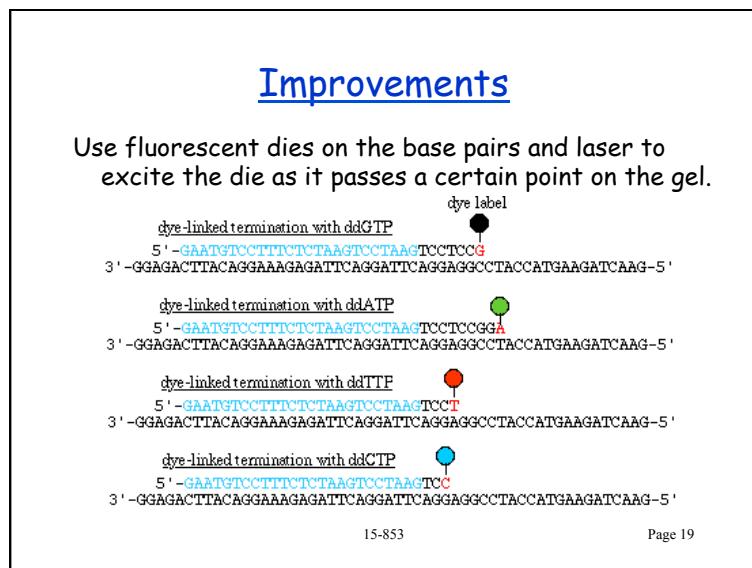
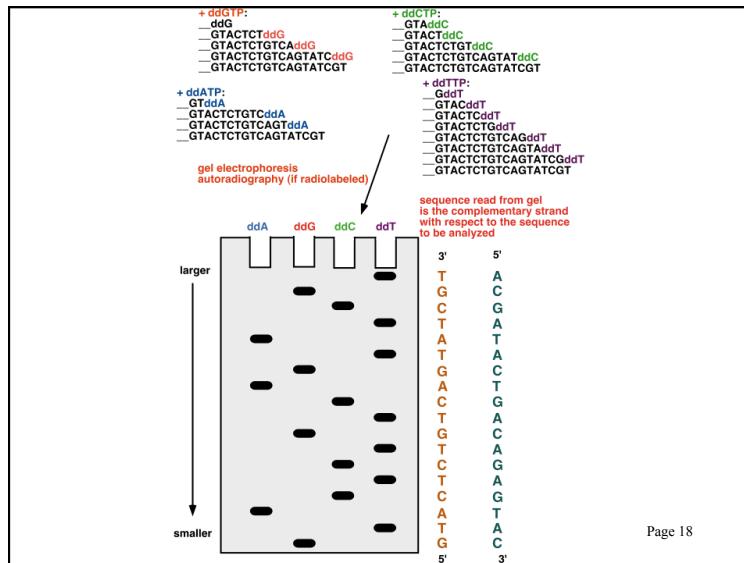
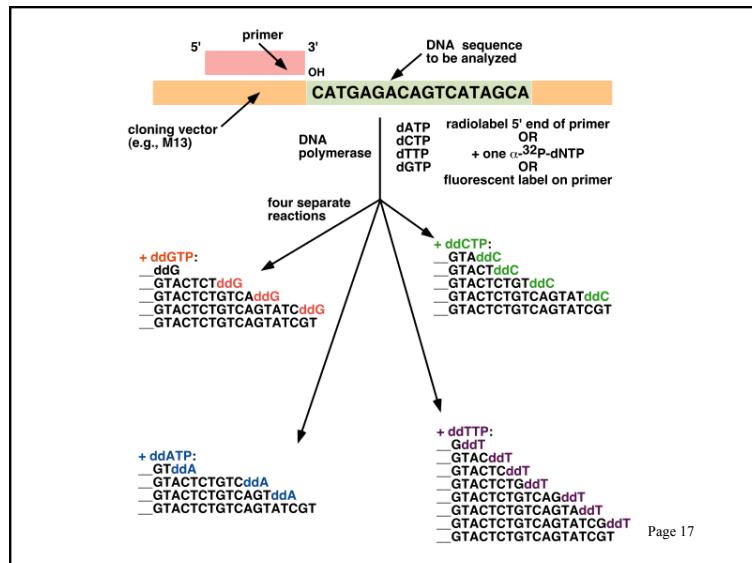
## Reading using lengths

Can use special base-pairs that stop growth: DDC, DDA, DDT, DDG. (terminator bases)

Will generate all prefixes that end in A, T, C or G.

15-853

Page 16





ABI 3700 sequencer

15-853

Page 21

## History of Sequencing

1971 Nobel prize for restriction enzymes

1973 First recombinant DNA

1980 Nobel prize for DNA sequencing

1988 Congress establishes Genbank

1995 First genomic sequence

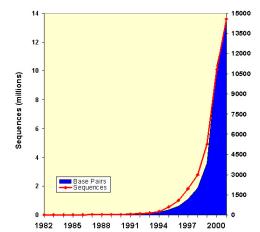
1998 First multicellular organism

2000 Fly genome

2000 First plant genome

2001 Human genome

2003 Mouse genome



22 million sequences

28 billion base pairs

Page 22

## Sequencing the Whole Genome

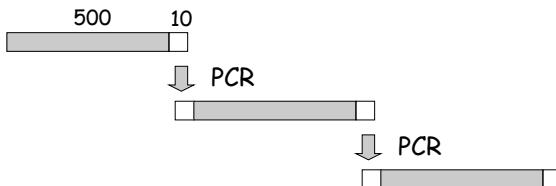
Problem: we only know how to sequence about 500 bps at a time in the lab.

1. Linear sequencing
2. The shotgun method
3. Hierarchical shotgun method
4. Whole genome and double-barreled shotgun methods

15-853

Page 23

## Linear Sequencing



Each step takes too long. Requires "wet" runs.  
 e.g. if each step took 4 hours, sequencing the human genome would take  
 $4 \times 3 \times 10^9 / 500 \text{ hours} = 3000 \text{ years}$   
 Also no interesting Computer Science ☺

15-853

Page 24

## The Shotgun Method

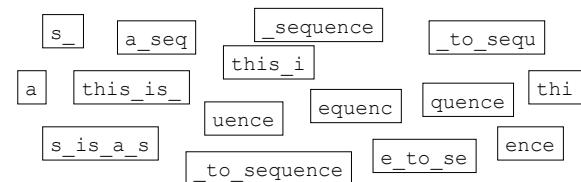
1. Make multiple copies of the sequence.
2. Randomly break sequences into parts (e.g. using radiation or chemicals).
3. Throw away parts that are too small or too large.
4. Read about 500bp from the end of each part
5. Try to put the information together to reconstruct the original sequence

15-853

Page 25

## Example

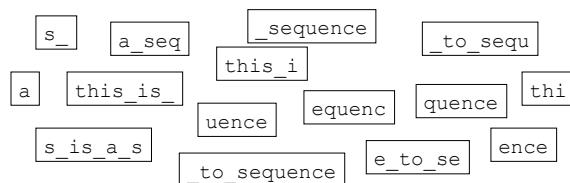
this\_is\_a\_sequence\_to\_sequence  
this\_is\_a\_sequence\_to\_sequence  
this\_is\_a\_sequence\_to\_sequence  
this\_is\_a\_sequence\_to\_sequence



15-853

Page 26

## Example

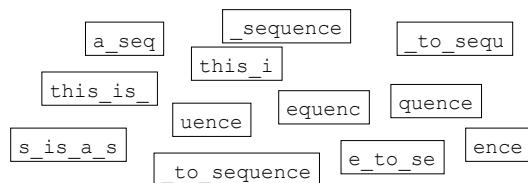


Remove strands that are too short (or too long)

15-853

Page 27

## Example

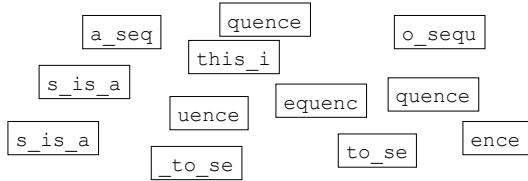


Sequence k characters from each (e.g. 6), from either end.

15-853

Page 28

### Example

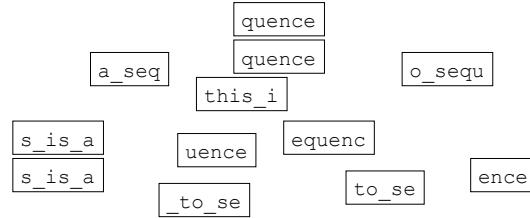


Find overlaps

15-853

Page 29

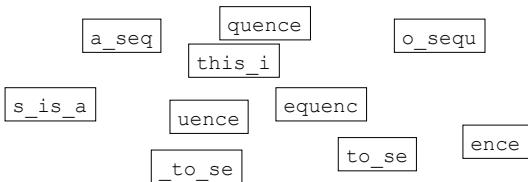
### Example



15-853

Page 30

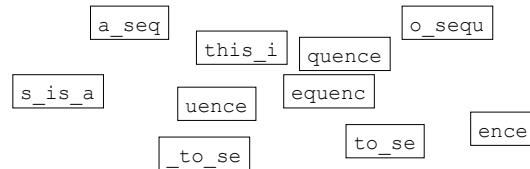
### Example



15-853

Page 31

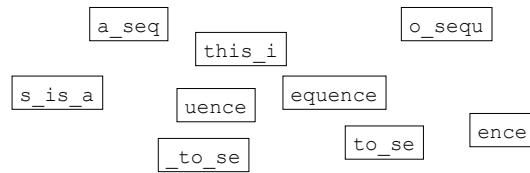
### Example



15-853

Page 32

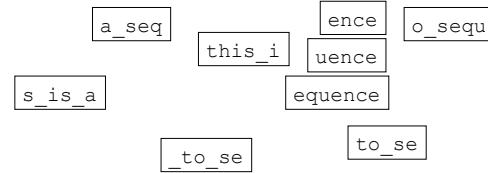
### Example



15-853

Page 33

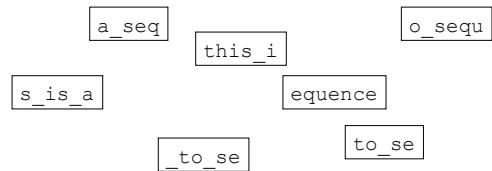
### Example



15-853

Page 34

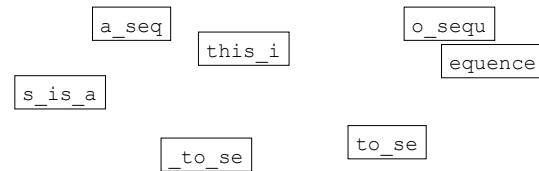
### Example



15-853

Page 35

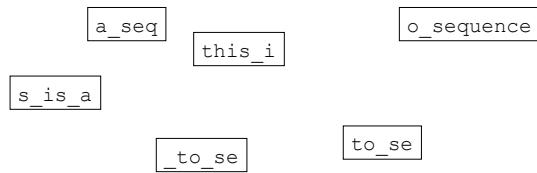
### Example



15-853

Page 36

### Example



15-853

Page 37

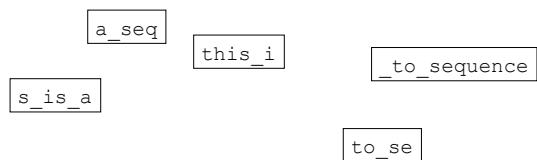
### Example



15-853

Page 38

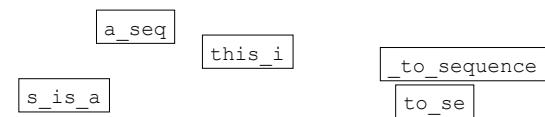
### Example



15-853

Page 39

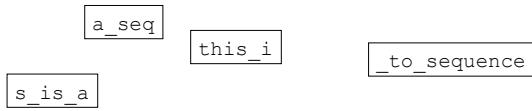
### Example



15-853

Page 40

## Example



15-853

Page 41

## Example



15-853

Page 42

## Example

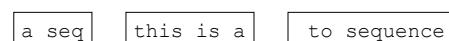


Having a single character overlap might not be enough to assume they overlap.

15-853

Page 43

## Example



15-853

Page 44

## Example

a\_seq    this\_is\_a    \_to\_sequence

We are left with **gaps**, and unsure matches.

Each covered region (e.g. this\_is\_a) is called a **contig**

Is there a systematic way to find or even define a  
“best solution”?

15-853

Page 45

## The SS Problem: an attempt

**The shortest superstring problem:** given a set of strings  $s_1, s_2, \dots, s_n$  find the shortest string  $S$  that contains all  $s_i$ .

NP-Hard, but can be reduced to TSP and solved approximately (nearly optimally in practice).

Even if easy to solve, are we done?

Our example gives:

this\_is\_a\_seq\_to\_sequence  
but this is the best we can do given the data.  
This problem is caused by repeats.  
Other problems?

15-853

Page 46

## Issues

In practice the data is noisy.

- Reads have up to a 1% error rate
- Samples could have contaminants
- Fragments can sometimes join up

The reads could be in either direction (front-to-back or back-to-front). Cannot distinguish.

15-853

Page 47

## Assembly in Practice

gatcgat\_ga  
attgactactatg

### Score all suffix-prefix pairs

- This can use a variant of the global alignment prob.  
It is the most expensive step ( $n^2$  scores).

### Repeat:

- Select best score and check for consistency
- If score is too low, quit
- If there is a good overlap, merge the two.

### Determine consensus:

- We know the ordering among strands, but since matches are approximate, we need to select bps.  
Can use, e.g., multiple alignment over windows.

15-853

Page 48

## Some Programs for Assembly

Phrap  
SEQAID  
CAP  
TIGR  
Celera assembler  
ARACHNE

After using one of these programs to generate a set of “contigs” with some gaps, one can use the linear method to fill in the gaps (assuming they are small).

`atgattagccatgtacgttt` `tccagcatcccagttacgtttatgcac` `tttagccaga`

15-853

Page 49

## Sequencing the Whole Genome

**Problem:** we only know how to sequence about 500 bps at a time in the lab.

1. Linear sequencing
2. The shotgun method
- 3. Hierarchical shotgun method
4. Whole genome and double-barreled shotgun methods

15-853

Page 50

## Shotgun on the Whole Genome?

### Problems:

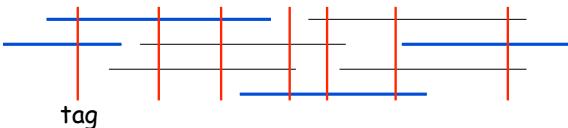
- Computationally very expensive
- 50% of genome consist of repeats. Causes major problems.
- Hard to partition work among multiple labs.

15-853

Page 51

## Hierarchical Shotgun

1. Generate clone Libraries (100K - 1M per clone)
2. Order the clones by finding “tags” that overlap multiple clones. Use these for ordering.
3. Identify a set of clones that cover the whole length (minimum tiling path)
4. Use shotgun technique on each identified clone
5. Put the results together.



15-853

Page 52

## 1. Clone Libraries

A **"BAC" library** will contain sequences of about 200K bps each. These can be cloned using "BAC Vectors" (Bacterial Artificial Chromosome)

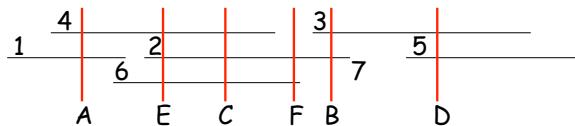
A **"YAC" library** will contain sequences of about 1M bps each. These can be cloned using "YAC Vectors" (Yeast Artificial Chromosome)

These are typically stored at a common site and can be ordered. Many can be purchased from companies.

15-853

Page 53

## 2. Ordering Clones



We have the clones, but we don't know their order or how they overlap.

Pick random small sequences that only appear once in one location covered by the library.

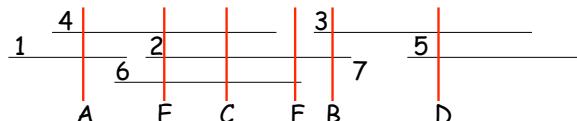
These are called STS (Sequence Tagged Sites)

Figure out which clones contain which STSs using PCR (use tag site to start copy...will only copy of the sequence contains the site).

15-853

Page 54

## 2. Ordering Clones (cont.)



**Goal:** Reorder the columns so that all the 1s in each row are contiguous.

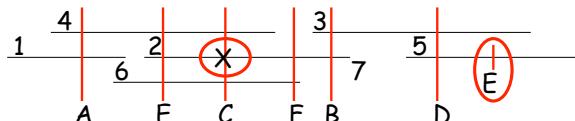
Can be done in  $O(n)$  time, where  $n$  is the number of entries in the array.

**But!!!**, what about errors?

15-853

Page 55

## 2. Ordering Clones (cont.)



	A	B	C	D	E	F
1	1	0	0	0	0	0
2	0	1	1	0	1	1
3	0	1	0	1	0	0
4	1	0	1	0	1	0
5	0	0	0	1	0	0
6	0	0	1	0	1	1

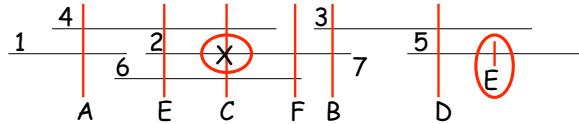


	A	E	C	F	B	D
1	1	0	0	0	0	0
2	0	1	0	1	1	0
3	0	0	0	0	1	1
4	1	1	1	0	0	0
5	0	1	0	0	0	1
6	0	0	1	1	0	0

15-853

Page 56

## 2. Ordering Clones (cont.)



Find ordering that minimizes the number of zero-one and one-zero transitions (i.e. errors).

This is NP-hard, but can be posed as a Traveling Salesman Problem (TSP).

Any ideas?

15-853

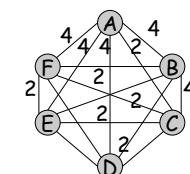
Page 57

## 2. Ordering Clones (cont.)

Create graph with one vertex per STS.

Edge weights = hamming distance (number of bits that differ).

	A	B	C	D	E	F
1	1	0	0	0	0	0
2	0	1	0	0	1	1
3	0	1	0	1	0	0
4	1	0	1	0	1	0
5	0	0	0	1	1	0
6	0	0	1	0	1	1



15-853

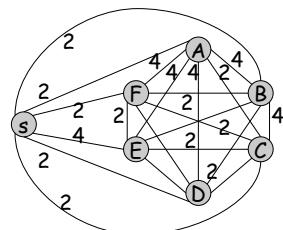
Page 58

## 2. Ordering Clones (cont.)

Add in source (s) node with weights equal to number of 1s in each row.

Solve TSP. Answer gives min number of transitions.

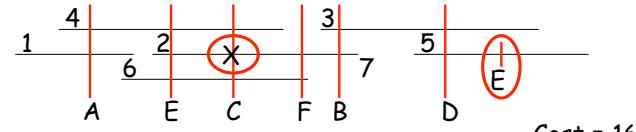
	A	B	C	D	E	F
1	1	0	0	0	0	0
2	0	1	0	0	1	1
3	0	1	0	1	0	0
4	1	0	1	0	1	0
5	0	0	0	1	1	0
6	0	0	1	0	1	1



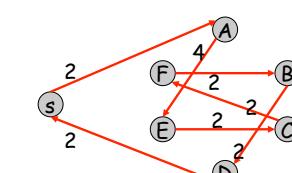
15-853

Page 59

## 2. Ordering Clones (cont.)



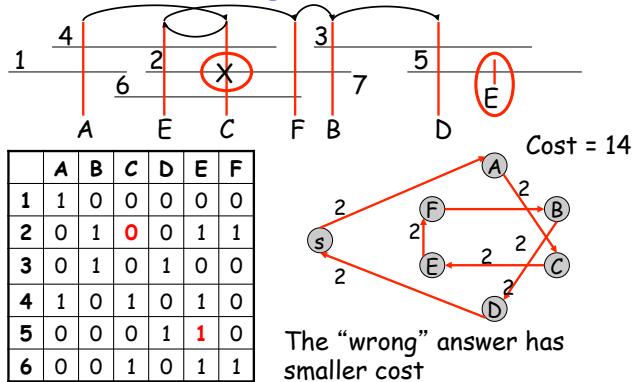
	A	B	C	D	E	F
1	1	0	0	0	0	0
2	0	1	0	0	1	1
3	0	1	0	1	0	0
4	1	0	1	0	1	0
5	0	0	0	1	1	0
6	0	0	1	0	1	1



15-853

Page 60

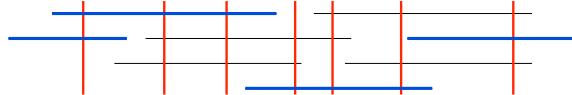
## 2. Ordering Clones (cont.)



15-853

Page 61

## 3. Find “Minimum Tiling Path”



**Minimum Tiling Path:** Find a set of clones that cover the whole length and for which the total number of bps is minimized.

Can be posed as a shortest path problem.

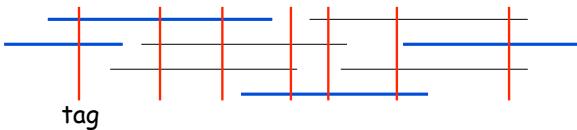
Any ideas?

15-853

Page 62

## Hierarchical Shotgun (revisited)

1. Generate clone Libraries (100K - 1M per clone)
2. Order the clones by finding “tags” that overlap multiple clones. Use these for ordering.
3. Identify a set of clones that cover the whole length (minimum tiling path)
4. Use shotgun technique on each identified clone
5. Put the results together.



15-853

Page 63

## Celera’s Method

### Whole genome shotgun:

Use shotgun method on whole genome.

Use **double-barreled** approach: some sequences of known length (e.g. 2-5K) are sequenced at both ends. These can be used to bridge across repeats.



In practice they used some mapping (hierarchical) data from the NIST effort, which was freely available. This was needed to deal with long repeats.

15-853

Page 64