$$0 \leq \|x^{k+1} - x^*\|^2 \leq \|x^k - x^*\|^2 - 2t_k\left[f(x^k) - f(x^*)\right] + t_k^2\|g^k\|^2$$

$$\leq \|x^0 - x^*\|^2 - 2\sum_{i=1}^{k} t_i\left[f(x^i) - f(x^*)\right] + L^2 \sum_{i=1}^{k} t_i^2$$

$$\leq D^2 - 2\sum_{i=1}^{k} t_i\left[f(x^i) - f(x^*)\right] + L^2 \sum_{i=1}^{k} t_i^2$$

$$\Rightarrow \quad 2\sum_{i=1}^{k} t_i\left[f(x_i) - f(x^*)\right] \leq D^2 + L^2 \sum_{i=1}^{k} t_i^2$$

$$\Rightarrow \quad \frac{\cancel{2}\sum_{i=1}^{k} t_i\left[f(x_i) - f(x^*)\right]}{\cancel{2}\sum_{i=1}^{k} t_i} \leq \frac{D^2 + L^2 \sum_{i=1}^{k} t_i^2}{2\sum_{i=1}^{k} t_i}$$

$$\vee\!|$$

$$f(\bar{x}) - f(x^*) \leq \frac{D^2 + L^2 \sum_{i=1}^{k} t_i^2}{2\sum_{i=1}^{k} t_i} \rightarrow 0 \quad \Leftrightarrow \quad \begin{array}{l} \sum t_i \rightarrow \infty \\ \sum t_i^2 < \infty \\ t_{i+1} \leq t_i \end{array}$$

$$\bar{x} = \frac{\sum_{i=1}^{k} t_i x^i}{\sum_{i=1}^{k} t_i}$$

$$\underset{x}{\operatorname{argmin}} \ \cancel{\Sigma}\ f(x_k) + g_k \cdot (x - x_k) + \frac{\|x - x_k\|^2}{2t_k}$$

$$= \underset{x}{\operatorname{argmin}} \ f(x_k) + g_k \cdot (x - x_k) \cancel{+} - \frac{x^T x_k}{t_k} + \frac{x^T x}{2t_k}$$

$$= \underset{x}{\operatorname{argmin}} \ t_k\left[f(x_k) + g_k \cdot (x - x_k)\right] - x^T(x_{k-1} - t_{k-1}g_{k-1}) + \frac{x^T x}{2}$$

$$= \underset{x}{\operatorname{argmin}} \ \frac{\sum_{i=1}^{k} t_i\left[f(x_i) + g_i \cdot (x - x_i)\right]}{\sum_{i=1}^{k} t_i} + \frac{\|x\|^2}{2\sum_{i=1}^{k} t_i}$$

$$\text{prox}_t(x) = \arg\min_z \frac{1}{2t}\|x-z\|^2 + h(z)$$

$$= \arg\min_z \cancel{\frac{x^Tx}{2t}} - \frac{x^Tz}{t} + \frac{z^Tz}{2t} + h(z)$$

$$\frac{1}{t} = \frac{\mu_k}{k}, \quad \frac{x}{t} = \hat{g}_k \Rightarrow x = \frac{k\hat{g}_k}{\mu_k}$$

v^* $\min f(x)$

$$f(x) = \max_g g^Tx - f^*(g) \qquad \text{[By convexity]}$$

$$v^* = \min_{\|x\| \leq D} f(x) \qquad \text{[Assume D large enough]}$$

$$= \min_{\|x\| \leq D} \max_g g^Tx - f^*(g)$$

$$= \max_g \min_{\|x\| \leq D} g^Tx - f^*(g) \qquad \text{[Strong Duality]}$$

$$= \max_g -f^*(g) - D\max_{\|z\| \leq 1} -z^Ty$$

$$= \max_g -f^*(g) - D\|-g\|_*$$

$$0 = v^* - v^* = \min_x f(x) - \max_g -f^*(g) - D\|-g\|_*$$

$$= \min_{x,g} f(x) + f^*(g) + D\|-g\|_*$$