$$\min_{X} \ \frac{1}{2}\sum_{i=1}^{n}(y_i - x_i)^2 + \lambda \sum |x_i - x_{i+1}|$$

$$x_i - y_i + \lambda(s_i - s_{i-1}) = 0 \qquad i = 1, \dots, n$$

$$s_i \in \partial |x_i - x_{i+1}| \qquad s_0 = s_n = 0.$$

$$\lambda = 0: \qquad x^*(0) = y$$

small $\lambda > 0$:

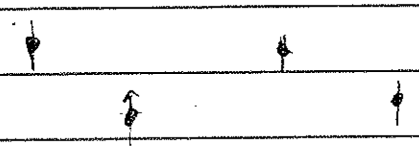
$$x_i^*(\lambda) = y_i - \lambda\left(\text{sign}(y_i - y_{i+1}) - \text{sign}(y_{i-1} - y_i)\right) \Leftarrow$$

$$x_i^* - y_i + \lambda(s_i - s_{i-1})$$

$$= \lambda\left[s_i - \text{sign}(y_i - y_{i+1}) - \left(s_{i-1} - \text{sign}(y_{i-1} - y_i)\right)\right]$$

$$= 0$$

$$\text{sign}(x_i^*(\lambda) - x_{i+1}^*(\lambda)) = \text{sign}(y_i - y_{i+1}) \quad \forall i$$



$$x_i^*(0) = y_i, \qquad x_i^*(\lambda) = a_i - \lambda b_i$$

D oriented incidence matrix

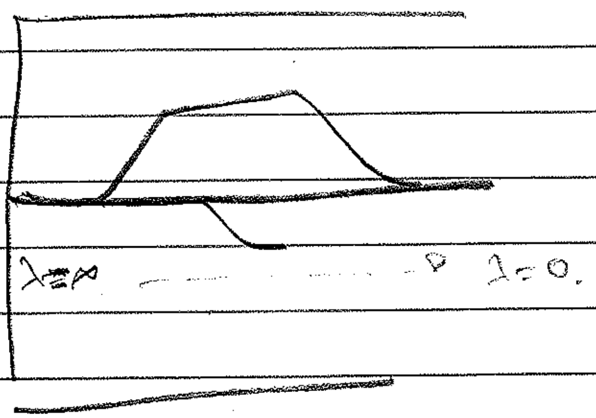
$E \times V$

DD^T is diagonally dominant

CI^Ts

$f(x) + \lambda g(x)$

$A^+(A^+)^T$



3^n

$\frac{1}{2}\|y - Ax\|_2^2 + \lambda\|x\|_2^2$ ridge regression

$x^*(\lambda) = (A^TA + \lambda I)^{-1} A^Ty$

$$a_i - b_i\lambda = a_{i+1} - b_{i+1}\lambda$$

$$\lambda = \frac{a_i - a_{i+1}}{b_i - b_{i+1}} \quad \text{paths } i \text{ and } i+1 \text{ cross}$$

$$\lambda_1 = \min_i \left\{ \frac{a_i - a_{i+1}}{b_i - b_{i+1}} \right\}$$

$$\min_x \ \frac{1}{2} \sum_i \sum_{j \in g_i} (y_j - x_{g_i})^2 + \lambda \sum_i | x_{g_i} - x_{g_{i+1}} |$$

$$0 = |g_i| \cdot x_{g_i} - \sum_{j \in g_i} y_j + \lambda (s_{g_i} - s_{g_{i-1}})$$

$x_i^*(\lambda_1)$ solves this at λ_1

for $\lambda > \lambda_1$ $\quad x^*_{g_i}(\lambda) = a_i - b_i\lambda$

$$|g_i| (a_i - b_i\lambda) - \sum_{j \in g_i} y_j + \lambda (s_{g_i} - s_{g_{i-1}}) = 0$$

$$a_i = \frac{1}{|g_i|} \sum_j y_j$$

$$b_i = \frac{1}{|g_i|} \Big[\operatorname{sign}(x^*_{g_i}(\lambda_1) - x^*_{g_{i+1}}(\lambda_1)) - \operatorname{sign}(x^*_{g_{i-1}}(\lambda_1) - x^*_{g_i}(\lambda)) \Big]$$

$$\underbrace{\min_x \ \frac{1}{2} \sum (y_i - x_i)^2}_{\|y - x\|_2^2} + \lambda \underbrace{\sum_{(i,j) \in E} | x_i - x_j |}_{\|y - Ax\|_2^2}$$