

$x$  c.w. minimizer, then for any  $y$

$$f(y) - f(x) = g(y) - g(x) + \sum h_i(y_i) - h_i(x_i)$$

$$\geq \nabla g(x)^T (y - x) + \sum h_i(y_i) - h_i(x_i)$$

$$= \underbrace{\sum \nabla_i g(x) (y_i - x_i) + h_i(y_i) - h_i(x_i)}_{\geq 0}$$

$$g(x) + \sum h_j(x_j)$$

$$\nabla_i g(x) + \partial h_i(x_i) \ni 0$$

$$-\nabla_i g(x) \in \partial h_i(x_i)$$

$$h_i(y_i) \geq h_i(x) - \nabla_i g(x) (y_i - x_i)$$

$$f(x) = \frac{1}{2} \|y - Ax\|_2^2$$

fix  $x_j$  all  $j \neq i$

$$\begin{aligned} 0 = \nabla_i f(x) &= A_i^T (Ax - y) \\ &= A_i^T (A_i x_i + A_{-i} x_{-i} - y) \end{aligned}$$

$$x_i = \frac{A_i^T (y - A_{-i} x_{-i})}{\|A_i\|^2} \quad i=1, \dots, n$$

$$x^+ = x - t A^T (Ax - y)$$

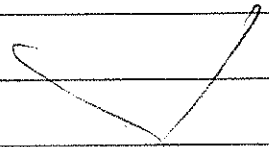
$$x_i = \frac{A_i^T (y - A_{-i} x_{-i})}{\|A_i\|^2}$$

$$\begin{aligned} &\downarrow \qquad \qquad \qquad \downarrow \\ &\qquad \qquad \qquad y - A x^{\text{old}} + A_i x_i^{\text{old}} \\ &\qquad \qquad \qquad \underbrace{\qquad \qquad \qquad}_{r} + A_i x_i^{\text{old}} \end{aligned}$$

$$= \frac{A_i^T r}{\|A_i\|^2} + x_i^{\text{old}}$$

$$\frac{1}{2} \|y - Ax\|^2 + \lambda \|x\|_1$$

$$\lambda \sum_{i=1}^n |x_i|$$



$$0 = A_i^T (Ax - y) + \lambda s_i \quad s_i \in \partial |x_i|$$

$$= \underbrace{A_i^T A}_{\|A_i\|^2} x_i + \underbrace{A_i^T A}_{\|A_i\|^2} x_{-i} - A_i^T y + \lambda s_i$$

$$x_i = \sum_{i=1}^n \frac{\lambda}{\|A_i\|^2} \left( \frac{A_i^T (y - A_{-i} x_{-i})}{\|A_i\|^2} \right)$$

$i=1, \dots, n$

$$x^+ = \sum_{\lambda/\epsilon} (x + \epsilon A^T (y - Ax))$$