x c.w. minimizer, then for any y

$$f(y) - f(x) = g(y) - g(x) + \sum h_i(y_i) - h_i(x_i)$$

$$\geq \nabla g(x)^T (y-x) + \sum h_i(y_i) - h_i(x_i)$$

$$= \sum \underbrace{\nabla_i g(x)(y_i - x_i) + h_i(y_i) - h_i(x_i)}_{\geq 0}$$

$$g(x) + \sum h_j(x_j)$$

$$\nabla_i g(x) + \partial h_i(x_i) \ni 0$$

$$-\nabla_i g(x) \in \partial h_i(x_i)$$

$$h_i(y_i) \geq h_i(x) - \nabla_i g(x)(y_i - x_i)$$

$$f(x) = \frac{1}{2}\|y - Ax\|_2^2$$

fix x_j all $j \neq i$

$$0 = \nabla_i f(x) = A_i^T(Ax - y)$$
$$= A_i^T(A_i x_i + A_{-i}x_{-i} - y)$$

$$x_i = \frac{A_i^T(y - A_{-i}x_{-i})}{\|A_i\|^2} \qquad i = 1, \ldots n$$

$$x^+ = x - t A^T(Ax - y)$$

$$x_i = \frac{A_i^T(y - A_{-i}x_{-i})}{\|A_i\|^2}$$

$$\underbrace{y - Ax^{old}}_{r} + A_i x_i^{old}$$
$$r + A_i x_i^{old}$$

$$= \frac{A_i^T r}{\|A_i\|^2} + x_i^{old}$$

$$\frac{1}{2}\|y - Ax\|^2 + \lambda \|x\|_1$$

$$\lambda \sum_{i=1}^{n} |x_i|$$

$$0 = A_i^T(Ax - y) + \lambda s_i \qquad s_i \in \partial |x_i|$$

$$= A_i^T A_i x_i + A_i^T A_{-i} x_{-i} - A_i^T y + \lambda s_i$$

$$x_i = S_{\frac{\lambda}{\|A_i\|^2}}\left(\frac{A_i^T(y - A_{-i}x_{-i})}{\|A_i\|^2}\right)$$

$i = 1, \ldots, n$

$$x^+ = S_{\lambda/t}\left(x + t A^T(y - Ax)\right)$$