$$\partial f^*(y) = \partial \max_x \{ y^T x - f(x) \}$$

$$= \text{conv} \bigcup_{x:\ f^*(y) = y^T x - f(x)} \{x\}$$

$$x \in \partial f^*(y) \qquad y \in \partial f(x)$$

$$x = \sum \alpha_i x_i$$
$$x_i:\ f^*(y) = y^T x_i - f(x_i)$$

$f$ strictly convex $\quad \nabla f^*(y) = \underset{x}{\text{argmin}}\ f(x) - y^T x$

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$$\min_x \ \tfrac{1}{2}\|y - Ax\|_2^2 + \lambda \|x\|_1$$

$$\min_{x,z} \ \tfrac{1}{2}\|y - z\|^2 + \lambda \|x\|_1 \quad \text{st } z = Ax$$

$$L(x, z, u) = \tfrac{1}{2}\|y - z\|^2 + \lambda\|x\|_1 + u^T(z - Ax)$$

$u^*$

wrt $z$ $\quad z - y + u^* = 0 \implies Ax^* = y - u^*$

wrt $x$ $\quad \lambda s = A^T u^*, \ s \in \partial \|x^*\|_1$

$$|A_i^T u^*| < \lambda \implies x_i^* = 0.$$

$$\min \|y-u\|^2 \quad \text{s.t.} \quad \underbrace{\|A^T u\|_\infty \le \lambda}_{u \in C = \{v : \|A^T v\|_\infty \le \lambda\}}$$

$$u^* = P_C(y)$$

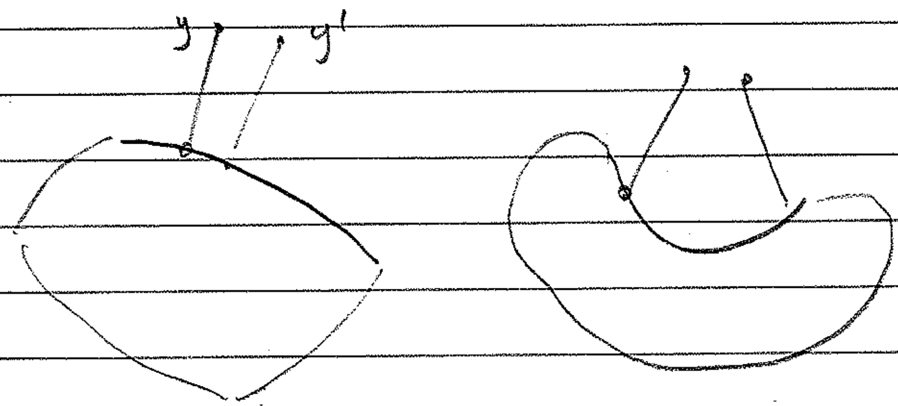
$$Ax^* = y - u^* = (I - P_C)(y)$$

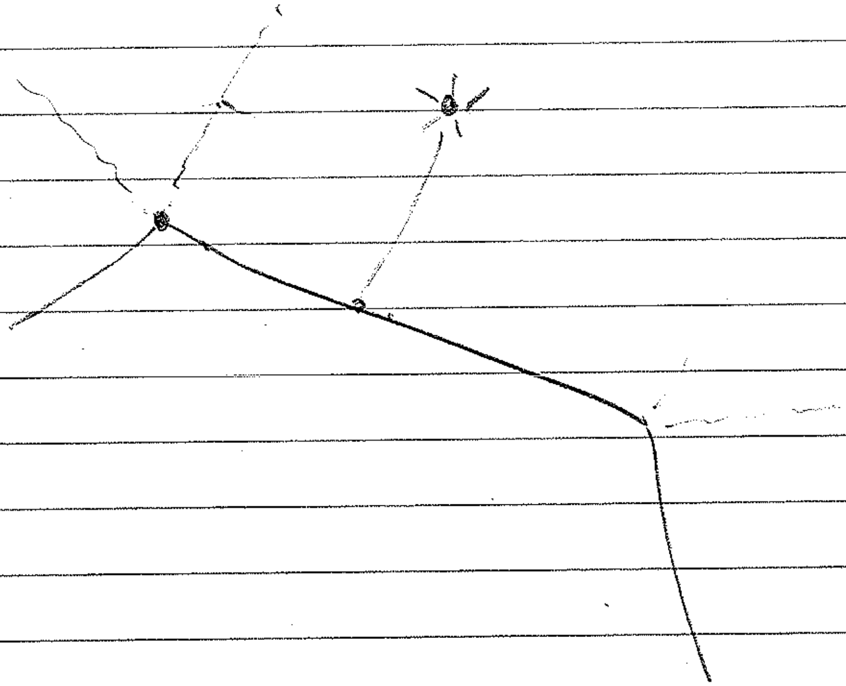
$$C = \bigcap_{i=1}^{p} \{u : A_i^T u \le \lambda\} \cap \{u : A_i^T u \ge -\lambda\}$$

polyhedron

$$C = \{u : \|A^T u\|_\infty \le \lambda\}$$
$$= \{u : A^T u \in \{v : \|v\|_\infty \le \lambda\}\}$$
$$= (A^T)^{-1}(\{v : \|v\|_\infty \le \lambda\})$$

$$\|Ax^*(y) - Ax^*(y')\| \le \|y - y'\|$$





$$f : A \to B$$

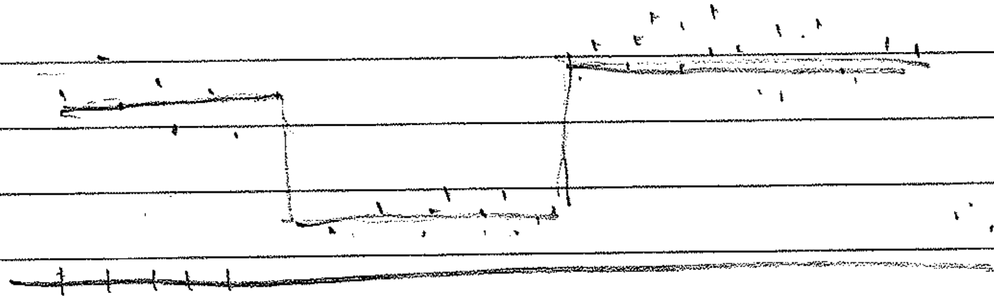
$$C \subseteq B \qquad f^{-1}(C) = \{x \in A : f(x) \in C\}$$

$$\text{null}(A_S^T) + \lambda (A_S^T)^{+} s$$

$$m_i = \max \left\{ \begin{array}{llll} \max & A_i^T u & \text{st.} & g(u) \geq \gamma \\ \min & A_i^T u & \text{st.} & g(u) \geq \gamma \end{array} \right\}$$

$$m_i < \lambda \Rightarrow |A_i^T u^*| < \lambda \Rightarrow x_i^* = 0.$$

$$m_i < \lambda \iff \text{SAFE RULE}$$



$$D = \begin{bmatrix} -1 & 1 & & & \\ & -1 & 1 & & \\ & & -1 & 1 & \\ & & & \ddots & \\ & & & -1 & 1 \end{bmatrix}$$

$$\|Dx\|_1 = \sum_{i=2}^{n} |x_i - x_{i+1}|$$

$$\min \|y-x\|^2 + \lambda \|Dx\|_1$$

$$\min \|y-x\|^2 + \lambda \|z\|_1 \qquad z = Dx$$

$$\text{prox}(z) = \min_u \|z-u\| \quad \text{s.t.} \quad \|u\|_\infty \le \lambda$$

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$$\min f(x) \quad \text{st} \quad Ax = b$$

$$L(x,u) = f(x) + u^T(Ax-b)$$

$$g(u) = f^*(-A^T u) \qquad \text{convex}$$

$$\partial g(u) = -A \partial f^*(-A^T u)$$

$$\partial g(u) = -A\,\partial f^*(-A^T u)$$

$$x \in \partial f^*(-A^T u)$$

$$\Longleftrightarrow \quad x \in \underset{z}{\operatorname{argmin}}\; f(z) + u^T A z$$

$$-Ax \in \partial g(u)$$

$u^{(0)}$

$$x^{(k)} \in \operatorname{argmin}\; f(z) + (u^{(k)})^T A z$$

$-Ax^{(k)}$ s.g.

$$\begin{aligned} u^{(k+1)} &= u^{(k)} - t_k(-Ax^{(k)} + b) \\ &= u^{(k)} + t_k(Ax^{(k)} - b) \end{aligned}$$

$$u^T A x = \sum u A_i^T x_i$$

$$A x = A_1 x_1 + \dots A_B x_B$$