$$\min \; f_i(x_i) - a_i v x_i$$
$$= f_i^*(a_i v)$$

$$\nabla f_i(x_i) = a_i v^*$$

$$\min_x \; f(x)$$

f^* constant

$$\min \; \frac{1}{2}\|y - Ax\|^2 + \lambda\|x\|_1$$

$$\Rightarrow \min \; \frac{1}{2}\|y - z\|^2 + \lambda\|x\|_1 \quad \text{s.t.} \quad z = Ax$$

$$\min_{x,z} \; \frac{1}{2}\|y - z\|^2 + \lambda\|x\|_1 + u^T(z - Ax)$$

$$\min_z \; \frac{1}{2}\|y\|^2 + \frac{1}{2}\|z\|^2 - (y-u)^T z \qquad \text{over } z$$

$$\frac{1}{2}\|y\|^2 - \frac{1}{2}\|y - u\|^2$$

$$\min_x \; \lambda\|x\|_1 - (A^T u)^T x \qquad \text{over } x$$

$$= \begin{cases} 0 & \|A^T u\|_\infty \le \lambda \\ -\infty & > \lambda \end{cases}$$

$$\max \ \tfrac{1}{2}\|y\|^2 - \tfrac{1}{2}\|y-u\|^2 \leftarrow$$
$$\text{st.} \quad \|A^T u\|_\infty \leq \lambda$$

$$\min \ \|y-u\|^2$$
$$\text{st} \quad \|A^T u\|_\infty \leq \lambda$$

$$L(x,z,u) = \tfrac{1}{2}\|y-z\|^2 + u^T(z - Ax)$$

$$0 = z - y + u^*$$

$$z^* = \boxed{Ax^* = y - u^*}$$

$$\min \ \tfrac{1}{2}\|y - Ax\|^2 + \lambda\|z\|_1 \quad \text{st.} \ x = z$$

$$f(x) + f^*(y) \geq x^T y$$

$$f^*(y) = \max_x \ y^T x - f(x)$$

$$\geq$$

$$\Rightarrow x \in \partial f^*(y) \iff y \in \partial f(x)$$
$$\iff f(x) + f^*(y) = x^T y.$$

$$\partial f^*(y) = \operatorname{conv}\Big(\bigcup_{\substack{x:\\ f^*(y) = y^T x - f(x)}} x \Big)$$

$$x \in \partial f^*(y) \iff x^T y = f(x) + f^*(y)$$
$$\iff x \text{ maximizes } y^T z - f(z) \text{ over } z$$

$$\nabla f^*(y) = \underset{x}{\operatorname{argmax}}\ y^T x - f(x)$$

$$f(x) = I_C(x) = \begin{cases} 0 & x \in C \\ \infty & x \notin C \end{cases}$$

$$f^*(y) = \max_{x \in C} y^T x = I_C^*(x)$$

$$f(x) = \|x\|$$
$$f^*(y) = \begin{cases} 0 & \text{if } \|y\|_* \le 1 \\ \infty & \text{if } \|y\|_* > 1 \end{cases}$$

$$-f^*(u) = \min_x \left(f(x) - u^T x\right)$$

$$\min_{x,z}\ f(x) + g(z) \quad \text{s.t. } x = z$$

$$f(x) + g(z) + u^T(z - x)$$
$$\min_{x,z}\ f(x) - u^T x + g(z) + u^T z$$

dual problem $\displaystyle \max_u\ -f^*(u) - g^*(-u)$

$$\min \; f(x) + I_C(x)$$

$$\max \; -f^*(u) - I_C^*(-u)$$

$$\min \; f(x) + \|x\|$$

$$\max \; -f^*(u) - I_{\{u : \|u\|_* \le 1\}}(-u)$$

$$\iff \max \; -f^*(u) \quad \text{s.t. } \|u\|_* \le 1$$

$$\min \; f(x) \quad \text{s.t. } \underset{u}{Ax \le b}, \; \underset{v}{Cx = d}$$

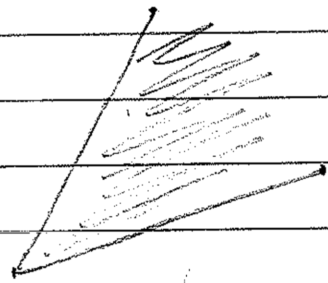
$$\max \; -f^*(-A^T u - C^T v) - b^T u - d^T v \quad u \ge 0.$$

$f^{**} = f$ if f closed & cvx

$$C = \{x : \|x\| \le 1\}$$

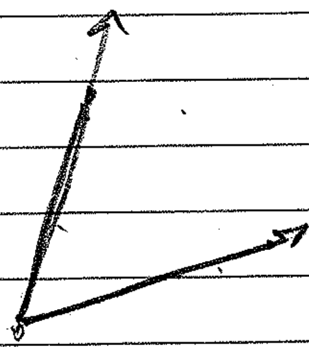
$$I_C^*(u) = \max_{x \in C} u^T x = \|u\|_*$$

$$\min f(x) \;\; \text{s.t. } x \in C \xrightarrow{\text{dual}} \max_u \; -f^*(u) - \|u\|_*$$

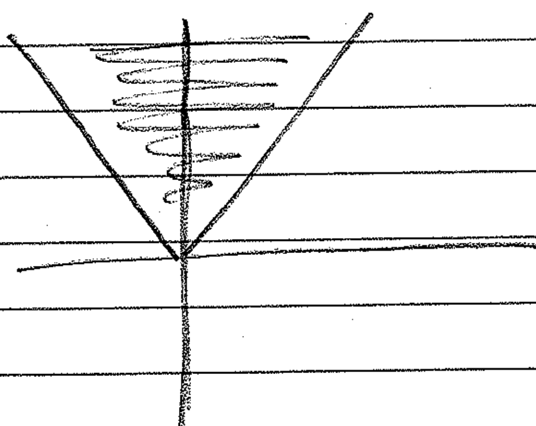


0 $\quad x_1, x_2 \in K \implies \theta_1 x_1 + \theta_2 x_1 \in K$

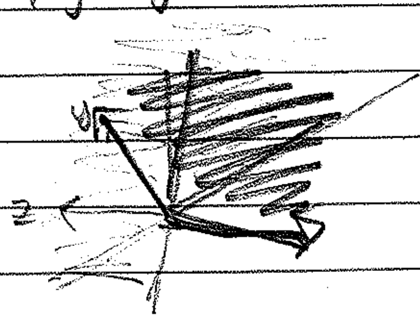
$\forall\, \theta_1, \theta_2 \geq 0.$



0



$$K^* = \{y : y^T x \geq 0 \quad \forall x \in K\}$$



$y \in K^*$

$\iff \{x : y^T x \geq 0\} \supseteq K$

$$\min_{x \in K} f(x)$$

$$\max -f^*(u) - I_K^*(-u)$$

$$I_K^*(-u) = \max_{x \in K} -u^T x = I_{K^*}(u)$$

$$K^* = \{y : y^T x \geq 0 \ \forall x \in K\}$$

$u \notin K^*$ then $\exists x \in K$ s.t. $u^T x < 0$

$u \in K^*$ then $u^T x \geq 0 \ \forall x \in K$

$$\max -f^*(u) \text{ s.t. } u \in K^*$$

$$K = \{x : x_i \geq 0 \ \forall x\}$$

$x \leq_K y \Rightarrow x_i \leq y_i \ \forall i$

write as $x \leq y$

$K = S_+^n$

$X \leq_{S_+^n} Y \Rightarrow Y - X$ p.s.d.

write as $X \preceq Y$