$$\min \quad px + qy$$

$$\text{s.t.} \quad x + y \geq 2 \qquad a \geq 0$$

$$x \geq 0 \qquad b \geq 0$$

$$y \geq 0 \qquad c \geq 0$$

$$a(x+y) + bx + cy \geq 2a$$

$$\underbrace{(a+b)}_{p}x + \underbrace{(a+c)}_{q}y \geq 2a$$

$$\max \quad 2a$$

$$a + b = p$$

$$a + c = q$$

$$a, b, c \geq 0.$$

$$\min \quad px + qy$$

$$\text{s.t.} \quad x \geq 0 \qquad a \geq 0$$

$$-y \geq -1 \qquad b \geq 0$$

$$3x + y = 2 \qquad c$$

$$ax - by + c(3x + y - 2) \geq -b + 0$$

$$\underbrace{(a+3c)}_{p}x + \underbrace{(-b+c)}_{q}y \geq 2c - b$$

$$\min \quad c^T x$$
$$\text{st.} \quad Ax = b \qquad u$$
$$Gx \le h \qquad v \ge 0$$

$$u^T(Ax-b) + v^T(Gx-h) \le 0$$
$$\underbrace{(-A^T u - G^T v)^T}_{c} x \ge \underline{-b^T u - h^T v}$$

$$\max \quad -b^T u - h^T v$$
$$\text{st.} \quad -A^T u - G^T v = c$$
$$v \ge 0.$$

$$\max \sum f_{sj}$$
$$\text{st.} \quad f_{ij} \ge 0 \qquad a_{ij} \ge 0$$
$$-f_{ij} \ge -c_{ij} \qquad b_{ij} \ge 0$$
$$\sum f_{ik} = \sum f_{kj} \qquad x_k$$

$$\sum_{(i,j)\in E} -a_{ij} f_{ij} + b_{ij}(f_{ij} - c_{ij})$$
$$+ \sum_{k \ne s,t} x_k \left(\sum_i f_{ik} - \sum_j f_{kj} \right) \le 0$$

~~LHS:~~

RHS:

s,j : $(-a_{sj} + b_{sj} + x_j)$ want $= 1$

i,t : $-a_{it} + b_{it} - x_i$ want $= 0$

i,j : $-a_{ij} + b_{ij} + x_j - x_i$ want $= 0$.

$\sum b_{ij} c_{ij}$

$$
\begin{aligned}
\min \quad & \sum b_{ij} c_{ij} \\
\text{st.} \quad & b_{ij} \geq 0 \\
& b_{sj} + x_j \geq 1 \\
& b_{it} - x_i \geq 0 \\
& b_{ij} + x_j - x_i \geq 0 \quad i \neq s,\ j \neq t
\end{aligned}
$$

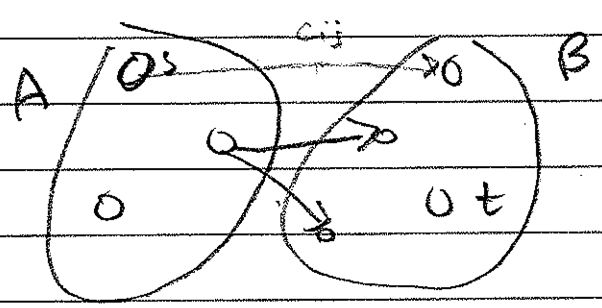
$$
\begin{aligned}
& x_s = 1, \quad x_t = 0. \\
& b_{ij} + x_j - x_i \geq 0 \quad \text{all } i,j
\end{aligned}
$$

suppose $x_i \in \{0,1\} \quad \forall i$

$A = \{i : x_i = 1\} \quad B = \{i : x_i = 0\}$

$b_{ij} \geq x_i - x_j = 1 \qquad i \in A \quad j \in B$

i,j st. not $(i \in A \text{ and } j \in B)$



$\min \sum b_{ij} c_{ij}$

max flow $\overset{=}{\le}$ optimal value
for dual problem

$=$ $\le$ min cut

primal optimal = dual optimal

$$\begin{array}{lll} \min & c^T x & \\ \text{s.t.} & Ax = b & u \\ & Gx \le h & v \ge 0 \end{array}$$

$$c^T x \ge \underbrace{c^T x + u^T(Ax-b) + v^T(Gx-h)}_{L(x,u,v)}$$

for any x feasible

C feasible set $= \{x : Ax = b,\ Gx \le h\}$.

$$\begin{aligned} f^* &\ge \min_{x \in C} L(x,u,v) \\ &\ge \min_{x \in \mathbb{R}^n} L(x,u,v) \\ &= g(u,v) \quad \text{for any } u,\ v \ge 0 \end{aligned}$$

$$\begin{aligned} g(u,v) &= \min_x \ c^T x + u^T(Ax-b) + v^T(Gx-h) \\ &= \min_x \ \underbrace{(A^T u + G^T v + c)}_{=0}{}^T x - b^T u - h^T v \end{aligned}$$

$$g(u,v) = \begin{cases} -b^T u - h^T v & \text{if } A^T u + G^T v + c = 0 \\ -\infty & \text{else} \end{cases}$$

$$\begin{array}{ll} \max & g(u,v) \\ \text{s.t.} & v \ge 0 \end{array} \iff \begin{array}{ll} \max & -b^T u - h^T v \\ \text{s.t.} & A^T u + G^T v + c = 0 \\ & v \ge 0 \end{array}$$

$$\begin{aligned} \min\ & f(x) \\ \text{s.t.}\ & h_i(x) \le 0 \qquad u_i \ge 0. \\ & l_j(x) = 0 \qquad v_j \end{aligned}$$

$$L(x,u,v) = f(x) + \sum u_i \underbrace{h_i(x)}_{\le 0} + \sum v_j \underbrace{l_j(x)}_{=0.} \le f(x)$$

$f(x) \ge L(x,u,v)$ $\quad$ x feasible

$u \ge 0.$

$L(\cdot, u)$ $\quad$ some $u \ge 0.$

$u \ge 0, v.$

$$f^* \ge \min_{x \in C} L(x,u,v) \ge \min_{x \in \mathbb{R}^n} L(x,u,v) = g(u,v)$$

dual function

$$\max_{u \ge 0} \ g(u,v)$$ Lagrange dual problem

$\min f \iff -\max(-f)$

cvx $\qquad$ ccv

$\min_{x \in \mathbb{R}^n} L(x,u,v)$

$$\min \quad \frac{1}{2}x^TQx + c^Tx \qquad f^*$$

$$\text{st} \quad \underset{v}{Ax=b}, \ \underset{u}{x \geq 0}$$

$Q \succ 0.$

$$L(x,u,v) = \frac{1}{2}x^TQx + c^Tx - u^Tx + v^T(Ax-b)$$

$$= \underbrace{\frac{1}{2}x^TQx + (c-u+A^Tv)^Tx - b^Tv}$$

$$x = -Q^{-1}(c-u+A^Tv)$$

$$\underbrace{g(u,v) = -\frac{1}{2}(c-u+A^Tv)^TQ^{-1}(c-u+A^Tv) - b^Tv}$$

$u \geq 0$, v.

$$\min \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}^T Q \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} + c^T x.$$

$$\text{s.t. } x \geq 0.$$

$$g(u) = \ldots$$
$$u \geq 0$$

$$\max_{u \geq 0} \; g(u,v) \longleftarrow \text{concave function}$$