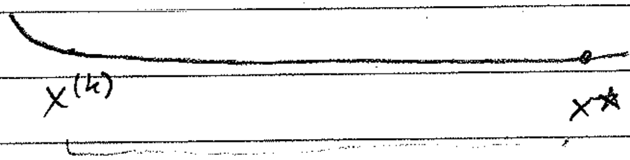
$$\sum t_k^2 < \infty, \quad \sum t_k = \infty.$$

$t_k = 1/k$ all k.



$$\frac{R^2 + G^2 \sum_{i=1}^{k} t_i^2}{2 \sum_{i=1}^{k} t_i}$$

$\boxed{t_i = R/(G\sqrt{k})}$ $i = 1, \ldots, k.$

$$\frac{R^{\cancel{2}} + \cancel{G^2}\,\frac{R^2}{\cancel{G^2}\cancel{k}}\cdot\cancel{k}}{\cancel{2}\,\frac{\cancel{R}}{G\sqrt{k}}\cdot k} = \boxed{\frac{RG}{\sqrt{k}}}$$

$$O(1/\sqrt{k})$$

$C_1, \ldots C_m$ closed, convex sets.

$x^* \in C_1 \cap \ldots C_m$

$$f(x) = \max_{i=1,\ldots m} \operatorname{dist}(x, C_i)$$

$$\operatorname{dist}(x, C) = \min_{u \in C} \|x - u\|$$

$$f(x^*) = 0 \iff x^* \in C_1 \cap \ldots C_m$$

unique point $P_C(x) = u^*$

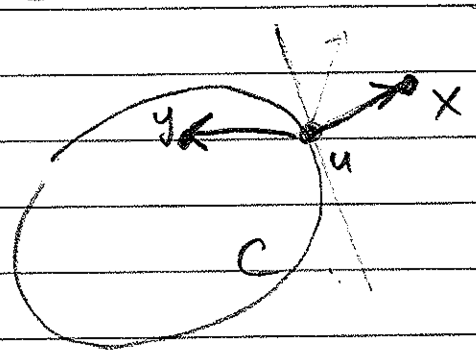
that minimizes $\|x-u\|$ over $u \in C$.

$\operatorname{dist}(x, C) = \|x - P_C(x)\|$.

For each i, $x \notin C_i$, $\dfrac{x - P_{C_i}(x)}{\|x - P_C(x)\|} \in \partial f_i(x)$

By $P_{C_i}(x) = u$ $\quad (x-u)^T(y-u) \le 0$

$\forall y \in C$.



$\min \frac{1}{2}\|x-u\|^2 + I_C(u)$

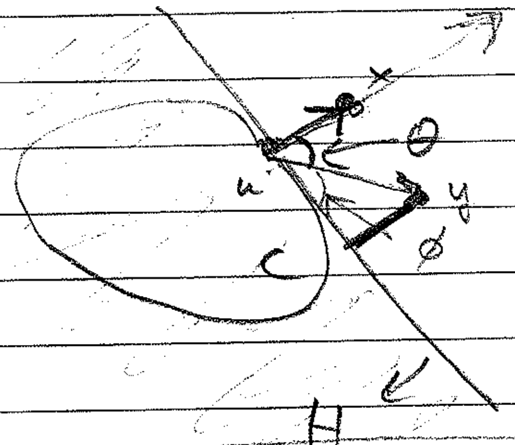
$u - x + N_C(u) \ni 0$.

$x - u \in N_C(u)$.

$$(x-u)^T u \geq (x-u)^T y \quad \forall y \in C.$$

$$(x-u)^T (y-u) \leq 0 \quad \forall y \in C.$$

$$C \subseteq H = \{ y : (x-u)^T (y-u) \leq 0 \}.$$



$$\begin{aligned}(x-u)^T(y-u) &= \|x-u\| \, \|y-u\| \cos\theta \\ &= \|x-u\| \, \|y-u\| \sin\phi\end{aligned}$$

$$\boxed{\mathrm{dist}(y, C) \geq \frac{(x-u)^T (y-u)}{\|x-u\|} \quad \forall y.}$$

$y \in H$: RHS is ≤ 0 ✓

$y \notin H$:
$$\begin{aligned}\mathrm{dist}(y, H) &= \frac{(x-u)^T (y-u)}{\|x-u\|} \\ &= \frac{\cancel{\|x-u\|} \, \|y-u\| \sin\phi}{\cancel{\|x-u\|}} \\ &= \mathrm{dist}(y, H)\end{aligned}$$

$$\begin{aligned}\mathrm{dist}(y, C) &\geq \frac{(x-u)^T (y-x+x-u)}{\|x-u\|} \\ &= \|x-u\| + \underbrace{\left(\frac{(x-u)^T}{\|x-u\|}\right)}_{g \, \in \, \partial \, \mathrm{dist}(x, C)} (y-x)\end{aligned}$$

$$f(x) = \max_{i=1,\dots m} f_i(x)$$

$$\partial f(x) = \mathrm{conv}\left(\bigcup_{j:\, f_j(x) = f(x)} \partial f_j(x) \right).$$

if $f_i(x) = f(x) \neq 0$, then

$$\frac{x - P_{C_i}(x)}{\| x - P_{C_i}(x) \|} \in \partial f(x).$$

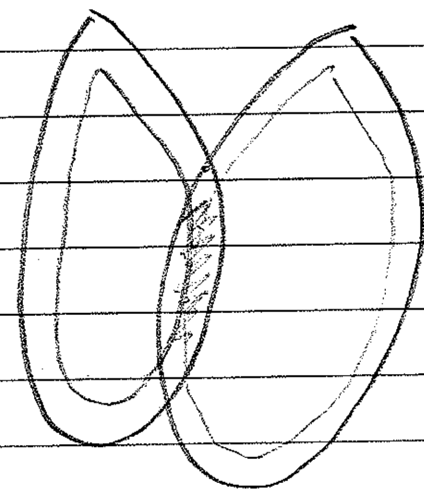
Polyak. $t_k = f(x^{(k-1)})$,

$$x^{(k)} = x^{(k-1)} - f(x^{(k-1)}) \frac{x^{(k-1)} - P_{C_i}(x^{(k-1)})}{\| x^{(k-1)} - P_{C_i}(x^{(k-1)}) \|}$$

where $x^{(k-1)}$ is farthest from C_i.

$$= \cancel{x^{(k-1)}} - \cancel{x^{(k-1)}} + P_{C_i}(x^{(k-1)})$$

$$= P_{C_i}(x^{(k-1)})$$



$x^{(0)}=0, \quad k=n-1$

$$f(x) = \max_{i=1,\dots,n} x_i + \frac{1}{2}\|x\|^2.$$

$$x^* = (-1/n, \dots -1/n), \quad f(x^*) = -\frac{1}{2n}$$

$$\underset{\shortparallel}{v} + x = 0.$$

$$(1/n, \dots 1/n) + (-1/n, \dots -1/n) = 0.$$

$$\in \mathrm{conv}\left(\begin{pmatrix}1\\0\\\vdots\\0\end{pmatrix}, \begin{pmatrix}0\\1\\\vdots\\0\end{pmatrix} \dots \begin{pmatrix}0\\0\\\vdots\\1\end{pmatrix}\right)$$

$$f(x^*) = -1/n + \frac{1}{2n} = -1/2n.$$

f is Lipschitz with $G = 1 + 1/\sqrt{n}$

on $\{x : \|x\| \le 1/\sqrt{n}\}$.

$v + x$ $\qquad\qquad\qquad\qquad$ $v \in \mathrm{conv}(e_1, \dots, e_n)$

$\|v + x\| \le \|v\| + \|x\|$

$\le \underbrace{1 + 1/\sqrt{n}}_{G}$ $\qquad\qquad$ $\underbrace{\{x : \|x\| \le 1/\sqrt{n}\}}_{R}$

$|f(x) - f(y)| \le G\|x - y\|.$

$f(x^*) = -1/2n.$

Oracle: $g = e_j + x$ where j smallest index for which $x_j = \max_i x_i$

i iterations

$x^{(i)}_{i+1} = \dots x^{(i)}_n = 0.$

at $x^{(0)} = 0$, $g^{(0)} = e_1$ $\qquad$ $e_1 = (1, 0 \dots 0)$

$e_j = (0, \dots 1, \dots 0)$

$\underbrace{\mathrm{span}\{g^{(0)}, g^{(1)}\} \subseteq \mathrm{span}\{e_1, e_2\}}$

$g^{(1)} \in \partial f(x^{(1)}) = v^{(1)} + x^{(1)}$

$\underbrace{x^{(1)} \in \mathrm{span}\{g^{(0)}\}} = \mathrm{span}\{e_1\}.$

$\mathrm{span}\{g^{(0)}, \dots g^{(i-1)}\} \subseteq \mathrm{span}\{e_1, \dots e_i\}$

$f(x^{(n-1)}) \ge 0,$ $\quad f(x^{(n-1)}) - f(x^*) \ge \frac{1}{2n} = \frac{RG}{2(1+\sqrt{n})}.$