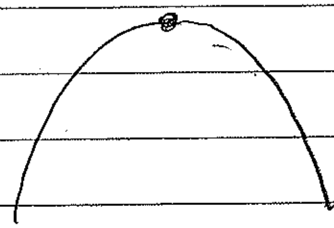
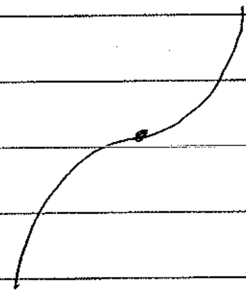




$$I_C(x) = \begin{cases} 0 & x \in C \\ \infty & x \notin C \end{cases}$$

$$\min_{x \in C} f(x) \iff \min_{x} f(x) + I_C(x).$$

$$g(x) = f(Ax+b).$$

$$f(x) = \|x\|_p = \left(\sum_{i=1}^{n} x_i^{p}\right)^{1/p}$$

$$= \max_{\|y\|_q \le 1} y^T x. \qquad \frac{1}{q} + \frac{1}{p} = 1.$$

$\|x\|_p = y^T x$ for some $\|y\|_q \le 1$.

$\partial \|x\|_p \ni y \ldots$

$$\Rightarrow \min_{x \in \mathbb{R}^n} f(x) = f(x^*)$$

$$\Longleftrightarrow \quad 0 \in \partial f(x^*) = \{\nabla f(x^*)\}$$

$$\forall y: \quad f(y) \geq f(x) + 0^T(y-x) = f(x).$$

$$\min_{x \in C} f(x) \Longleftrightarrow \min_{x \in \mathbb{R}^n} \underline{f(x) + I_C(x)}$$

$$0 \in \partial(f(x^*) + I_C(x^*))$$

$$= \underbrace{\partial f(x^*)} + \underbrace{\partial I_C(x^*)}$$

$$\min \underbrace{\tfrac{1}{2}\|y-x\|^2}_{f(x)} + \lambda \underbrace{\|x\|_1}_{\sum_{i=1}^{n}|x_i|} \qquad \lambda \geq 0.$$

$$\partial f(x) \ni \underbrace{x-y. + \lambda s}$$

$$\underbrace{\begin{aligned} s_i &= \operatorname{sign}(x_i) & \text{if } x_i \neq 0. \\ &\in [-1,1] & \text{if } x_i = 0. \end{aligned}}$$

$$\underbrace{x^*(y) = S_\lambda(y)} \quad \text{soft-thresholding}$$

$$[S_\lambda(y)]_i = \begin{cases} y_i - \lambda & y_i > \lambda \\ 0 & -\lambda \leq y_i \leq \lambda \\ y_i + \lambda & y_i < -\lambda \end{cases}$$

$y_i > \lambda$

$g_i \qquad x_i^* - y_i = -\lambda + \lambda \cdot 1 \qquad y_i - \lambda.$

$x_i = y_i - \lambda. \qquad = 0.$

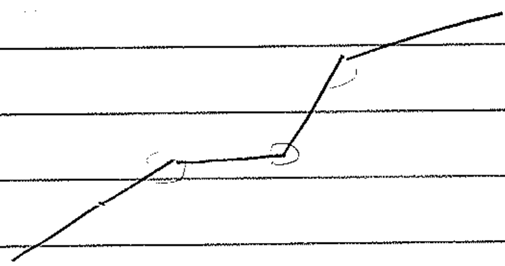
$y_i < -\lambda$ similar.

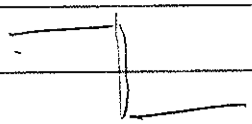
$-\lambda < y_i < \lambda.$

$$x_i - y_i = -y_i + \lambda\left(\frac{y_i}{\lambda}\right) = 0.$$

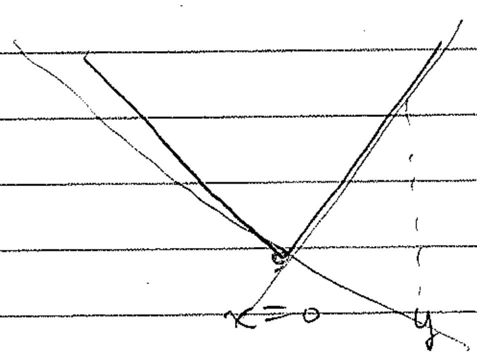
$\sum t_k^2 < \infty$, $\sum t_k = \infty$

$t_k = 1/k$.





$\| g_x - g_y \| \leq L \|x - y\| \quad \forall x, y$.



$|-1-1| < L|y|$.

Proof

$$\|x^{(k+1)} - x^\star\|^2 = \|x^{(k)} - t_k g^{(k)} - x^\star\|^2$$

$$= \|x^{(k)} - x^\star\|^2 - 2t_k (g^{(k)})^T (x^{(k)} - x^\star) + t_k^2 \|g^{(k)}\|^2$$

$$f(x^\star) \geq f(x^{(k)}) + g^{(k)} (x^\star - x^{(k)})$$

$$-(g^{(k)})^T (x^{(k)} - x^\star) \leq -(f(x^{(k)}) - f(x^\star))$$

$$\leq \|x^{(k)} - x^\star\|^2 - 2t_k (f(x^{(k)}) - f(x^\star)) + t_k \|g^{(k)}\|^2$$

$$\leq \|x^{(1)} - x^\star\|^2 - 2\sum_{i=1}^{k} t_i (f(x^{(i)}) - f(x^\star)) + \sum_{i=1}^{k} t_i^2 \|g^{(i)}\|^2$$

$$0 \leq R^2 + \sum_{i=1}^{k} t_i^2 G^2 - 2\sum_{i=1}^{k} t_i (f(x^{(i)}) - f(x^\star))$$

$$f(x_{best}^{(k)})$$

$$2\sum_{i=1}^{k} t_i (f(x^{(i)}) - f(x^\star)) \leq R^2 + \sum_{i=1}^{k} t_i^2 G^2$$

$$\left(f(x_{best}^{(k)}) - f(x^\star) \right) \leq \frac{R^2 + \sum_{i=1}^{k} t_i^2 G^2}{2\sum_{i=1}^{k} t_i}$$

If $t_i = t$

$$\frac{R^2 + G^2 t^2 k}{2tk} \longrightarrow \frac{G^2 t}{2}$$

as $k \to \infty$

$$\sum t_i^2 < \infty, \quad \sum t_i = \infty$$

$$\frac{R^2 + G^2 \sum t_i^2}{\sum t_i} \longrightarrow 0.$$

e.g. take $t_i = 1/i$