$$\min_{y} \ \hat{f}(y) = f(x) + \nabla f(x)^T (y - x) + \frac{1}{2t} \|y - x\|^2$$

$$\Leftrightarrow \quad \min_{y} \ \frac{1}{t} \| \ y - (\underbrace{x - t\nabla f(x)}) \|^2$$

$$y = x - t\nabla f(x).$$

$$\nabla^2 f \preceq LI$$

$$A \succeq 0.$$

$$x^T A x \geq 0 \quad \forall x.$$

$\boxed{\text{Proof.}}$

$$\forall\, x, y \quad f(y) = f(x) + \nabla f(x)^T (y-x) + \tfrac{1}{2}(x-y)^T \nabla f^2(z)(x-y)$$

for some. $z \in [x, y]$ Taylor remainder thm

$$\nabla^2 f \preceq LI$$

$$(x-y)^T(\nabla^2 f - LI)(x-y) \leq 0 \quad \forall\, x, y.$$

$$L\|x-y\|^2 \geq (x-y)^T \nabla^2 f(z)(x-y)$$

- $\boxed{f(y) \leq f(x) + \nabla f(x)^T (y-x) + \frac{L}{2}\|x-y\|^2}$

$$y = x - t\nabla f(x).$$

$$\underbrace{f(x - t\nabla f(x))}_{x^+} \leq f(x) - \nabla f(x)^T t \nabla f(x) + \frac{Lt^2}{2}\|\nabla f(x)\|^2$$

- $= f(x) - \left(1 - \frac{Lt}{2}\right) t \|\nabla f(x)\|^2$

- $t \leq 1/L \qquad \left(1 - \frac{Lt}{2}\right) \geq 1 - \frac{1}{2} = \frac{1}{2}.$

$$\rightarrow \boxed{f(x^+) \leq f(x) - \frac{t}{2}\|\nabla f(x)\|^2}$$

$$\leq f(x^*) + \nabla f(x)^T (x - x^*) - \frac{t}{2}\|\nabla f(x)\|^2$$

$$= f(x^*) + \frac{1}{2t}\left(\|x - x^*\|^2 - \underbrace{\|x - x^* - t\nabla f(x)\|^2}_{\|x^+ - x^*\|^2}\right)$$

$f(x^*) \geq f(x) - \nabla f(x)^T (x - x^*)$

$f(x) \leq f(x^*) + \nabla f(x)^T (x - x^*)$

$$f(x^+) \le f(x^*) + \frac{1}{2t}\left(\|x - x^*\|^2 - \|x^+ - x^*\|^2\right).$$

$$\underline{\sum_{i=1}^{k}\left(f(x^{(i)}) - f(x^*)\right)} \le \frac{1}{2t}\sum_{i=1}^{k}\left(\|x^{(i-1)} - x^*\|^2 - \|x^{(i)} - x^*\|^2\right)$$

$$= \frac{1}{2t}\left(\|x^{(0)} - x^*\|^2 - \|x^{(k)} - x^*\|^2\right)$$

$$\le \underline{\frac{1}{2t}\|x^{(0)} - x^*\|^2}.$$

$f(x^{(i)})$ is nonincreasing.

$$f(x^{(k)}) - f(x^*) \le \frac{1}{k}\sum_{i=1}^{k}\left(f(x^{(i)}) - f(x^*)\right)$$

$$\le \frac{\|x^{(0)} - x^*\|^2}{2tk}.$$

$f(x) = \frac{1}{2}\|y - Ax\|^2$

$\nabla f(x) = -A^T(y - Ax).$

$\nabla^2 f(x) = A^T A.$

$A^T A \preceq LI$, take $L = \sigma_{max}^2(A)$.

$u \in \partial \|\nabla f(x)\|_r.$