convex optimization problem

$$\min \quad f(x)$$

$$\text{s.t.} \quad g_i(x) \le 0 \quad i = 1, \dots m$$

$$h_j(x) = 0 \quad j = 1, \dots n$$

f convex.

g_i convex.

h_j linear

$$h_j(x) = a_j^T x + b_j$$

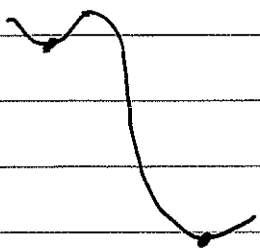
$$h_j(x) \le 0 \quad \cancel{h_j(x) \ge 0}.$$

$$-h_j(x) \le 0.$$

x^* feasible

$f(x^*) \le f(y) \quad \forall\, y \text{ s.t. } \|x^* - y\| \le R$

note: x^* must be globally optimal. why?



Suppose z feasible

and $f(z) < f(x^*)$

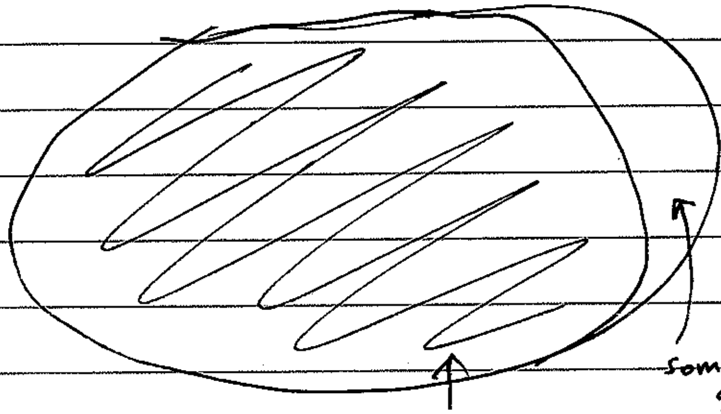
$\|x^* - z\| > R$

$\alpha = \dfrac{R}{\|x^* - z\|} \qquad y = (1-\alpha)\,x^* + \alpha z$

$$\begin{aligned}
\|x^* - y\| &= \|\alpha(x^* - z)\| \\
&= \alpha\,\|x^* - z\| \\
&= R
\end{aligned}$$

$$\begin{aligned}
f(y) &= f((1-\alpha)x^* + \alpha z) \\
&\le (1-\alpha)\,f(x^*) + \alpha f(z) \\
&< (1-\alpha)\,f(x^*) + \alpha f(x^*) \\
&= f(x^*).
\end{aligned}$$

contradiction



problems know how to solve in stats/ML

some rare gems in here!

Convex.

First ~~methods~~ order methods	Second-order, related	coord. methods	dual methods
⋮	⋮	⋮	⋮
fast, useful algo's	"	"	"

when to use what?