Introduction

10-725 Optimization
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Administrivia

- http://www.cs.cmu.edu/~ggordon/10725-F12/
- http://groups.google.com/group/10725-f12

Administrivia

- Prerequisites: no formal ones, but class will be fast-paced
- Algorithms: basic data structures & complexity
- Programming: we assume you can do it
- Linear algebra: matrices are your friends
- ML/stats: source of motivating examples
- *Most important: formal thinking*

Administrivia

- Coursework: 5 HWs, scribing, midterm, project
- Project: use optimization to do something cool!
  - groups of 2–3 (no singletons please)
  - proposal, milestone, final poster session, final paper
- Final poster session: Tue or Wed, Dec 11 or 12, starting at about 3PM in NSH atrium, lasting 3 hrs

Administrivia

- Scribing
  - multiple scribes per lecture (coordinate one writeup); required to do once during term
  - sign up now to avoid timing problems
- Late days: you have 5 to use wisely
  - in lieu of any special exceptions for illness, travel, holidays, etc.—your responsibility to allocate
  - some deadlines will be non-extendable

Administrivia

- Working together
  - great to have study groups
  - always write up your own solutions, ***closed*** notes
  - disclose collaborations on front page of HW

Administrivia

- Office hours
- Recitations: none this week
- Audit forms: please audit r.t. just sitting in
  - except: postdocs & faculty welcome to sit in
- Waitlist: there shouldn't be one
- Videos

Most important

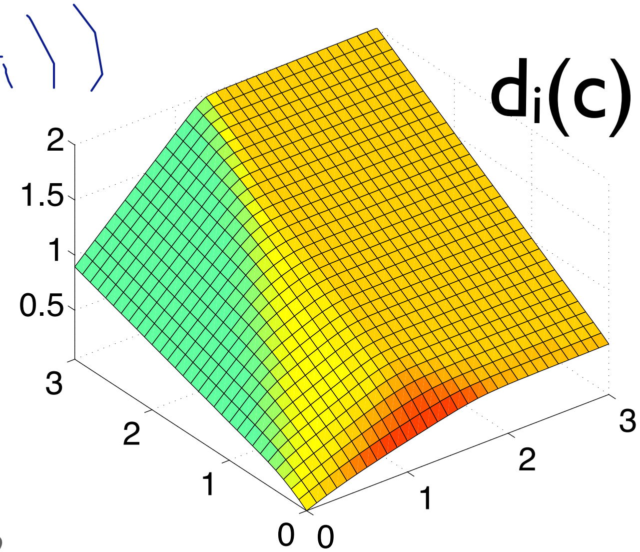
- Work hard, have fun!

Optimization example

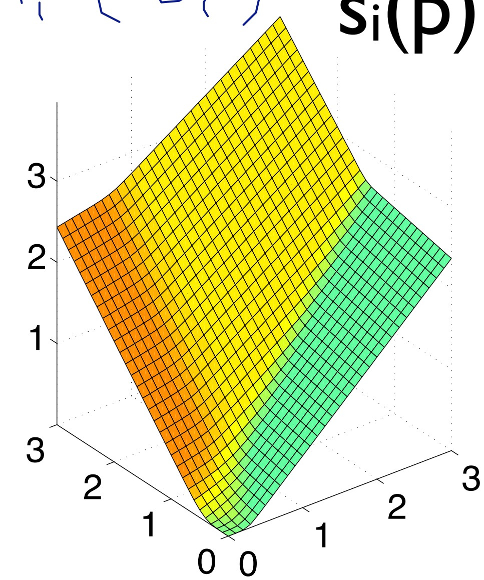
- Simple economy: m agents, n goods
  - each agent: production $p_i \in R^n$, consumption $c_i \in R^n$
- Cost of producing p_i for agent i: $s_i(p_i)$
- Utility of consuming c_i for agent i: $d_i(c_i)$

$$\max_{p_i, c_i} \sum_i \left(d_i(c_i) - s_i(p_i)\right)$$

$$\sum_i c_i = \sum_i p_i$$



$d_i(c)$



$s_i(p)$

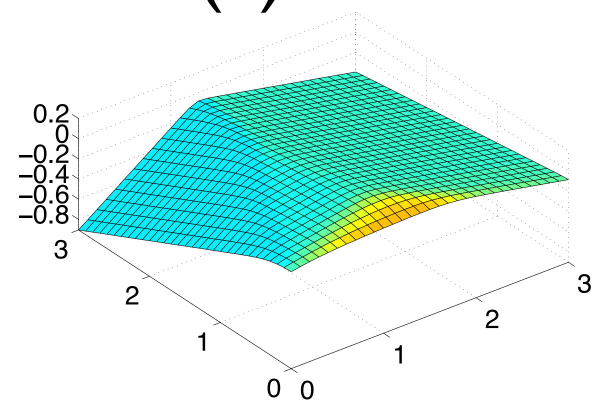
Walrasian equilibrium

$$\max \sum_i [d_i(c_i) - s_i(p_i)] \text{ s.t. } \sum_i p_i = \sum_i c_i$$

- Idea: put price λ_j on good j; agents optimize production/consumption independently

$\max_{p_i, c_i} \sum_i (d_i(c_i) - \lambda \cdot c_i) - (s_i(p_i) - \lambda \cdot p_i)$

$d_i(c) - \lambda^T c$



  - high price → produce ↑, consume ↓
  - low price → produce ↓, consume ↑
  - "just right" prices → constraint satisfied

Algorithm: tâtonnement

$$\max \sum_i [d_i(c_i) - s_i(p_i)] \text{ s.t. } \sum_i p_i = \sum_i c_i$$

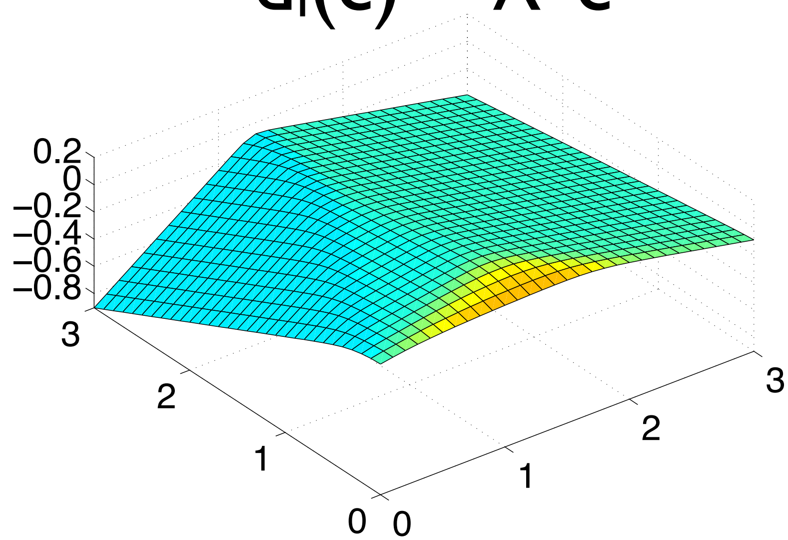
$\lambda \leftarrow [0\ 0\ 0\ \ldots]^T$

for k = 1, 2, …

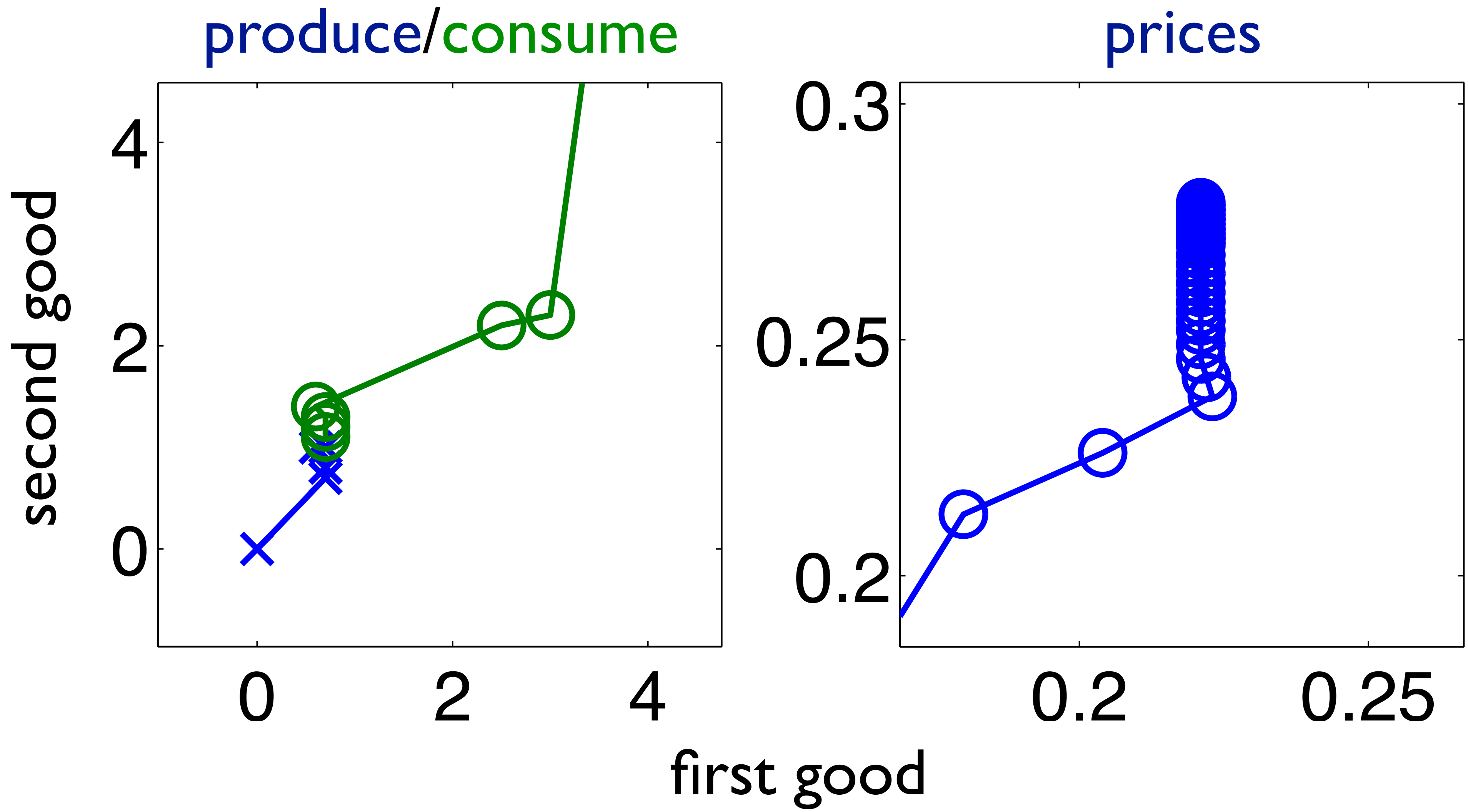
- each agent solves for p_i and c_i at prices λ
- $\lambda \leftarrow \lambda + t_k(c - p)$

> 0 $c = \sum_i c_i$ $\sum_i p_i$

$d_i(c) - \lambda^T c$



Results for a random market



Why is tâtonnement cool?

- Algorithm is nearly obvious, given setup
  - Leon Walras (1874), based on ideas of Antoine Augustin Cournot (1838)
- But analysis (Arrow and Debreu, 1950s) is subtle: needs concepts from later in this course
  - duality, dual decomposition, convergence rates of gradient descent
- Variants need even more subtlety

"Typical" problem

- Minimize $f(x)$ s.t. $g_i(x) \le 0$, $i \in INEQ$; $h_i(x) = 0$, $i \in EQ$

- e.g.: f() and g_i() all linear: LP
- e.g.: f() and g_i() all convex: convex program
- e.g.: f() linear, g_1() is –min(eig(reshape(x, k, k))): SDP

any linear h_i $X = X^T$

Ubiquitous (and pretty cool)

- LPs at least as old as Fourier
- first practical algorithm: simplex (Dantzig, 1947)
  - for a long time, best runtime bounds were exponential, but practical runtime observed good
- many thought LPs were NP-hard
- Kachiyan (1979), Karmarkar (1984): LP in P
- Spielman & Teng (2002): simplex solves "most" LPs in poly time
- LPs are P-complete: "hardest" poly-time problem

Optimization for ML & stats

- Lots of ML & stats based on optimization
  - regression, PCA, max likelihood, SVM, POMDP
- Exceptions?
  - integration posterior, hypothesis testing? nonparametrics, belief propagation, game theory? property testing?
- Advantages
  - fast generic algo's
    expressive
    connect objective to stats
    standard xforms

Choices

- Set up problem
- Transformations: duality, relaxations, approximations
- Algorithms:
  - first order, interior point, ellipsoid, cutting plane
  - smooth v. nonsmooth v. some combination
  - eigensystems
  - message passing / relaxation

usually many choices, ***widely*** *different performance (runtime, solution quality, …)*

Consequences

- First order (gradient descent, FISTA, Nesterov's method) v. higher order (Newton, log barrier, ellipsoid, affine scaling)
  - # iters poly in $1/\epsilon$ vs. in $\log(1/\epsilon)$
  - cost of each iteration: $O(n)$ or less, vs. $O(n^3)$ or so
- Balanced (#constrs ≈ #vars) or not?
  - e.g., ellipsoid handles #constrs = ∞

Consequences

- Sparsity? Locality? Other special structure?
  - in solution, in active constraints, in matrices describing objective or constraints
- E.g., $Ax = b$: how fast can we compute Ax?
- E.g., simplex vs. log barrier

Consequences

- What degree of "niceness"?
  - differentiable, strongly convex, self-concordant, submodular
- Can we split $f(x) = g(x) + h(x)$?
- Is $f(x)$ "close to" a smooth fn?
- Care more about practical implementation or analysis?

Some more examples

- Image segmentation
- Perceptron, SVM
- MPE in graphical model
- Linear regression
- Lasso (group, graphical, …)
- Parsing, grammar learning
- Sensor placement in a sensor network
- Equilibria in games (CE, EFCE, polymatrix)
- Maximum entropy
- Network flow
- TSP
- Experimental design
- Compressed sensing
- …

Example: playing poker

- http://www.cs.cmu.edu/~ggordon/poker/
- Problem: compute a minimax equilibrium
- Even this simple game has 2^{26} strategies/player
- We reduce to an LP with ~100 variables
- Similar methods have been used for competition-level 2-player limit Texas Hold'em
  - abstract the game by clustering information sets
  - buy a really big workstation, run for days

Dynamic walking

[Schkolnik, Levashov, Manchester, Tedrake, 2010]

http://groups.csail.mit.edu/locomotion/movies/LittleDog_MIT_dynamic_short.f4v