Lecture 12
Level Sets &
Parametric Transforms

sec. 8.5.2 & ch. 11 of *Machine Vision* by Wesley E. Snyder & Hairong Qi

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16-725 (CMU RI) : BioE 2630 (Pitt)

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A Quick Review

- The movement of boundary points on an active contour can be governed by a partial differential equation (PDE)
- PDE's operate on discrete "time steps"
  - One time step per iteration
- Snake points move normal to the curve
  - The normal direction is recalculated for each iteration.
- Snake points move a distance determined by their speed.

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Typical Speed Function

- Speed is usually a combination (product or sum) of internal and external terms:
  - $s(x,y) = s_I(x,y)\ s_E(x,y)$
- Internal (shape) speed:
  - e.g., $s_I(x,y) = 1 - \|\varepsilon\kappa(x,y)\|$
  - where $\kappa(x,y)$ measures the snake's curvature at (x,y)
- External (image) speed:
  - e.g., $s_E(x,y) = (1+\Delta(x,y))^{-1}$
  - where $\Delta(x,y)$ measures the image's edginess at (x,y)

Can be pre-computed from the input image

- Note that $s(x,y)$ above is always positive.
  - Such a formulation would allow a contour to grow but not to shrink.

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Active Contours using PDEs: Typical Problems

- Curvature measurements are very sensitive to noise
  - They use 2nd derivatives
- They don't allow an object to split
  - This can be a problem when tracking an object through multiple slices or multiple time frames.
  - A common problem with branching vasculature or dividing cells
- How do you keep a curve from crossing itself?
  - One solution: only allow the curve to grow

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Level Sets

- A philosophical/mathematical framework:
  - Represent a curve (or surface, etc.) as an isophote in a “special” image, denoted ψ, variously called the:
    - Merit function
    - Embedding
    - Level-set function
  - Manipulate the curve indirectly by manipulating the level-set function.

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Active Contours using PDEs on Level Sets

- The PDE active-contour framework can be augmented to use a level-set representation.
- This use of an implicit, higher-dimensional representation addresses the active-contour problems mentioned 2 slides back.

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Level Sets: An Example from the ITK Software Guide

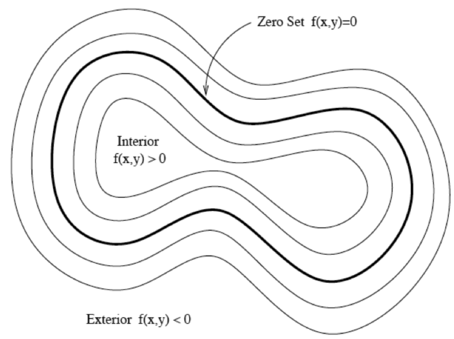

Figure 9.13: Concept of zero set in a level set.

Figures 9.13 from the ITK Software Guide v 2.4, by Luis Ibáñez, et al.

Note: ITK has inside positive; some other papers & Snyder text have inside negative

Level Sets and the Distance Transform (DT)

- DT is applied to a binary or segmented image
  - Typically applied to the contour's *initialization*
  - Outside the initial contour, we typically negate the DT
- Records at each pixel the distance from that pixel to the nearest boundary.
- **The 0-level set of the initialization's DT is the original boundary**

	1					1	1	
	1					1	1	
	1	1	1		1	**2**	1	
	1	**2**	**2**	1	**2**	**2**	1	
		1	**2**	**2**	**3**	**2**	1	
		1	**2**	**3**	**2**	**2**	1	
		1	**2**	**2**	1	**2**	1	
		1	**2**	1		1	1	
			1			1		
			1					

Level-Set Segmentation: Typical Procedure

- Create an initial contour
  - Many level-set segmentation algorithms require the initialization to be inside the desired contour
- Initialize ψ: $$\psi(x,y) = \begin{cases} -DT(x,y) & \text{if } (x,y) \text{ is outside the contour} \\ DT(x,y) & \text{if } (x,y) \text{ is inside the contour} \end{cases}$$
- Use a PDE to incrementally update the segmentation (by updating ψ)
  - Level Set Eq: dψ/dt = velocity * gradient_mag(ψ):
- Stop at the right time
  - This can be tricky; more later.

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Measuring curvature and surface normals

- One of the advantages of level sets is that they can afford good measurements of curvature
- Because the curve is represented implicitly as the 0-level set, it can be fit to ψ with sub-pixel resolution
- Surface normals are collinear with the gradient of ψ. (why?)
- See Snyder 8.5 for details on computing curvature (κ).

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Allowing objects to split or merge

- Suppose we want to segment vasculature from CT with contrast
- Many segmentation algorithms only run in 2D
  - So we need to slice the data
  - But we don't want to initialize each slice by hand

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Allowing objects to split or merge

- Solution:
  - Initialize 1 slice by hand
  - Segment that slice
  - Use the result as the initialization for neighboring slices
- But vasculature branches
  - One vessel on this slice might branch into 2 vessels on the next slice
  - Segmentation methods that represent a boundary as a single, closed curve will break here.

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Allowing objects to split or merge

- Level Sets represent a curve implicitly
- Nothing inherently prevents the 0-level set of ψ from representing multiple, distinct objects.
- Most level-set segmentation algorithms naturally handle splitting or merging
  - PDEs are applied and calculated locally

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Active Surfaces

- Level Sets can represent surfaces too!
- ψ now fills a volume
- The surface is still implicitly defined as the zero level set.
- The PDE updates “every” point in the volume
  - (To speed up computation, on each iteration we can update only pixels that are close to the 0 level set)
- Being able to split and merge 3D surfaces over time can be very helpful!

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ITK's Traditional PDE Formulation

$$\frac{d}{dt}\psi = -\alpha \mathbf{A}(\mathbf{x}) \cdot \nabla\psi - \beta P(\mathbf{x})|\nabla\psi| + \gamma Z(\mathbf{x})\kappa|\nabla\psi|$$

- $\mathbf{A}$ is an advection term
  - Draws the 0-level set toward image edginess
- P is a propagation (expansion or speed) term
  - The 0-level set moves slowly in areas of edginess in the original image
- Z is a spatial modifier term for the mean curvature κ
- α, β, and γ are weighting constants
- Many algorithms don't use all 3 terms

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A Very Simple Example (ITK Software Guide 4.3.1)

- Initialize inside the object
- Propagation:
  - Slow down near edges
  - Is always positive (growth only)
- Stop at the "right" time
  - Perform enough iterations (time steps) for the curve to grow close to the boundaries
  - Do not allow enough time for the curve to grow past the boundaries
- This method is very fast!

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A More Complex Example (ITK Software Guide 4.3.3)

- Geodesic Active Contours Segmentation
- Uses an advection term, $\mathbf{A}$
  - Draws the curve toward edginess in the input image
  - Things no longer "blow up" if we run too long
- Now, we can simply stop when things converge (sufficiently small change from one time step to the next).
  - Still, it's a good idea to program a maximum number of allowed time steps, in case things don't converge.

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Some General Thoughts about Level Sets

- Remember, Level Sets are nothing more than a way of representing a curve (or surface, hypersurface, etc.)
- Level-Sets do have some advantages (e.g, splitting/merging)
- But, Level-Sets otherwise work no better than any other method.
  - Look at the many examples in the ITK software guide; their results often leave a little or a lot to be desired

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Level Set References

- Snyder, 8.5.2
- *Insight into Images*, ch. 8
- *ITK Software Guide*, book 2, 4.3
- "The" book:
  - *Level Set Methods and Fast Marching Methods: Evolving Interfaces in Computational Geometry, Fluid Mechanics, Computer Vision and Materials Science,* by J.A. Sethian, Cambridge University Press, 1999.
  - Also see: http://math.berkeley.edu/~sethian/2006/level_set.html
- All of the above reference several scientific papers.

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Snyder ch. 11: Parametric Transforms

- Goal: Detect geometric features in an image
- Method: Exchange the role of variables and parameters
- References: Snyder 11 & ITK Software Guide book 2, 4.4

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Geometric Features?

- For now, think of geometric features as shapes that can be graphed from an equation.
- Line: $\mathbf{y} = m\mathbf{x} + b$
- Circle: $R^2 = (\mathbf{x}-x_{center})^2 + (\mathbf{y}-y_{center})^2$

(variables are shown in **bold purple**, parameters are in black)

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Why Detect Geometric Features?

- Guide segmentation methods
  - Automated initialization!
- Prepare data for registration methods
- Recognize anatomical structures

From the ITK Software Guide v 2.4, by Luis Ibáñez, et al., p. 596

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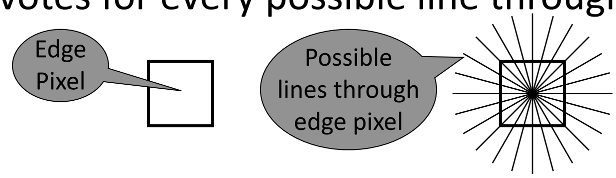
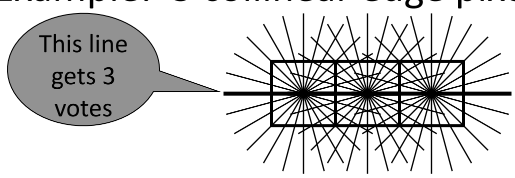
How do we do this again?

- Actually, each edge pixel "votes"
- If we are looking for lines, each edge pixel votes for every possible line through itself:



- Example: 3 collinear edge pixels:



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How to Find All Possible Shapes for each Edge Pixel

- Exchange the role of variables and parameters:
- Example for a line: $y = \mathbf{m}x + \mathbf{b}$
  (variables are shown in **bold purple**)
- Each edge pixel in the image:
  - Has its own (x, y) coordinates
  - Establishes its own equation of $(\mathbf{m},\mathbf{b})$

This is the set of all possible shapes through that edge point

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How to Implement Voting

- With an accumulator
  - Think of it as an image in parameter space
  - Its axes are the new variables (which were formally parameters)
  - But, writing to a pixel increments (rather than overwriting) that pixel's value.
- Graph each edge pixel's equation on the accumulator (in parameter space)
- Maxima in the accumulator are located at the parameters that fit the shape to the image.

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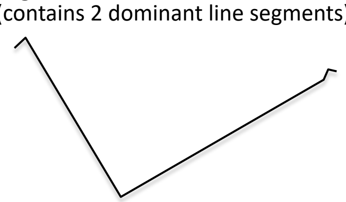
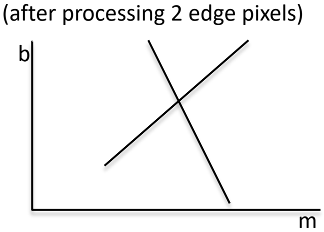
Example 1: Finding Lines

- If we use $\mathbf{y} = \mathrm{m}\mathbf{x} + \mathrm{b}$
- Then each edge pixel results in a line in parameter space: $\mathbf{b} = -\mathbf{m}\mathrm{x} + \mathrm{y}$

Edge Detection Results
(contains 2 dominant line segments)



Accumulator Intermediate Result
(after processing 2 edge pixels)



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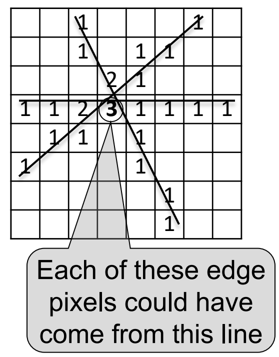
Example 1: Finding Lines

- A closer look at the accumulator after processing 2 and then 3 edge pixels
- The votes from each edge pixel are graphed as a line in parameter space
- Each accumulator cell is incremented each time an edge pixel votes for it
  - I.e., each time a line in parameter space passes through it



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Example 2: Finding Lines… A Better Way

- What's wrong with the previous example?
  - Consider vertical lines: $m = \infty$
  - My computer doesn't like infinite-width accumulator images. Does yours?
- For parametric transforms, we need a different line equation, one with a bounded parameter space.

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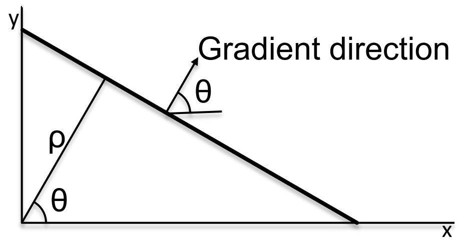
Example 2: Finding Lines… A Better Way

- A better line equation for parameter voting:

$$\rho = x \cos \theta + y \sin \theta$$

- $\rho \leq$ the input image diagonal size
  - But, to make math easy, ρ can be - too.
- θ is bounded within $[0,2\pi]$



See *Machine Vision* Fig. 11.5 for example of final accumulator for 2 noisy lines

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Computational Complexity

- This can be really slow
  - Each edge pixel yields a lot of computation
  - The parameter space can be huge
- Speed things up:
  - Only consider parameter combinations that make sense…
  - Each edge pixel has an apx. direction attached to its gradient, after all.

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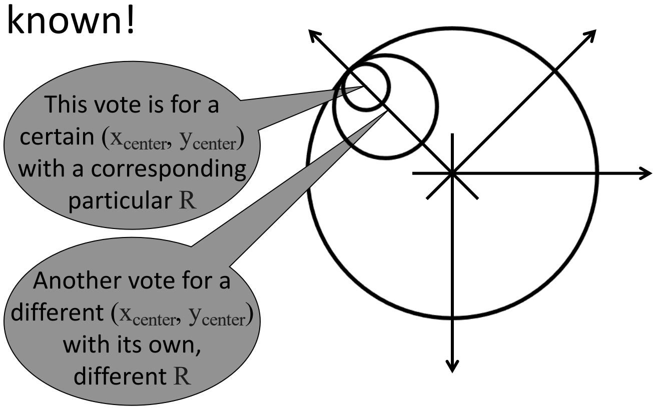
Example 3: Finding Circles

- Equation: $R^2 = (x - x_{center})^2 + (y - y_{center})^2$
- Must vote for 3 parameters if R is not known!



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Example 4: General Shapes

- What if our shape is weird, but we can draw it?
  - Being able to draw it implies we know how big it will be
- See Snyder 11.4 for details
- Main idea:
  - For each boundary point, record its coordinates in a local reference frame (e.g., at the shape's center-of-gravity).
  - Itemize the list of boundary points (on our drawing) by the direction of their gradient

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