15-462 Computer Graphics I
Lecture 16

Ray Tracing

Ray Casting
Ray-Surface Intersections
Barycentric Coordinates
Reflection and Transmission
[Angel, Ch 13.2-13.3] [Handout]

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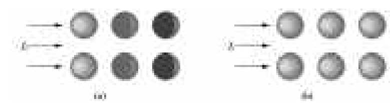
http://www.cs.cmu.edu/~fp/courses/graphics/

Local vs. Global Rendering Models

- Local rendering models (graphics pipeline)
  - Object illuminations are independent
  - No light scattering between objects
  - No real shadows, reflection, transmission
- Global rendering models
  - Ray tracing (highlights, reflection, transmission)
  - Radiosity (surface interreflections)

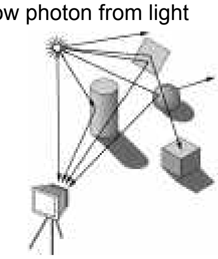


Object Space vs. Image Space

- Graphics pipeline: for each object, render
  - Efficient pipeline architecture, on-line
  - Difficulty: object interactions
- Ray tracing: for each pixel, determine color
  - Pixel-level parallelism, off-line
  - Difficulty: efficiency, light scattering
- Radiosity: for each two surface patches, determine diffuse interreflections
  - Solving integral equations, off-line
  - Difficulty: efficiency, reflection

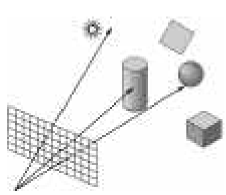
Forward Ray Tracing

- Rays as paths of photons in world space
- Forward ray tracing: follow photon from light sources to viewer
- Problem: many rays will not contribute to image!



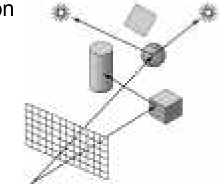
Backward Ray Tracing

- Ray-casting: one ray from center of projection through each pixel in image plane
- Illumination
  1. Phong (local as before)
  2. Shadow rays
  3. Specular reflection
  4. Specular transmission
- (3) and (4) require recursion



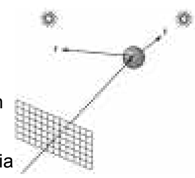
Shadow Rays

- Determine if light “really” hits surface point
- Cast shadow ray from surface point to light
- If shadow ray hits opaque object,no contribution
- Improved diffuse reflection



Reflection Rays

- Calculate specular component of illumination
- Compute reflection ray (recall: backward!)
- Call ray tracer recursively to determine color
- Add contributions
- Transmission ray
  – Analogue for transparent or translucent surface
  – Use Snell's laws for refraction
- Later:
  – Optimizations, stopping criteria



Ray Casting

- Simplest case of ray tracing
- Required as first step of recursive ray tracing
- Basic ray-casting algorithm
  – For each pixel (x,y) fire a ray from COP through (x,y)
  – For each ray & object calculate closest intersection
  – For closest intersection point **p**
    - Calculate surface normal
    - For each light source, calculate and add contributions
- Critical operations
  – Ray-surface intersections
  – Illumination calculation

Outline

- Ray Casting
- Ray-Surface Intersections
- Barycentric Coordinates
- Reflection and Transmission

Ray-Surface Intersections

- General implicit surfaces
- General parametric surfaces
- Specialized analysis for special surfaces
  – Spheres
  – Planes
  – Polygons
  – Quadrics
- Do not decompose objects into triangles!
- CSG (Constructive Solid Geometry)
  – Construct model from building blocks (later lecture)

Rays and Parametric Surfaces

- Ray in parametric form
  – Origin $\mathbf{p}_0 = [x_0\ \ y_0\ \ z_0\ \ 1]^T$
  – Direction $\mathbf{d} = [x_d\ \ y_d\ \ z_d\ \ 0]^t$
  – Assume **d** normalized ($x_d^2 + y_d^2 + z_d^2 = 1$)
  – Ray $\mathbf{p}(t) = \mathbf{p}_0 + \mathbf{d}\ t$ for $t > 0$
- Surface in parametric form
  – Point $\mathbf{q} = g(u, v)$, possible bounds on u, v
  – Solve $\mathbf{p} + \mathbf{d}\ t = g(u, v)$
  – Three equations in three unknowns (t, u, v)

Rays and Implicit Surfaces

- Ray in parametric form
  – Origin $\mathbf{p}_0 = [x_0\ \ y_0\ \ z_0\ \ 1]^T$
  – Direction $\mathbf{d} = [x_d\ \ y_d\ \ z_d\ \ 0]^t$
  – Assume **d** normalized ($x_d^2 + y_d^2 + z_d^2 = 1$)
  – Ray $\mathbf{p}(t) = \mathbf{p}_0 + \mathbf{d}\ t$ for $t > 0$
- Implicit surface
  – Given by $f(\mathbf{q}) = 0$
  – Consists of all points **q** such that $f(\mathbf{q}) = 0$
  – Substitute ray equation for **q**: $f(\mathbf{p}_0 + \mathbf{d}\ t) = 0$
  – Solve for t (univariate root finding)
  – Closed form (if possible) or numerical approximation

Ray-Sphere Intersection I

- Common and easy case
- Define sphere by
  - Center **c** = $[x_c \;\; y_c \;\; z_c \;\; 1]^T$
  - Radius r
  - Surface f(**q**) = $(x - x_c)^2 + (y - y_c)^2 + (z - z_c)^2 - r^2 = 0$
- Plug in ray equations for x, y, z:

$$x = x_0 + x_d t \qquad y = y_0 + y_d t \qquad z = z_0 + z_d t$$

$$(x_0 + x_d t - x_c)^2 + (y_0 + y_d t - y_c)^2 + (z_0 + z_d t - z_c)^2 = r^2$$

Ray-Sphere Intersection II

- Simplify to

$$a t^2 + b t + c = 0$$

where

$$a = x_d^2 + y_d^2 + z_d^2 = 1 \quad \text{since } |\mathbf{d}| = 1$$
$$b = 2(x_d(x_0 - x_c) + y_d(y_0 - y_c) + z_d(z_0 - z_c))$$
$$c = (x_0 - x_c)^2 + (y_0 - y_c)^2 + (z_0 - z_c)^2 - r^2$$

- Solve to obtain t_0 and t_1

$$t_{0,1} = \frac{-b \pm \sqrt{b^2 - 4c}}{2}$$

Check if $t_0, t_1 > 0$ (ray)
Return $\min(t_0, t_1)$

Ray-Sphere Intersection III

- For lighting, calculate unit normal

$$\mathbf{n} = \frac{1}{r}[(x_i - x_c) \; (y_i - y_c) \; (z_i - z_c) \; 0]^T$$

- Negate if ray originates inside the sphere!
- Note possible problems with roundoff errors

Simple Optimizations

- Factor common subexpressions
- Compute only what is necessary
  - Calculate $b^2 - 4c$, abort if negative
  - Compute normal only for closest intersection
  - Other similar optimizations [Handout]

Inverse Mapping for Texture Coords.

- How do we determine texture coordinates?
- Inverse mapping problem
- No unique solution
- Reconsider in each case
  - For different basic surfaces
  - For surface meshes
  - Still an area of research

Ray-Polygon Intersection I

- Assume planar polygon
  1. Intersect ray with plane containing polygon
  2. Check if intersection point is inside polygon
- Plane
  - Implicit form: ax + by + cz + d = 0
  - Unit normal: **n** = $[a \;\; b \;\; c \;\; 0]^T$ with $a^2 + b^2 + c^2 = 1$
- Substitute:

$$a(x_0 + x_d t) + b(y_0 + y_d t) + c(z_0 + z_d t) + d = 0$$

- Solve: $$t = \frac{-(a x_0 + b y_0 + c z_0 + d)}{a x_d + b y_d + c z_d}$$

Ray-Polygon Intersection II

- Substitute t to obtain intersection point in plane
- Test if point inside polygon
- For example, use even-odd rule or winding rule
  - Easier in 2D (project) and for triangles (tesselate)

Ray-Polygon Intersection III

- Rewrite using dot product

$$t = \frac{-(ax_0 + by_0 + cz_0 + d)}{ax_d + by_d + cz_d} = \frac{-(\mathbf{n} \cdot \mathbf{p}_0 + d)}{\mathbf{n} \cdot \mathbf{d}}$$

- If $\mathbf{n} \cdot \mathbf{d} = 0$, no intersection
- If $t \leq 0$ the intersection is behind ray origin
- Point-in-triangle testing critical for polygonal models
- Project onto planes x = 0, y = 0, or z = 0 for point-in-polygon test; can be precomputed

Ray-Quadric Intersection

- Quadric f(**p**) = f(x, y, z) = 0, where f is polynomial of order 2
- Sphere, ellipsoid, paraboloid, hyperboloid, cone, cylinder
- Closed form solution as for sphere
- Important case for modelling in ray tracing
- Combine with CSG

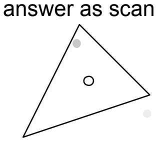
[see Handout]

Outline

- Ray Casting
- Ray-Surface Intersections
- Barycentric Coordinates
- Reflection and Transmission

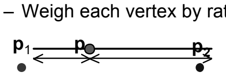
Interpolated Shading for Ray Tracing

- Assume we know normals at vertices
- How do we compute normal of interior point?
- Need linear interpolation between 3 points
- Barycentric coordinates
- Yields same answer as scan conversion



Barycentric Coordinates in 1D

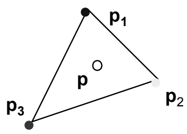
- Linear interpolation
  - $\mathbf{p}(t) = (1 - t)\mathbf{p}_1 + t\,\mathbf{p}_2$, $0 \leq t \leq 1$
  - $\mathbf{p}(t) = \alpha\,\mathbf{p}_1 + \beta\,\mathbf{p}_2$ where $\alpha + \beta = 1$
  - **p** is between $\mathbf{p}_1$ and $\mathbf{p}_2$ iff $0 \leq \alpha, \beta \leq 1$
- Geometric intuition
  - Weigh each vertex by ratio of distances from ends



- α, β are called barycentric coordinates

Barycentric Coordinates in 2D

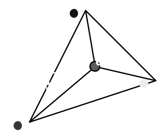
- Given 3 points instead of 2



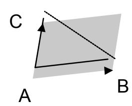
- Define 3 barycentric coordinates, α, β, γ
- $\mathbf{p} = \alpha\, \mathbf{p}_1 + \beta\, \mathbf{p}_2 + \gamma\, \mathbf{p}_3$
- $\mathbf{p}$ inside triangle iff $0 \leq \alpha, \beta, \gamma \leq 1$, $\alpha + \beta + \gamma = 1$
- How do we calculate α, β, γ given $\mathbf{p}$?

Barycentric Coordinates for Triangle

- Coordinates are ratios of triangle areas



Computing Triangle Area



- In 3 dimensions
  - Use cross product
  - Parallelogram formula
  - Area(ABC) = $(1/2)|(B - A) \times (C - A)|$
  - Optimization: project, use 2D formula
- In 2 dimensions
  - Area(x-y-proj(ABC)) = $(1/2)((b_x - a_x)(c_y - a_y) - (c_x - a_x)(b_y - a_y))$

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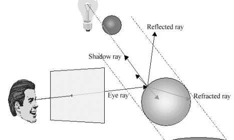
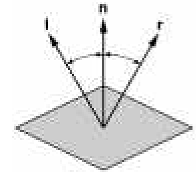
Recursive Ray Tracing

- Calculate specular component
  - Reflect ray from eye on specular surface
  - Transmit ray from eye through transparent surface
- Determine color of incoming ray by recursion
- Trace to fixed depth
- Cut off if contribution below threshold



Angle of Reflection

- Recall: incoming angle = outgoing angle
- $\mathbf{r} = 2(\mathbf{l} \cdot \mathbf{n})\, \mathbf{n} - \mathbf{l}$
- For incoming/outgoing ray negate $\mathbf{l}$!
- Compute only for surfaces with actual reflection
- Use specular coefficient
- Add specular and diffuse components



Transmitted Light

- Index of refraction is relative speed of light
- Snell's law
  - η_l = index of refraction for upper material
  - η_t = index of refraction for lower material

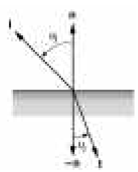
[U = θ]

$$\frac{\sin(\theta_l)}{\sin(\theta_t)} = \frac{\eta_t}{\eta_l} = \eta$$

$$\mathbf{t} = -\frac{1}{\eta}\mathbf{l} - (\cos(\theta_t) - \frac{1}{\eta}\cos(\theta_l))\mathbf{n}$$

where $\cos(\theta_l) = \mathbf{l} \cdot \mathbf{n}$

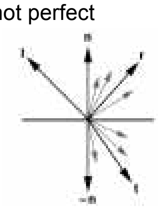
and $\cos^2(\theta_t) = 1 - \frac{1}{\eta^2}(1 - \mathbf{l} \cdot \mathbf{n})$



Note: negate l or t for transmission!

Translucency

- Diffuse component of transmission
- Scatter light on other side of surface
- Calculation as for diffuse reflection
- Reflection or transmission not perfect
- Use stochastic sampling



Ray Tracing Preliminary Assessment

- Global illumination method
- Image-based
- Pluses
  - Relatively accurate shadows, reflections, refractions
- Minuses
  - Slow (per pixel parallelism, not pipeline parallelism)
  - Aliasing
  - Inter-object diffuse reflections

Ray Tracing Acceleration

- Faster intersections
  - Faster ray-object intersections
    - Object bounding volume
    - Efficient intersectors
  - Fewer ray-object intersections
    - Hierarchical bounding volumes (boxes, spheres)
    - Spatial data structures
    - Directional techniques
- Fewer rays
  - Adaptive tree-depth control
  - Stochastic sampling
- Generalized rays (beams, cones)

Raytracing Example I

www.povray.org

Raytracing Example II

www.povray.org

Raytracing Example II

Saito, Saturn Ring

Raytracing Example IV

www.povray.org

Summary

- Ray Casting
- Ray-Surface Intersections
- Barycentric Coordinates
- Reflection and Transmission

Preview

- Spatial data structures
- Ray tracing optimizations
- Assignment 6 out today
- Assignment 7 out after spring break