

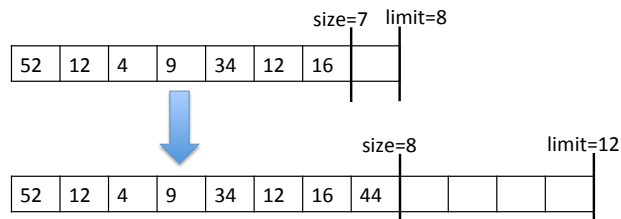
15-122: Principles of Imperative Computation

Recitation 6: Array Disarray

Rob Simmons, Josh Zimmerman

Unbounded arrays

When implementing unbounded arrays on an embedded device, a programmer is concerned that doubling the size of the array when we reach its limit may use precious memory resources too aggressively. So she decides to see if she can increase it by a factor of $\frac{3}{2} = 1.5$ instead, rounding down if the result is not an integral number.



This means that it won't make sense for the limit to be less than _____, because otherwise you might resize the array and get an array that wasn't any bigger. This needs to be reflected in the data structure invariant!

```
1 struct arr_header {
2   int size;
3   int limit;
4   string[] data;
5 };
6 typedef struct arr_header* arr;
7
8 bool is_arr_expected_length(int[] A, int limit) {
9   //@assert \length(A) == limit;
10  return true;
11 }
12
13 bool is_uba(struct arr_header* A) {
14
15 _____
16 _____
17 _____
18 _____
19 _____
20 }
```

Checkpoint 0

Implement the `arr_new(n)` function for this version of unbounded arrays, where `n` is the initial size of the array. Give appropriate preconditions and postconditions, and make sure that your contracts guard against overflow. Your implementation of `arr_new` should handle the case that `n` is very small.

Checkpoint 1

Right after an array resize, we should assume we'll have no tokens in reserve for an array with size k and length $3k/2$ (let's assume k is even).

We might have to resize again after as few as _____ `arr_add` operations.

That next resize would force us to use _____ tokens to copy everything into a larger array (with size $9k/4$). The adds that we do in the meantime add elements to the last third of the array.

Each cell in that last third therefore needs to have _____ tokens associated with it.

This gives `uba_add` an amortized cost of _____ tokens, because that we need one token to do the initial write whenever we call `uba_add`.

Checkpoint 2

Our analysis indicates that a smaller resizing factor gives us a higher amortized cost, even if it's still in $O(1)$. This indicates that doing n operations on this array, while still in $O(n)$, has a higher constant attached to it. Does this make sense?

You will find in the course of your study in algorithms that, like in this example, achieving higher space efficiency often necessitates a tradeoff in time efficiency, and vice versa.

Checkpoint 3

Repeat this analysis for the case where we triple the size of the array (both if we are only able to use whole tokens and if we're allowed to have an amortized cost of a fraction of a token).