15-462 Computer Graphics I
Lecture 14

Clipping and Scan Conversion

Line Clipping
Polygon Clipping
Clipping in Three Dimensions
Scan Conversion (Rasterization)
[Angel 7.3-7.6, 7.8-7.9]

March 19, 2002
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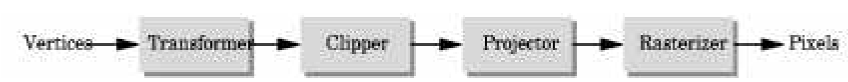
http://www.cs.cmu.edu/~fp/courses/graphics/

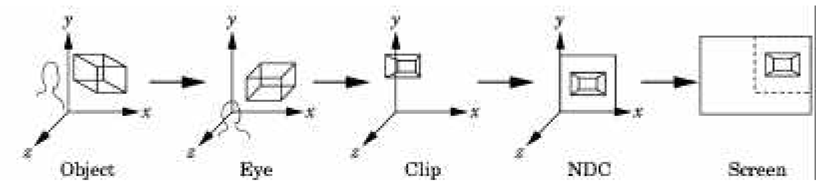
The Graphics Pipeline, Revisited



- Must eliminate objects outside viewing frustum
- Tied in with projections
  - Clipping: object space (eye coordinates)
  - Scissoring: image space (pixels in frame buffer)
- Introduce clipping in stages
  - 2D (for simplicity)
  - 3D (as in OpenGL)
- In a later lecture: scissoring

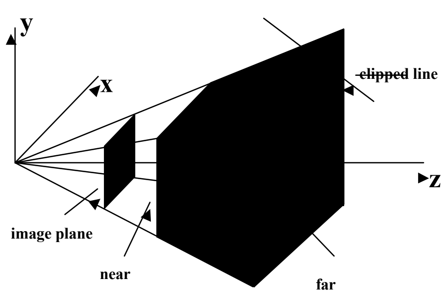
Transformations and Projections

- Sequence applied in many implementations
  1. Object coordinates to
  2. Eye coordinates to
  3. Clip coordinates to
  4. Normalized device coordinates to
  5. Screen coordinates



Clipping Against a Frustum

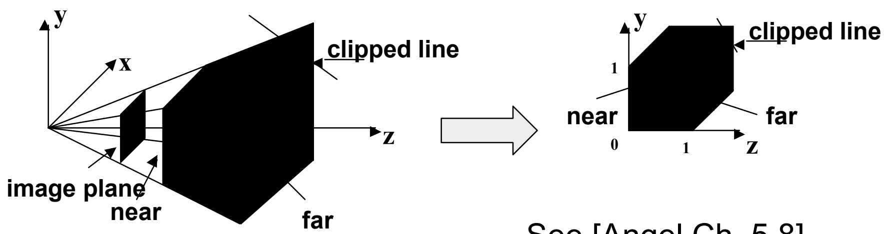
- General case of frustum (truncated pyramid)



- Clipping is tricky because of frustum shape

Perspective Normalization

- Solution:
  - Implement perspective projection by perspective normalization and orthographic projection
  - Perspective normalization is a homogeneous tfm.



See [Angel Ch. 5.8]

The Normalized Frustum

- OpenGL uses -1 $\leq$ x,y,z $\leq$ 1 (others possible)
- Clip against resulting cube
- Clipping against programmer-specified planes is different and more expensive
- Often a useful programming device

The Viewport Transformation

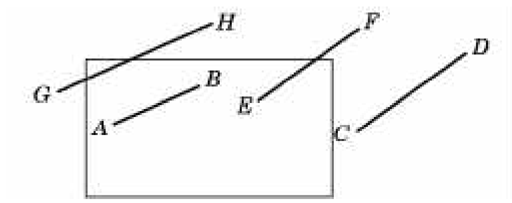
- Transformation sequence again:
  1. Camera: From object coordinates to eye coords
  2. Perspective normalization: to clip coordinates
  3. Clipping
  4. Perspective division: to normalized device coords.
  5. Orthographic projection (setting $z_p = 0$)
  6. Viewport transformation: to screen coordinates
- Viewport transformation can distort
- Often in OpenGL: resize callback

Line-Segment Clipping

- General: 3D object against cube
- Simpler case:
  – In 2D: line against square or rectangle
  – Before scan conversion (rasterization)
  – Later: polygon clipping
- Several practical algorithms
  – Avoid expensive line-rectangle intersections
  – Cohen-Sutherland Clipping
  – Liang-Barsky Clipping
  – Many more [see Foley et al.]

Clipping Against Rectangle

- Line-segment clipping: modify endpoints of lines to lie within clipping rectangle
- Could calculate intersections of line (segments) with clipping rectangle (expensive)



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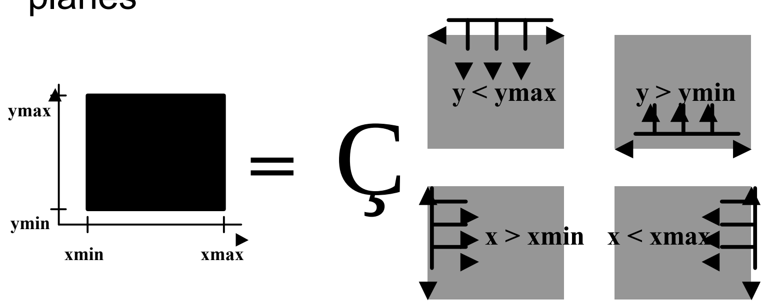
Cohen-Sutherland Clipping

- Clipping rectangle as intersection of 4 half-planes



- Encode results of four half-plane tests
- Generalizes to 3 dimensions (6 half-planes)

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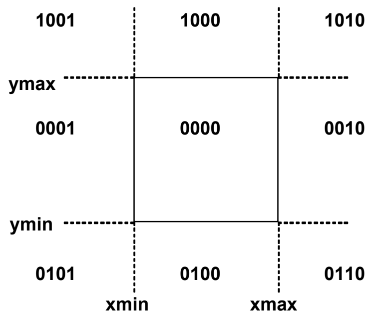
Outcodes

- Divide space into 9 regions
- 4-bit outcode determined by comparisons

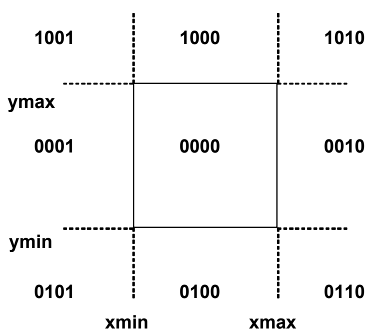


b_o: $y > y_{max}$
b_1: $y < y_{min}$
b_2: $x > x_{max}$
b_3: $x < x_{min}$

- o_1 = outcode(x_1,y_1) and o_2 = outcode(x_2,y_2)

Cases for Outcodes

- Outcomes: accept, reject, subdivide



$o_1 = o_2 = 0000$: accept
o_1 & $o_2 \neq 0000$: reject
$o_1 = 0000$, $o_2 \neq 0000$: subdiv
$o_1 \neq 0000$, $o_2 = 0000$: subdiv
o_1 & $o_2 = 0000$: subdiv

Cohen-Sutherland Subdivision

- Pick outside endpoint ($o \neq 0000$)
- Pick a crossed edge ($o = b_0b_1b_2b_3$ and $b_k \neq 0$)
- Compute intersection of this line and this edge
- Replace endpoint with intersection point
- Restart with new line segment
  - Outcodes of second point are unchanged
- Must converge (roundoff errors?)

Liang-Barsky Clipping

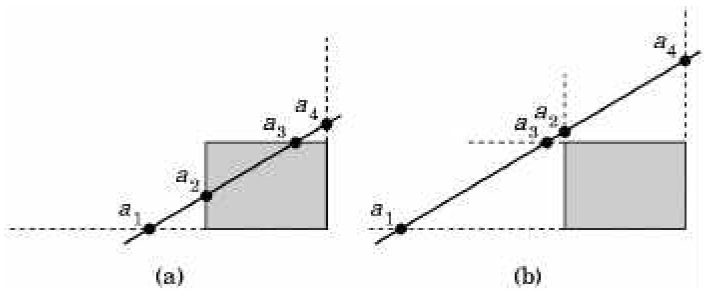
- Starting point is parametric form

$$\mathbf{p}(\alpha) = (1-\alpha)\mathbf{p}_1 + \alpha\mathbf{p}_2, \quad 0 \leq \alpha \leq 1$$

$$\begin{aligned} x(\alpha) &= (1-\alpha)x_1 + \alpha x_2 \\ y(\alpha) &= (1-\alpha)y_1 + \alpha y_2 \end{aligned}$$

- Compute four intersections with extended clipping rectangle
- Will see that this can be avoided

Ordering of intersection points



- Order the intersection points
- Figure (a): 1 > α_4 > α_3 > α_2 > α_1 > 0
- Figure (b): 1 > α_4 > α_2 > α_3 > α_1 > 0

Liang-Barsky Efficiency Improvements

- Efficiency improvement 1:
  - Compute intersections one by one
  - Often can reject before all four are computed
- Efficiency improvement 2:
  - Equations for α_3, α_2

$$y_{max} = (1 - \alpha_3)y_1 + \alpha_3 y_2$$
$$x_{min} = (1 - \alpha_2)x_1 + \alpha_2 x_2$$

$$\alpha_3 = \frac{y_{max} - y_1}{y_2 - y_1}. \quad \alpha_2 = \frac{x_{min} - x_1}{x_2 - x_1}$$

  - Compare α_3, α_2 without floating-point division

Line-Segment Clipping Assessment

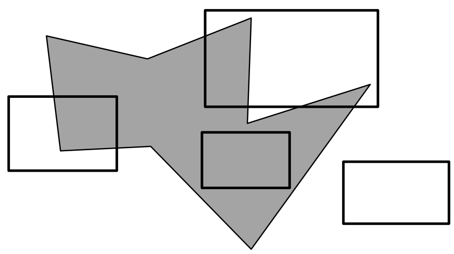
- Cohen-Sutherland
  - Works well if many lines can be rejected early
  - Recursive structure (multiple subdiv) a drawback
- Liang-Barsky
  - Avoids recursive calls (multiple subdiv)
  - Many cases to consider (tedious, but not expensive)
  - Used more often in practice (?)

Outline

- Line-Segment Clipping
  - Cohen-Sutherland
  - Liang-Barsky
- Polygon Clipping
  - Sutherland-Hodgeman
- Clipping in Three Dimensions
- Scan Conversion
  - DDA algorithm
  - Bresenham's algorithm

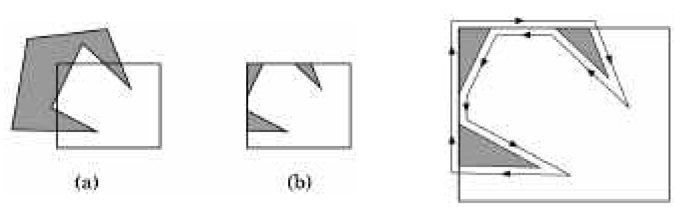
Polygon Clipping

- Convert a polygon into one ore more polygons
- Their union is intersection with clip window
- Alternatively, we can first tesselate concave polygons (OpenGL supported)

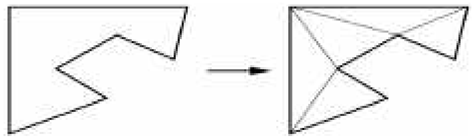


Concave Polygons

- Approach 1: clip and join to a single polygon

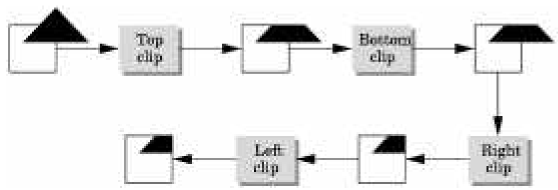


- Approach 2: tesselate and clip triangles



Sutherland-Hodgeman I

- Subproblem:
  - Input: polygon (vertex list) and single clip plane
  - Output: new (clipped) polygon (vertex list)
- Apply once for each clip plane
  - 4 in two dimensions
  - 6 in three dimension
  - Can arrange in pipeline

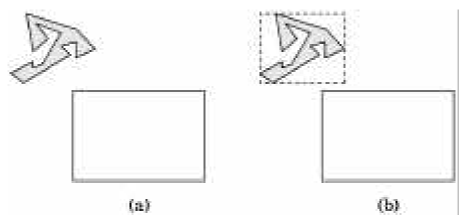


Sutherland-Hodgeman II

- To clip vertex list (polygon) against half-plane:
  - Test first vertex. Output if inside, otherwise skip.
  - Then loop through list, testing transitions
    - In-to-in: output vertex
    - In-to-out: output intersection
    - out-to-in: output intersection and vertex
    - out-to-out: no output
  - Will output clipped polygon as vertex list
- May need some cleanup in concave case
- Can combine with Liang-Barsky idea

Other Cases and Optimizations

- Curves and surfaces
  - Analytically if possible
  - Through approximating lines and polygons otherwise
- Bounding boxes
  - Easy to calculate and maintain
  - Sometimes big savings



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  - Cohen-Sutherland
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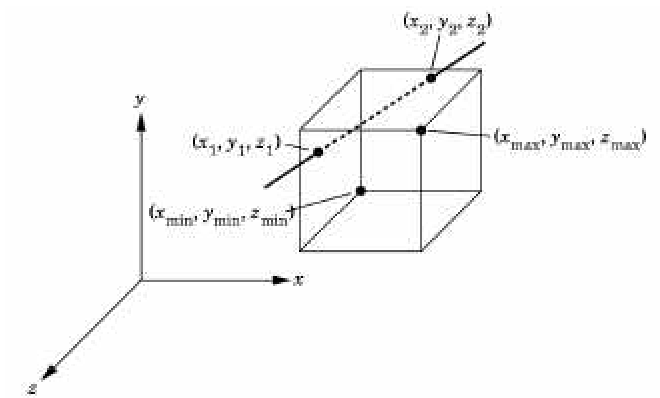
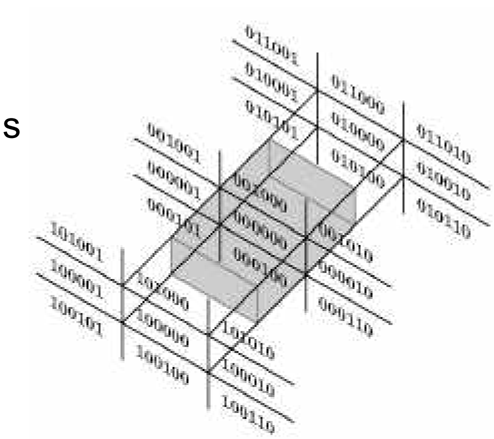
Clipping Against Cube

- Derived from earlier algorithms
- Can allow right parallelepiped



Cohen-Sutherland in 3D

- Use 6 bits in outcode
  - b_4: $z > z_{max}$
  - b_5: $z < z_{min}$
- Other calculations as before



Liang-Barsky in 3D

- Add equation $z(\alpha) = (1-\alpha) z_1 + \alpha z_2$
- Solve, for $\mathbf{p}_0$ in plane and normal $\mathbf{n}$:

$$y_{max} = (1-\alpha_3)y_1 + \alpha_3 y_2$$
$$x_{min} = (1-\alpha_2)x_1 + \alpha_2 x_2$$
$$\alpha_3 = \frac{y_{max} - y_1}{y_2 - y_1}. \quad \alpha_2 = \frac{x_{min} - x_1}{x_2 - x_1}$$

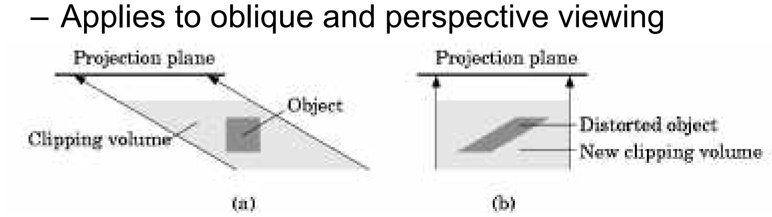
- Yields

$$\alpha = \frac{\mathbf{n} \cdot (\mathbf{p}_0 - \mathbf{p}_1)}{\mathbf{n} \cdot (\mathbf{p}_2 - \mathbf{p}_1)}$$

- Optimizations as for Liang-Barsky in 2D

Perspective Normalization

- Intersection simplifies for orthographic viewing
  - One division only (no multiplication)
  - Other Liang-Barsky optimizations also apply
- Otherwise, use perspective normalization
  - Reduces to orthographic case
  - Applies to oblique and perspective viewing



Normalization of oblique projections

Summary: Clipping

- Clipping line segments to rectangle or cube
  - Avoid expensive multiplications and divisions
  - Cohen-Sutherland or Liang-Barsky
- Clipping to viewing frustum
  - Perspective normalization to orthographic projection
  - Apply clipping to cube from above
- Client-specific clipping
  - Use more general, more expensive form
- Polygon clipping
  - Sutherland-Hodgeman pipeline

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Rasterization

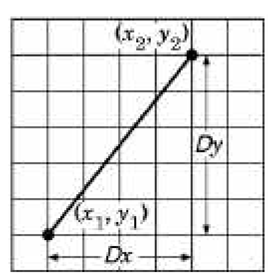
- Final step in pipeline: rasterization (scan conv.)
- From screen coordinates (float) to pixels (int)
- Writing pixels into frame buffer
- Separate z-buffer, display, shading, blending
- Concentrate on primitives:
  - Lines
  - Polygons (Thursday)

DDA Algorithm

- DDA ("Digital Differential Analyzer")
- Represent

$$y = mx + h \quad \text{where} \quad m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{\Delta y}{\Delta x}$$

- Assume $0 \leq$ m ≤ 1
- Exploit symmetry
- Distinguish special cases

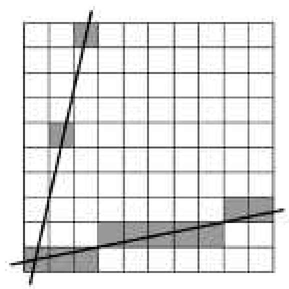


DDA Loop

- Assume write_pixel(int *x*, int *y*, int *value*)

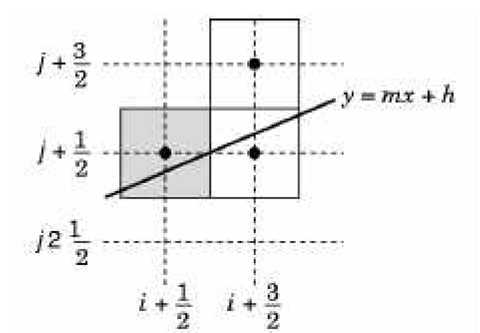
```
For (ix = x1; ix <= x2; ix++)
{
   y += m;
   write_pixel(ix, round(y), color);
}
```

- Slope restriction needed
- Easy to interpolate colors



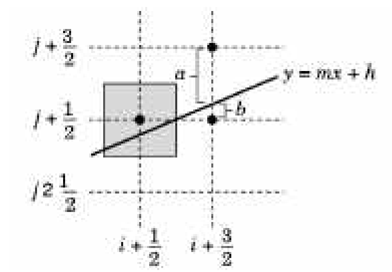
Bresenham's Algorithm I

- Eliminate floating point addition from DDA
- Assume again $0 \leq m \leq 1$
- Assume pixel centers halfway between ints



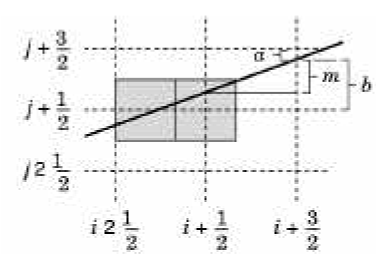
Bresenham’s Algorithm II

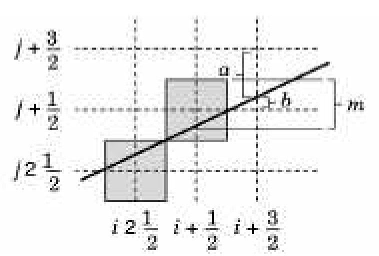
- Decision variable a – b
  - If a – b > 0 choose lower pixel
  - If a – b $\leq$ 0 choose higher pixel
- Goal: avoid explicit computation of a – b
- Step 1: re-scale d = $(x_2 - x_1)(a - b) = \Delta x(a - b)$
- d is always integer



Bresenham’s Algorithm III

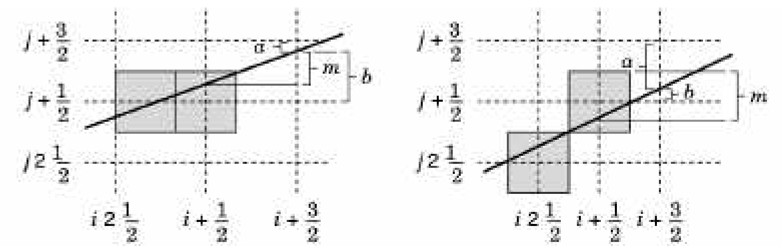
- Compute d at step k +1 from d at step k!
- Case: j did not change (d_k > 0)
  - a decreases by m, b increases by m
  - (a – b) decreases by 2m = 2(Δy/Δx)
  - Δx(a-b) decreases by 2Δy





Bresenham's Algorithm IV

- Case: j did change ($d_k \leq 0$)
  - a decreases by m-1, b increases by m-1
  - (a – b) decreases by 2m – 2 = 2(Δy/Δx – 1)
  - Δx(a-b) decreases by 2(Δy - Δx)



Bresenham's Algorithm V

- So $d_{k+1} = d_k - 2\Delta y$ if $d_k > 0$
- And $d_{k+1} = d_k - 2(\Delta y - \Delta x)$ if $d_k \leq 0$
- Final (efficient) implementation:

```
void draw_line(int x1, int y1, int x2, int y2) {
    int x, y = y0;
    int dx = 2*(x2-x1), dy = 2*(y2-y1);
    int dydx = dy-dx, D = (dy-dx)/2;

    for (x = x1 ; x <= x2 ; x++) {
        write_pixel(x, y, color);
        if (D > 0) D -= dy;
        else {y++; D -= dydx;}
    }
}
```

Bresenham's Algorithm VI

- Need different cases to handle other m
- Highly efficient
- Easy to implement in hardware and software
- Widely used

Summary

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  - Liang-Barsky
- Polygon Clipping
  - Sutherland-Hodgeman
- Clipping in Three Dimensions
- Scan Conversion
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Preview

- Scan conversion of polygons
- Anti-aliasing
- Other pixel-level operations
- Assignment 5 due Thursday
- Assignment 6 (written) out Thursday