

15-462 Computer Graphics I
Lecture 12

Physically-Based Modeling

Overview
Newtonian Particles
Spring Forces
Solving Particle Systems
Constraints
[Angel, Ch 11.1-11.5]

March 12, 2002
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<http://www.cs.cmu.edu/~fp/courses/graphics/>

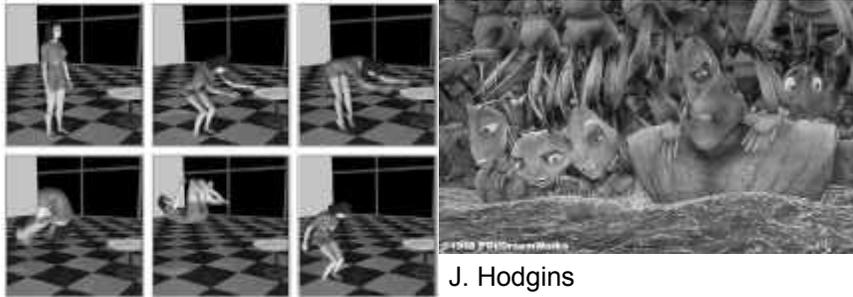
Motivation

- Animation
 - Frame-by-frame or keyframing (artist's intuition)
 - Performance-based (motion capture)
 - Physically-based (dynamic simulation)
 - Trade-offs and combinations
- Interaction
 - Model deformation and acquisition (haptics)

Dynamic Simulation

- Realistic motion
- High-level control
- Passive vs active systems
- Control and constraints

Antz PDI/Dreamworks



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3

Scientific Computation and Graphics

- Scientific computation
 - Specified error tolerance
 - Must be numerically accurate to tolerance
 - Often difficult to compute and unstable
- Computer graphics
 - Viewer's tolerance dominates
 - Need not be numerically accurate
 - Can be faster to compute
- Cross-fertilization
 - Scientific visualization
 - Physically-based modeling

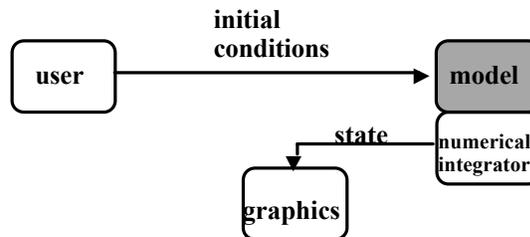
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4

Passive Systems

- No muscles or motors (no internal sources of energy)



- Examples
 - Particle systems
 - Leaves
 - Water spray
 - Clothing

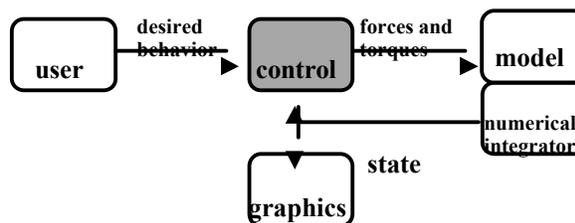
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5

Active Systems

- Internal source of energy or control



- Examples
 - Running human
 - Swimming fish
 - Driven car
 - Airplane

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6

Dynamics

- Generating motion by applying physical laws
 - Typical: Newton's laws, Hook's law
 - Particles, soft objects, rigid bodies
- Simulates physical phenomena
 - Gravity
 - Momentum (inertia)
 - Collisions
 - Friction
 - Fluid flow (drag, turbulence, surface tension, ...)
 - Solidity, flexibility, elasticity
 - Fracture

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7

Outline

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8

Newtonian Particles

- Basis for many forms of simulation
- $m\mathbf{a} = \mathbf{f}$
 - Scalar m : mass
 - Vector \mathbf{a} : acceleration
 - Vector \mathbf{f} : force
- Particle determined by position and velocity

$$\mathbf{p} = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \quad \mathbf{v} = \begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \end{bmatrix} = \begin{bmatrix} \frac{dx}{dt} \\ \frac{dy}{dt} \\ \frac{dz}{dt} \end{bmatrix}$$

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9

Newton's Law

- Six coupled first-order differential equations
$$\dot{\mathbf{p}} = \mathbf{v}$$
$$\dot{\mathbf{v}} = \frac{1}{m}\mathbf{f}(t)$$
- Rendering of each particle is orthogonal issue
 - Just a dot
 - Person or animal
- Can simulate many independent particles

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10

Simple Pseudocode Template

- Assume n particles

```
Float time, delta;
Float state[6n], force[3n];
State = get_initial_state();
For (time = init; time < final; time += delta)
{
    force = force_function(state, time);
    state = ode(force, state, time, delta);
    render(state, time);
}
```

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11

Independent Particles

- Example: gravity for particle i :

$$\mathbf{f}_i(\mathbf{p}_i, \mathbf{v}_i) = \mathbf{f}_i = \mathbf{g} = \begin{bmatrix} 0 \\ -g \\ 0 \end{bmatrix}$$

- Simple approximate computation

$$\mathbf{v}' = \mathbf{v} + \mathbf{g}\Delta t$$

$$\mathbf{p}' = \mathbf{p} + \frac{\mathbf{v} + \mathbf{v}'}{2} \Delta t$$

- May be enough, depending on application
- Numerical integration is more general solution

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Particle Systems, Example I

- Clip from Karl Sims, *Particle Dreams*, 1988



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Particle Systems, Example II

- Clip from *Star Trek II: The Wrath of Khan*



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Particle Systems: Other examples

- Clouds
- Smoke
- Fire
- Waterfalls
- Fireworks
- *Batman Returns*, using Reynolds' flocking algs.
- *The Lion King*, Wildebeest stampede

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Particle Systems

- Creation
 - Number
 - Position and velocity
 - Shape, size, material properties
 - Lifetime
- Update of position and velocity
- Deletion
- Rendering style
 - Motion blur
 - Compositing

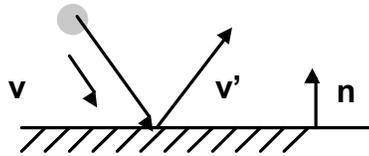
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16

Other Forces

- Unary
 - Gravity: $\mathbf{f} = m \mathbf{g}$
 - Viscous drag: $\mathbf{f} = -k \mathbf{v}$
- N-ary
 - Spring: $\mathbf{f} = -k \mathbf{d}$
- Spatial interaction force
 - Collisions
 - $\mathbf{v}' = \mathbf{v} - 2(\mathbf{v} \cdot \mathbf{n})\mathbf{n}$



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17

Outline

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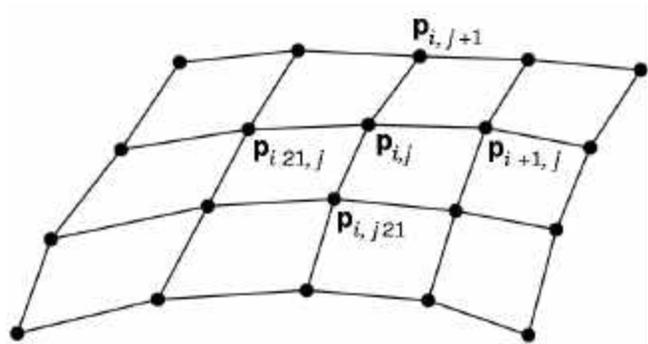
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18

Spring-Mass Systems

- Examples: cloth in 2D, jello in 3D
- In general, $O(n^2)$, sometimes can limit to $O(n)$



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19

Spring-Mass Systems: Example



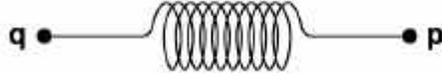
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20

Hook's Law

- Simple linear spring force approximation



- Hooke's law

$$\mathbf{f} = -k_s(|\mathbf{d}| - s) \frac{\mathbf{d}}{|\mathbf{d}|}$$

\mathbf{f} = force
 k_s = spring constant
 $\mathbf{d} = \mathbf{p} - \mathbf{q}$ (distance)
 s = resting length

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21

Damping Term

- Damping or drag depends on velocity
- Acts in direction of spring force
- Proportional to projection of velocity vector onto distance vector

$$\mathbf{f} = - \left(k_s(|\mathbf{d}| - s) + k_d \frac{\dot{\mathbf{d}} \cdot \mathbf{d}}{|\mathbf{d}|} \right) \frac{\mathbf{d}}{|\mathbf{d}|}$$

$$\dot{\mathbf{d}} = \dot{\mathbf{p}} - \dot{\mathbf{q}}$$

k_d = damping constant

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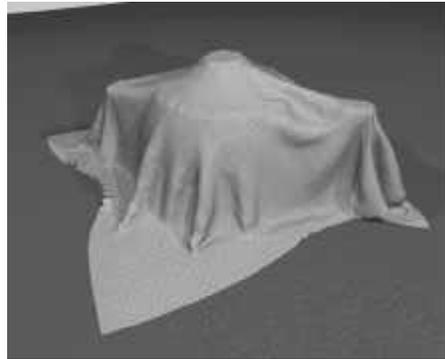
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Cloth

- Example of spring-mass system
- Many different varieties

Breen, 1995



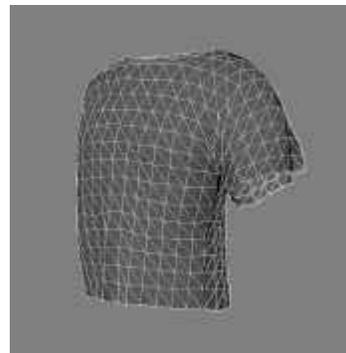
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Mesh Size vs Computation Time

- Higher resolution is slower to compute
- Resolution in space vs resolution in time
- Example: modeling for clothing



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Collisions for Clothing

- Computationally very expensive
- Using bounding box hierarchy to reduce tests



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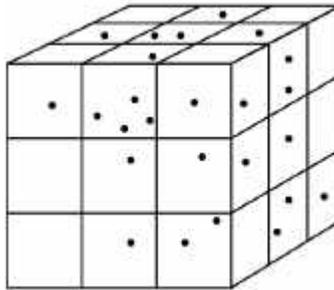
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Attractive and Repulsive Forces

- Avoid collisions (e.g., stampede)
- Example: force is inversely proportional to square of distance

$$f = -k_r \frac{d}{|d|^3}$$



- Divide space into cells to avoid $O(n^2)$ calcn.

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26

Outline

- Overview
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Ordinary Differential Equations

- Applicable when forces are simple
$$\dot{\mathbf{u}} = \mathbf{g}(\mathbf{u}, t)$$
- \mathbf{u} represents both position and velocity vectors
- \mathbf{g} includes external forces
- Use numerical ODE solvers
- Approximate \mathbf{u} over short time h

$$\int_t^{t+h} \dot{\mathbf{u}} d\tau = \mathbf{u}(t+h) - \mathbf{u}(t) = \int_t^{t+h} \mathbf{g}(\mathbf{u}, \tau) d\tau$$

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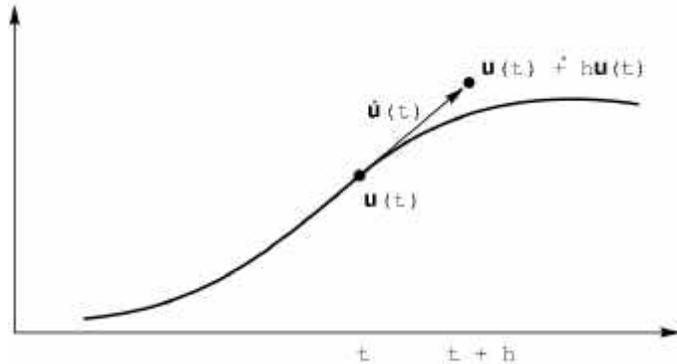
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Euler's Method

- If h is small, approximate \mathbf{g} over $[t, t+h]$ by $\mathbf{g}(t)$

$$\mathbf{u}(t+h) \approx \mathbf{u}(t) + h\mathbf{g}(\mathbf{u}(t), t)$$



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29

Assessing Euler's Method

- Particularly easy to implement

$$\mathbf{u}(t+h) = \mathbf{u}(t) + h\mathbf{g}(\mathbf{u}(t), t) + O(h^2)$$

- Accuracy depends on step size
- Accumulating errors (numerical instability)
- Specialized method for certain applications
 - Cloth, clothing, soft object

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30

Runge-Kutta Method

- Straightforward improvement

$$\mathbf{u}(t+h) = \mathbf{u}(t) + \int_t^{t+h} \mathbf{g}(\mathbf{u}, \tau) d\tau$$

- Use average of \mathbf{g} over $[t, t+h]$ to approx integral

$$\int_t^{t+h} \mathbf{g}(\mathbf{u}, \tau) d\tau \approx \frac{h}{2}(\mathbf{g}(\mathbf{u}(t), t) + \mathbf{g}(\mathbf{u}(t+h), t+h))$$

- Need to evaluate \mathbf{g} twice
- Error only $O(h^3)$, other orders similar

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31

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- Overview
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32

Constraints

- Hard constraints
 - Collisions
 - Contact force
 - Joints
- Soft constraints
 - Preservation of energy
 - Other examples?

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33

Collisions

- Often expensive to compute (bounding boxes)
- Elastic collision
 - Particle retains all energy
 - Reflected like a light beam
 - Need surface normal
- Inelastic collision
 - Coefficient of restitution: fraction of normal velocity
 - Example: simple bouncing ball
- Example: particles inside a sphere

[Angel, Ch 11.5.2]

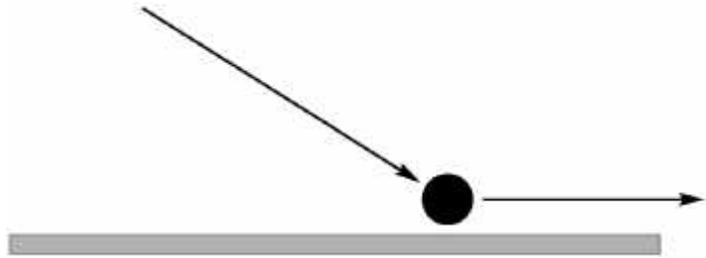
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Contact Forces

- Useful for programming assignment 5
- Decompose force vector into normal and tangential component
- Apply frictional term in tangential component



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35

Summary

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Physically-Based Modeling

- Only scratched the surface (so to speak)
- Active and exciting research area
- Example: Doug James, CMU faculty candidate



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37

Preview

- Thursday: Texture Mapping [Shayan Sarkar]
- Next week: the “end” of the graphics pipeline
- Assignment 5 due next week
- Pick up midterm now

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38