15-462 Computer Graphics I
Lecture 11

Midterm Review

Assignment 3 Movie
Midterm Review
Midterm Preview

February 26, 2002
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http://www.cs.cmu.edu/~fp/courses/graphics/

Announcements

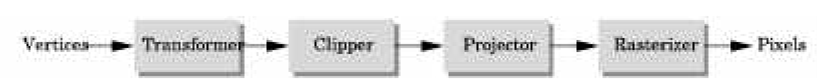
- Assignment 4 due Thursday before lecture
- Lecture by John Ketchpaw
- Midterm next Tuesday
  - In class
  - Closed book
  - One double-sided sheet of notes permitted
  - Everything covered in lecture so far
- Assignment 3 movies
  - Some flaws may be problems in production software
  - Enjoy!

1. Course Overview Revisited

- Modeling: how to represent objects
- Animation: how to control and represent motion
- Rendering: how to create images
- OpenGL graphics library

2. Basic Graphics Programming

- The graphics pipeline



- Pipelines and parallelism
- Latency vs throughput
- Efficiently implementable in hardware
- Not so efficiently implementable in software
- Course approach: walk the pipeline left-to-right

Graphics Functions

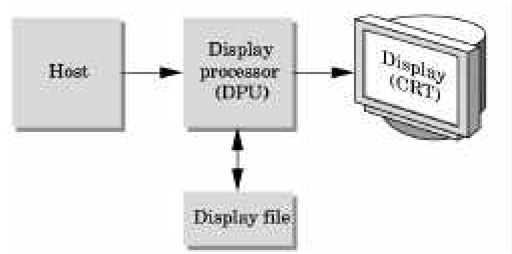
- Primitive functions (points, lines, polygons)
- Attribute functions (color, lighting, material)
- Transformation functions (homogeneous coord)
- Viewing functions (projections)
- Input functions (callbacks)
- Control functions (GLUT library calls)

3. Interaction

- Client/Server Model
- Callbacks
- Double Buffering
- Hidden Surface Removal

Client/Server Model

- Graphics hardware and caching



- Important for efficiency
- Need to be aware where data are stored
- Examples: vertex arrays, display lists

Hidden Surface Removal

- Classic problem of computer graphics
- What is visible after clipping and projection?
- Object-space vs image-space approaches
- Object space: depth sort (Painter’s algorithm)
- Image space: ray cast (z-buffer algorithm)
- Related: back-face culling

4. Transformations

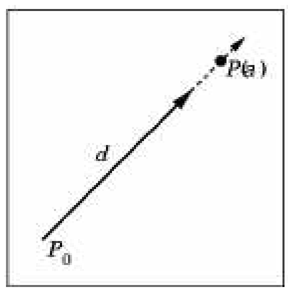
- Vector Spaces
- Affine and Euclidean Spaces
- Frames
- Homogeneous Coordinates
- Transformation Matrices
- OpenGL Transformation Matrices

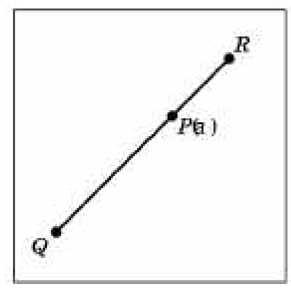
Geometric Interpretations

- Lines and line segments
- Convexity
- Dot product and projections
- Cross product and normal vectors
- Planes

Lines and Line Segments

- Parametric form of line: $P(\alpha) = P_0 + \alpha\, d$



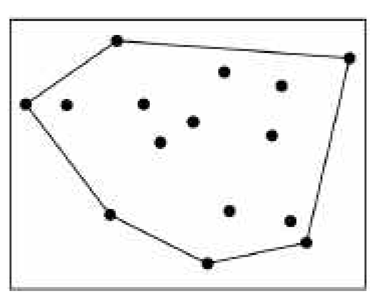


- Line segment between Q and R:
  $P(\alpha) = (1-\alpha)\, Q + \alpha\, R$ for $0 \leq \alpha \leq 1$

Convex Hull

- Convex hull defined by

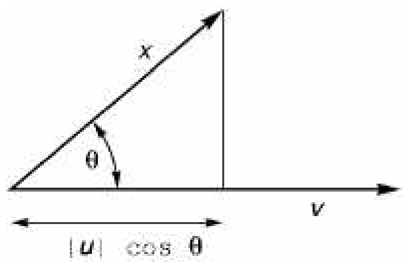
  $P = \alpha_1 P_1 + \cdots + \alpha_n P_n$
  for $a_1 + \cdots + a_n = 1$
  and $0 \leq a_i \leq 1$, i = 1, ..., n



Projection

- Dot product projects one vector onto other

  $u \cdot v = |u|\ |v| \cos(\theta)$



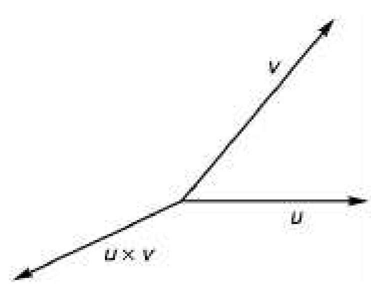
[diagram correction: x = u]

Normal Vector

- Cross product defines normal vector

  $u \times v = n$

  $|u \times v| = |u|\ |v|\ |\sin(\theta)|$
- Right-hand rule



Plane

- Plane defined by point P_0 and vectors u and v
- u and v cannot be parallel
- Parametric form: $T(\alpha, \beta) = P_0 + \alpha u + \beta v$
- Let $n = u \times v$ be the normal
- Then $n \cdot (P - P_0) = 0$ iff P lies in plane

Homogeneous Coordinates

- In affine space, $P = \alpha_1 v_1 + \alpha_2 v_2 + \alpha_3 v_3 + P_0$
- Define $0 \cdot P = 0$, $1 \cdot P = P$
- Points $[\alpha_1 \;\; \alpha_2 \;\; \alpha_3 \;\; 1]^T$
- Vectors $[\delta_1 \;\; \delta_2 \;\; \delta_3 \;\; 0]^T$
- Change of frame

$$\mathbf{M} = \begin{bmatrix} \gamma_{11} & \gamma_{12} & \gamma_{13} & 0 \\ \gamma_{21} & \gamma_{22} & \gamma_{23} & 0 \\ \gamma_{31} & \gamma_{32} & \gamma_{33} & 0 \\ \gamma_{41} & \gamma_{42} & \gamma_{43} & 1 \end{bmatrix}$$

Affine Transformations

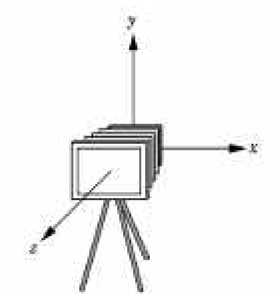
- Compose
  - Rotations, translations, scalings
  - Express in homogeneous coods (4 $\times$ 4 matrices)
- Apply from right to left!
  - $\mathbf{R}\,\mathbf{p} = (\mathbf{R}_z\,\mathbf{R}_y\,\mathbf{R}_x)\,\mathbf{p} = \mathbf{R}_z\,(\mathbf{R}_y\,(\mathbf{R}_x\,p))$
  - Postmultiplication in OpenGL
- Think in terms of composition
  - Translation to and from origin
  - Remember geometric intuition

5. Viewing and Projection

- Camera Positioning
- Parallel Projections
- Perspective Projections

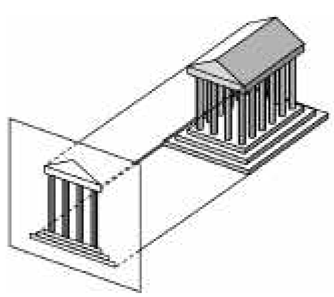
Camera in Modeling Coordinates

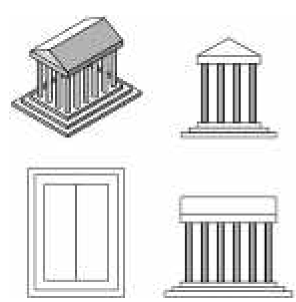
- Camera position is identified with a frame
- Either move and rotate the objects
- Or move and rotate the camera
- Those views are inverses!
  - Each transformation
  - Order of transformation
  - gluLookAt utility



Orthographic Projections

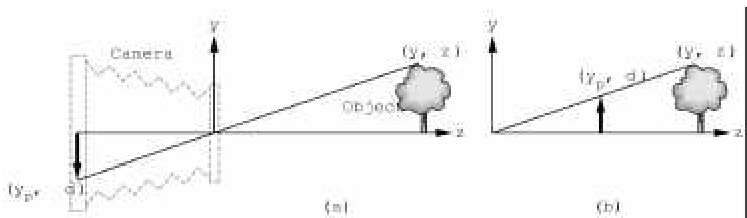
- Projectors perpendicular to projectoin plane
- Simple, but not realistic





Perspective Viewing

- Characterized by foreshortening
- More distant objects appear smaller



- $y/z = y_p/d$ so $y_p = y/(z/d)$
- Note this is non-linear!
- Need homogeneous coordinates

Perspective Projection Matrix

- Represent multiple of point

$$(z/d)\begin{bmatrix} \frac{x}{z/d} \\ \frac{y}{z/d} \\ d \\ 1 \end{bmatrix} = \begin{bmatrix} x \\ y \\ z \\ z/d \end{bmatrix}$$

- Solve

$$\mathrm{M}\begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} = \begin{bmatrix} x \\ y \\ z \\ z/d \end{bmatrix} \quad \text{with} \quad \mathrm{M} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 1/d & 0 \end{bmatrix}$$

6. Hierarchical Models

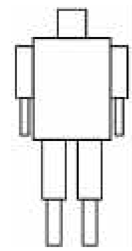
- Matrix and attribute stacks
- Save and restore state
- Exploit natural hierarchical structure for
  - Efficient rendering
  - Example: bounding boxes (later in course)
  - Concise specification of model parameters
  - Example: joint angles
  - Physical realism

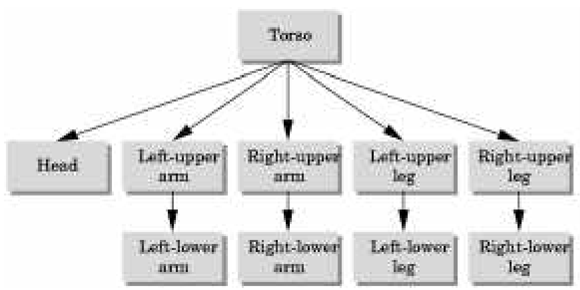
Hierarchical Objects and Animation

- Drawing functions are time-invariant
- Can be easily stored in display list
- Change parameters of model with time
- Redraw when idle callback is invoked

Complex Objects

- Tree rather than linear structure
- Interleave along each branch
- Use push and pop to save state





Unified View of Computer Animation

- Models with parameters
  - Polygon positions, control points, joint angles, ...
  - *n* parameters define n-dimensional state space
- Animation defined by path through state space
  - Define initial state, repeat:
  - Render the image
  - Move to next point (following motion curves)
- Animation = specifying state space trajectory

Animation vs Modeling

- Modeling: what are the parameters?
- Animation: how do we vary the parameters?
- Sometimes boundary not clear
- Build models that are easy to control
- Hierarchical models often easy to control

Basic Animation Techniques

- Traditional (frame by frame)
- Keyframing
- Procedural techniques
- Behavioral techniques
- Performance-based (motion capture)
- Physically-based (dynamics)

7. Lighting and Shading

- Approximate physical reality
- Ray tracing:
  - Follow light rays through a scene
  - Accurate, but expensive (off-line)
- Radiosity:
  - Calculate surface inter-reflection approximately
  - Accurate, especially interiors, but expensive (off-line)
- Phong Illumination model:
  - Approximate only interaction light, surface, viewer
  - Relatively fast (on-line), supported in OpenGL

Light Sources and Material Properties

- Appearance depends on
  - Light sources, their locations and properties
  - Material (surface) properties
  - Viewer position
- Ray tracing: from viewer into scene
- Radiosity: between surface patches
- Phong Model: at material, from light to viewer

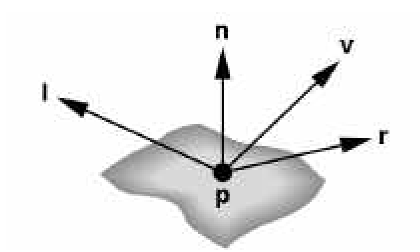
Types of Light Sources

- Ambient light: no identifiable source or direction
- Point source: given only by point
- Distant light: given only by direction
- Spotlight: from source in direction
  - Cut-off angle defines a cone of light
  - Attenuation function (brighter in center)
- Light source described by a luminance
  - Each color is described separately
  - $\mathbf{I} = [I_r \;\; I_g \;\; I_b]^T$ (I for intensity)
  - Sometimes calculate generically (applies to r, g, b)

Phong Illumination Model

- Calculate color for arbitrary point on surface
- Compromise between realism and efficiency
- Local computation (no visibility calculations)
- Basic inputs are material properties and l, n, v:

l = vector to light source
n = surface normal
v = vector to viewer
r = reflection of l at p
(determined by l and n)



Summary of Phong Model

- Light components for each color:
  - Ambient (L_a), diffuse (L_d), specular (L_s)
- Material coefficients for each color:
  - Ambient (k_a), diffuse (k_d), specular (k_s)
- Distance q for surface point from light source

$$I = \frac{1}{a + bq + cq^2}(k_d L_d (\mathbf{l} \cdot \mathbf{n}) + k_s L_s (\mathbf{r} \cdot \mathbf{v})^{\alpha}) + k_a L_a$$

l = vector from light
n = surface normal
r = l reflected about n
v = vector to viewer

Normal Vectors

- Critical for Phong model (diffuse and specular)
- Must calculate accurately
  - From geometry (e.g., differential calculus)
  - From approximating surface (e.g., Bezier patch)
- Pitfalls
  - Unit length (some OpenGL support)
  - Surface boundary

8. Shading in OpenGL

- Polygonal shading
- Material properties
- Approximating a sphere [example]

Polygonal Shading

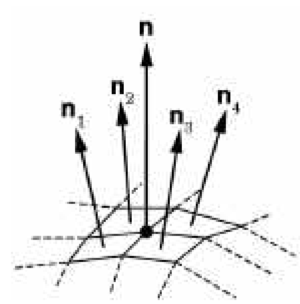
- Curved surfaces are approximated by polygons
- How do we shade?
  - Flat shading
  - Interpolative shading
  - Gouraud shading
  - Phong shading (different from Phong illumination)
- Two questions:
  - How do we determine normals at vertices?
  - How do we calculate shading at interior points?

Gouraud Shading

- Special case of interpolative shading
- How do we calculate vertex normals?
- Gouraud: average all adjacent face normals

$$\mathbf{n} = \frac{\mathbf{n}_1 + \mathbf{n}_2 + \mathbf{n}_3 + \mathbf{n}_4}{|\mathbf{n}_1 + \mathbf{n}_2 + \mathbf{n}_3 + \mathbf{n}_4|}$$

- Requires knowledge about which faces share a vertex

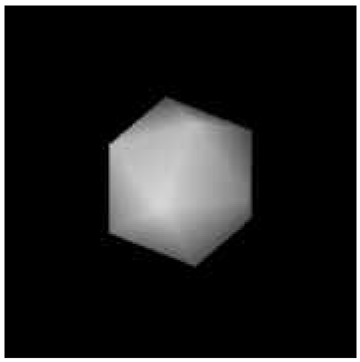


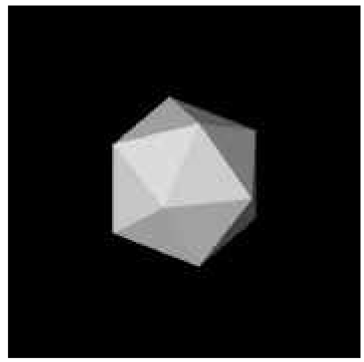
Data Structures for Gouraud Shading

- Sometimes vertex normals can be computed directly (e.g. height field with uniform mesh)
- More generally, need data structure for mesh
- Key: which polygons meet at each vertex

Drawing a Sphere

- Recursive subdivision technique quite general
- Interpolation vs flat shading effect





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Recursive Subdivision

- General method for building approximations
- Research topic: construct a good mesh
  - Low curvature, fewer mesh points
  - High curvature, more mesh points
  - Stop subdivision based on resolution
  - Some advanced data structures for animation
  - Interaction with textures
- Here: simplest case
- Approximate sphere by subdividing icosahedron

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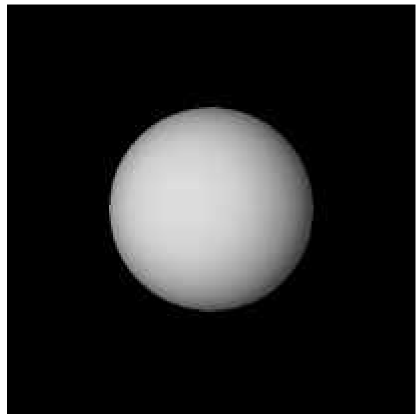
Subdivision Example

- Icosahedron after 3 subdivisions (fast converg.)

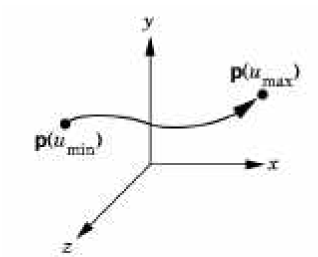



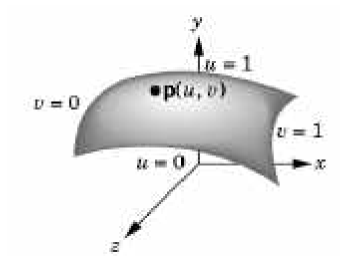
9. Curves and Surfaces

- Parametric Representations
  - Also used: implicit representations
- Cubic Polynomial Forms
- Hermite Curves
- Bezier Curves and Surfaces

Parametric Forms

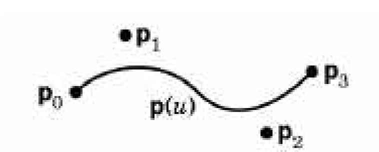
- Parameters often have natural meaning
- Easy to define and calculate
  - Tangent and normal
  - Curves segments (for example, $0 \leq u \leq 1$)
  - Surface patches (for example, $0 \leq u,v \leq 1$)

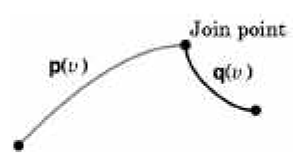




Approximating Surfaces

- Use parametric polynomial surfaces
- Important concepts:
  - Join points for segments and patches
  - Control points to interpolate
  - Tangents and smoothness
  - Blending functions to describe interpolation
- First curves, then surfaces





Cubic Polynomial Form

- Degree 3 appears to be a useful compromise
- Curves:

$$\mathbf{p}(u) = \mathbf{c}_0 + \mathbf{c}_1 u + \mathbf{c}_2 u^2 + \mathbf{c}_3 u^3 = \sum_{k=0}^{3} \mathbf{c}_k u^k$$

- Each c_k is a column vector $[c_{kx} \ \ c_{ky} \ \ c_{kz}]^T$
- From control information (points, tangents) derive 12 values c_{kx}, c_{ky}, c_{kz} for $0 \leq k \leq 3$
- These determine cubic polynomial form

Geometry Matrix

- Calculate approximating polynomial from control point with geometry matrix M

$$\mathbf{p}(u) = \mathbf{c}_0 + \mathbf{c}_1 u + \mathbf{c}_2 u^2 + \mathbf{c}_3 u^3$$

$$\begin{bmatrix} \mathbf{c}_0 \\ \mathbf{c}_1 \\ \mathbf{c}_2 \\ \mathbf{c}_3 \end{bmatrix} = \mathbf{M} \begin{bmatrix} \mathbf{p}_0 \\ \mathbf{p}_1 \\ \mathbf{p}_2 \\ \mathbf{p}_3 \end{bmatrix}$$

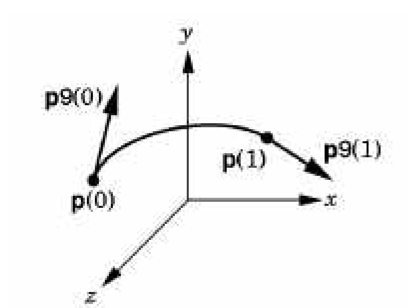
- Each form of interpolation has its own geometry matrix

Standard Methods

- Hermite curves
  - Given by 2 points, 2 tangents
  - C^1 continuity, intersect control points
- Bezier curves
  - Given by 4 control points
  - Intersects 2, others approximate tangent
- Bezier surface patches
  - Given by 16 control points
  - Intersects 4 corners, other approximate tangents

Hermite Curves

- Another cubic polynomial curve
- Specify two endpoints and their tangents



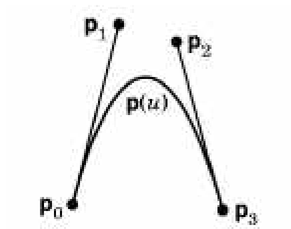
[diagram correction p9 = p']

Bezier Curves

- Widely used in computer graphics
- Approximate tangents by using control points

$$p'(0) = 3(p_1 - p_0)$$

$$p'(1) = 3(p_3 - p_2)$$

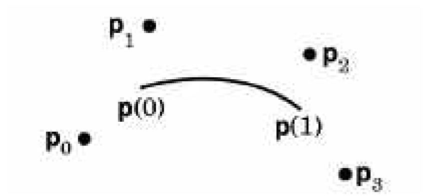


10. Splines

- Approximating more than 4 control points
- Piecing together a longer curve or surface

B-Splines

- Use 4 points, but approximate only middle two



- Draw curve with overlapping segments
  0-1-2-3, 1-2-3-4, 2-3-4-5, 3-4-5-6, etc.
- Curve may miss all control points
- Smoother at joint points

Cubic B-Splines

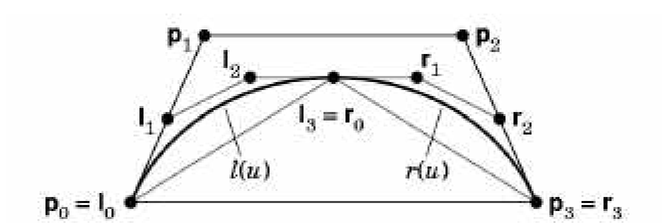
- Need m+2 control points for m cubic segments
- Computationally 3 times more expensive
- C^2 continuous at each interior point
- Derive as follows:
  - Consider two overlapping segments
  - Enforce C^0 and C^1 continuity
  - Employ symmetry
  - C^2 continuity follows

Rendering by Subdivision

- Divide the curve into smaller subpieces
- Stop when “flat” or at fixed depth
- How do we calculate the sub-curves?
  - Bezier curves and surfaces: easy (next)
  - Other curves: convert to Bezier!

Subdividing Bezier Curves

- Given Bezier curve by p_0, p_1, p_2, p_3
- Find l_0, l_1, l_2, l_3 and r_0, r_1, r_2, r_3
- Subcurves should stay the same!



Preview I

- Physically based models
  - Particle systems
  - Spring forces (cloth)
  - Collisions and constraints
- Rendering
  - Clipping, bounding boxes
  - Line drawing
  - Scan conversion
  - Anti-aliasing

Preview II

- Textures and pixels
  - Texture mapping
  - Bump maps
  - Environment maps
  - Opacity and blending
  - Filtering
  - Image transformation
- Ray tracing
  - Spatial data structures
  - Bounding volumes

Preview III

- Radiosity
  - Inter-surface reflections
  - Ray casting
- Scientific visualization
  - Height fields and contours
  - Isosurfaces
  - Marching cubes
  - Volume rendering
  - Volume textures

Announcements

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- Lecture by John Ketchpaw
- Midterm next Tuesday
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  - Everything covered in lecture so far