

Estimating Approximate Incentive Compatibility

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Berkeley (EECS) → Stanford (MS&E and CS)

Working paper, preliminary version in
Conference on Economics and Computation (EC)

Incentive compatibility (IC)

Agents maximize utility by reporting type truthfully
Fundamental concept in mechanism design

Many real-world mechanisms are **not** IC

Discriminatory auctions

Multi-unit variant of first-price auction

Not incentive compatible

Used to sell treasury bills since 1929

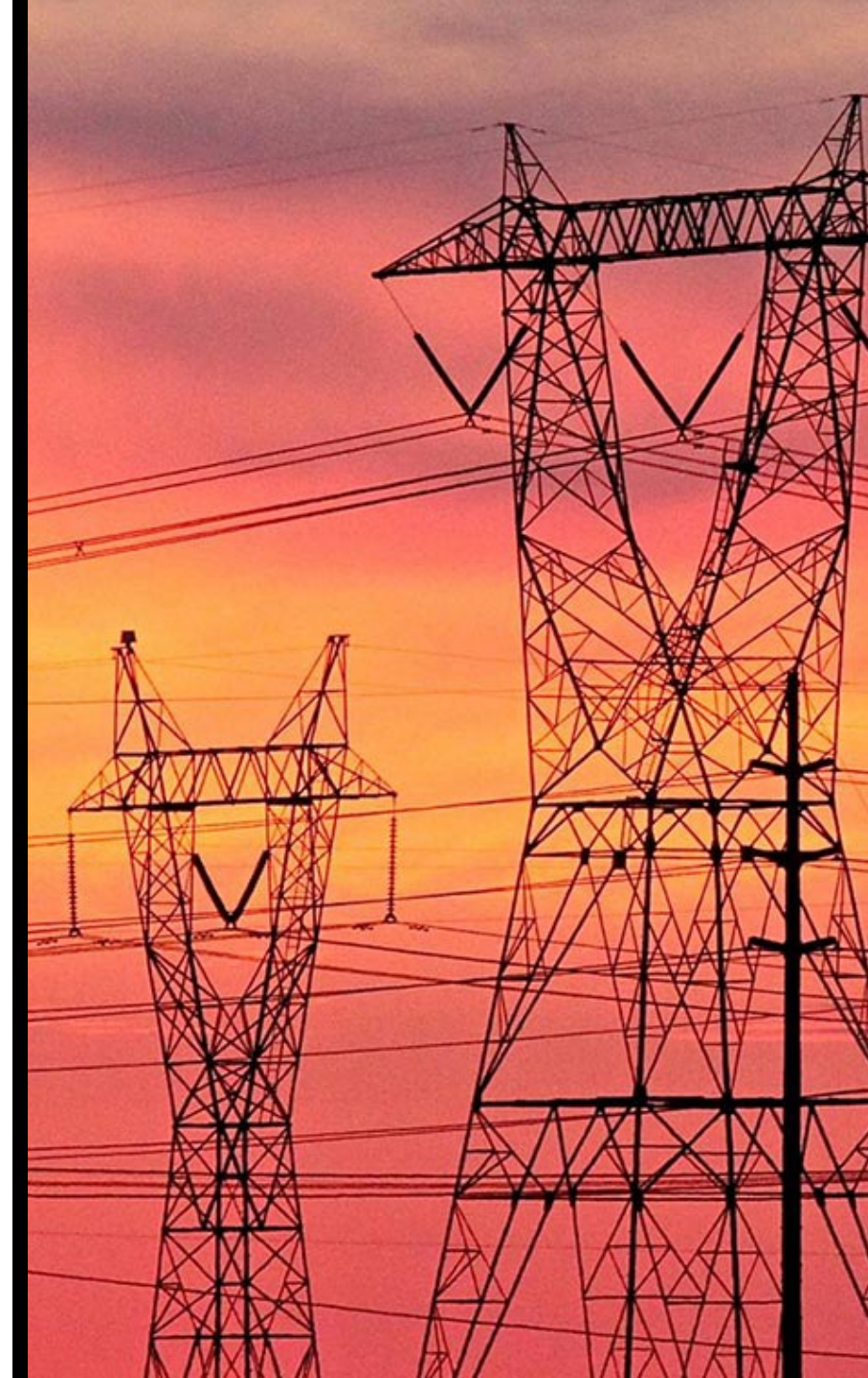


Discriminatory auctions

Multi-unit variant of first-price auction

Not incentive compatible

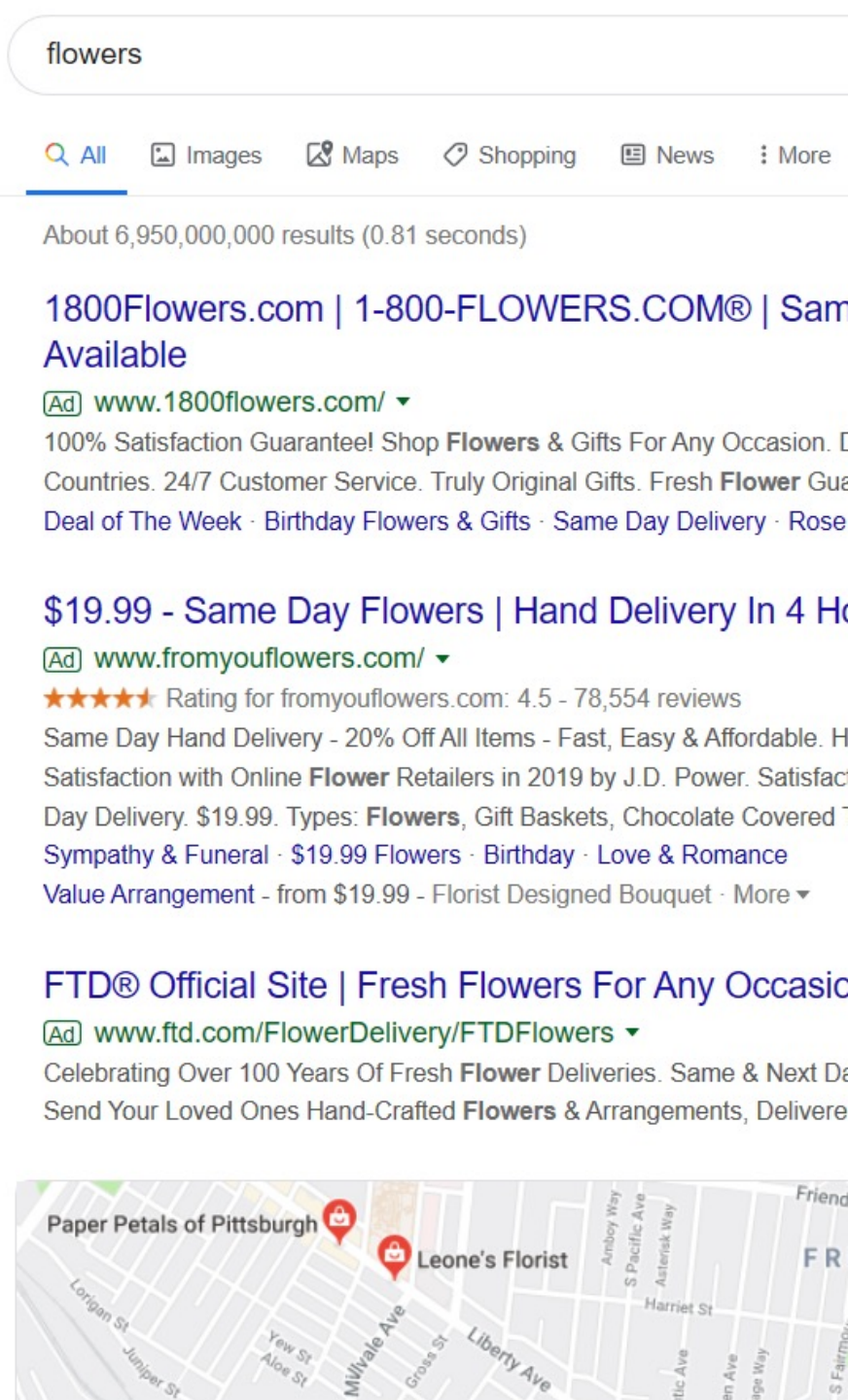
Used to sell treasury bills since 1929
and electricity in the UK



Generalized 2nd-price

Used for sponsored search

Not incentive compatible



Multi-item, multi-agent auctions

Nearly all fielded combinatorial auctions
(such as sourcing auctions)
aren't incentive compatible



Why aren't real-world auctions IC?

Rules are **easier** to explain

Bids used to tune **future** parameters

Might leak **private** values

Agents not **risk** neutral

Approximate incentive compatibility

Auction is γ -IC when for each agent i :

If everyone except agent i is truthful,

she can only increase exp. utility by γ when lies about type

Kothari, Parkes, Suri, EC'03; Archer, Papadimitriou, Talwar, Tardos, Internet Mathematics '04; Conitzer and Sandholm, IJCAI'07; Dekel, Fischer, Procaccia, JCSS'10; Lubin, Parkes, Current Science '12; Mennle and Seuken, EC'14; Dütting, Fischer, Jirapinyo, Lai, Lubin, Parkes TEAC'15; Azevedo, Budish, Review of Economic Studies '18; Feng, Narasimhan, Parkes, AAMAS'18; Golowich, Narasimhan, Parkes, IJCAI'18; Dütting, Feng, Narasimhan, Parkes, Ravindranath, ICML'19

Approximate incentive compatibility

Auction is **γ -IC** when for each agent i :

If everyone except agent i is truthful,

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Overriding goal: Given **samples** from dist. over agents' types, estimate IC approximation factor (γ) using samples

Complements literature on sample-based **revenue maximization**

Likhodedov, Sandholm, AAAI'04, '05; Balcan, Blum, Hartline, Mansour, FOCS'05; Elkind, SODA'07; Cole, Roughgarden, STOC'14; Mohri, Medina, ICML'14; Huang, Mansour, Roughgarden, EC'15; Morgenstern, Roughgarden, NeurIPS'15, COLT'16; Roughgarden, Schrijvers, EC'16; Devanur, Huang, Psomas, STOC'16; Balcan, Sandholm, **V.**, NeurIPS'16; Gonczarowski, **Nisan**, STOC'17; Cai, Daskalakis, FOCS'17; Balcan, Sandholm, **V.**, EC'18; ...

Why estimate IC approximation factor?

Some mechanisms might have terrible **worst case** IC apx factor,
but are (nearly) IC for distribution over agents' types at hand

Use mechanism might have discarded as non-IC,
remaining optimistic that agents will be truthful

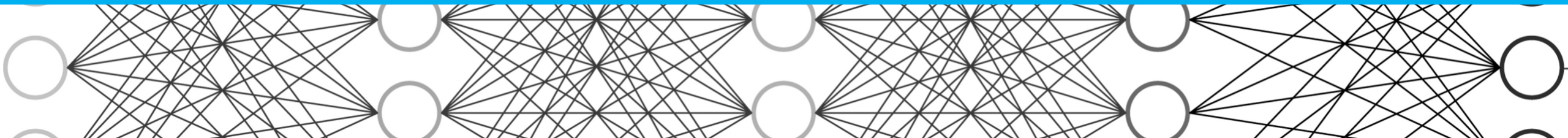
Why estimate IC approximation factor?

In mechanism design via machine learning:

Add constraint requiring this estimate be small

[Feng, Narasimhan, Parkes, AAMAS'18; Golowich, Narasimhan, Parkes, IJCAI'18; Dütting, Feng, Narasimhan, Parkes, Ravindranath, ICML'19]

Is resulting mechanism (nearly) IC?



Background

n agents with types in $[0,1]^M$

Standard assumption:

Agents' types drawn from probability distribution $(\mathbf{t}_1, \dots, \mathbf{t}_n) \sim \mathcal{D}$

In this talk, \mathcal{D} is product distribution: $\mathbf{t}_i \sim \mathcal{D}_i$ and $\mathcal{D} = \mathcal{D}_1 \times \dots \times \mathcal{D}_n$

$$\mathcal{D}_{-i} = \times_{j \neq i} \mathcal{D}_j$$

Incentive compatibility

Auction is **incentive compatible** (IC) if for any agent:

In expectation over other's types,

utility maximized by reporting type truthfully,

so long as others also truthful

Utility of agent i : $u(t_i, \tilde{t}_i, \mathbf{t}_{-i})$

Incentive compatibility

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True type

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Reported type

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Others' types

Incentive compatibility

Auction is **incentive compatible** (IC) if for any agent:

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Mechanism is IC if for any agent i and any t_i, \tilde{t}_i ,

$$\mathbb{E}_{t_{-i}}[u(t_i, t_i, \mathbf{t}_{-i})] \geq \mathbb{E}_{t_{-i}}[u(t_i, \tilde{t}_i, \mathbf{t}_{-i})]$$

Utility from
truthful report

Utility from
strategic report

"Ex-interim" IC

Approximate incentive compatibility

Auction is **γ -IC** if for any agent i and any t_i, \tilde{t}_i ,

$$\mathbb{E}_{\mathbf{t}_{-i}}[u(t_i, t_i, \mathbf{t}_{-i})] \geq \mathbb{E}_{\mathbf{t}_{-i}}[u(t_i, \tilde{t}_i, \mathbf{t}_{-i})] - \gamma$$

Utility from
truthful report

Utility from
strategic report

Kothari, Parkes, Suri, EC'03; Archer, Papadimitriou, Talwar, Tardos, Internet Mathematics '04; Conitzer and Sandholm, IJCAI'07; Dekel, Fischer, Procaccia, JCSS'10; Lubin, Parkes, Current Science '12; Mennle and Seuken, EC'14; Dütting, Fischer, Jirapinyo, Lai, Lubin, Parkes TEAC'15; Azevedo, Budish, Review of Economic Studies '18; Feng, Narasimhan, Parkes, AAMAS'18; Golowich, Narasimhan, Parkes, IJCAI'18; Dütting, Feng, Narasimhan, Parkes, Ravindranath, ICML'19

Our goal: Estimate IC approximation factor (γ) using samples

Our estimate

Our estimate (first try):

Maximum utility agent i can gain by misreporting her type, on average over samples $\mathbf{t}_{-i}^{(1)}, \dots, \mathbf{t}_{-i}^{(N)} \sim \mathcal{D}_{-i}$:

$$\max_{t_i, \tilde{t}_i \in \mathbb{R}^M} \left\{ \frac{1}{N} \sum_{j=1}^N u \left(t_i, \tilde{t}_i, \mathbf{t}_{-i}^{(j)} \right) - u \left(t_i, t_i, \mathbf{t}_{-i}^{(j)} \right) \right\}$$

Utility from strategic report **Utility from truthful report**

Our estimate

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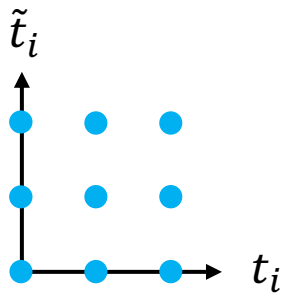
Not convex
and many
discontinuities

Our estimate

Our estimate $\hat{\gamma}$:

Maximum utility agent i can gain by misreporting her type,
on average over samples $\mathbf{t}_{-i}^{(1)}, \dots, \mathbf{t}_{-i}^{(N)} \sim \mathcal{D}_{-i}$,
if true & reported types from uniform grid \mathcal{G}

$$\hat{\gamma} = \max_{t_i, \tilde{t}_i \in \mathcal{G}} \left\{ \frac{1}{N} \sum_{j=1}^N u \left(t_i, \tilde{t}_i, \mathbf{t}_{-i}^{(j)} \right) - u \left(t_i, t_i, \mathbf{t}_{-i}^{(j)} \right) \right\}$$



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Challenge:

Might miss pairs of true & reported types with large utility gains

Our estimate

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Key question:

$$|\gamma - \hat{\gamma}| \leq ?$$

Uniform grid

Challenge:

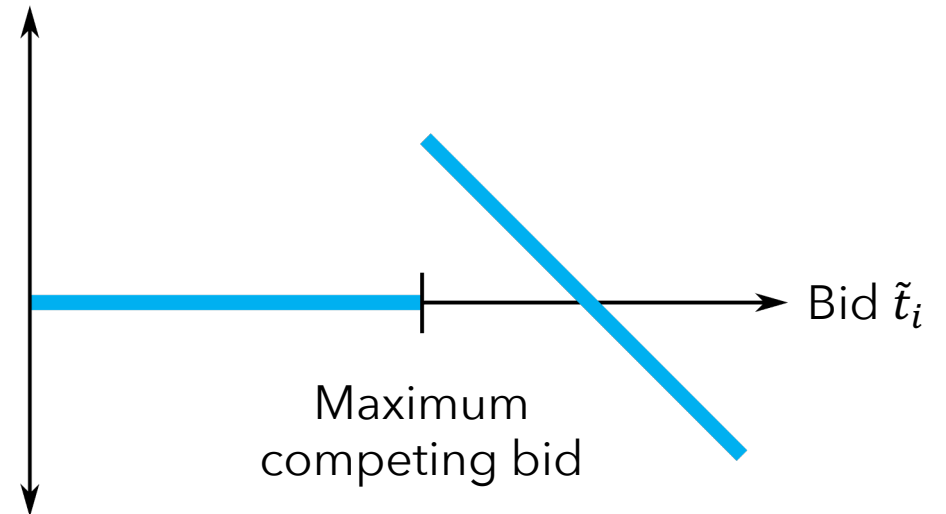
Utility functions are volatile

First-price auction:

Highest bidder wins

Pays highest bid

First-price auction
utility $u(t_i, \mathbf{t}_{-i})$



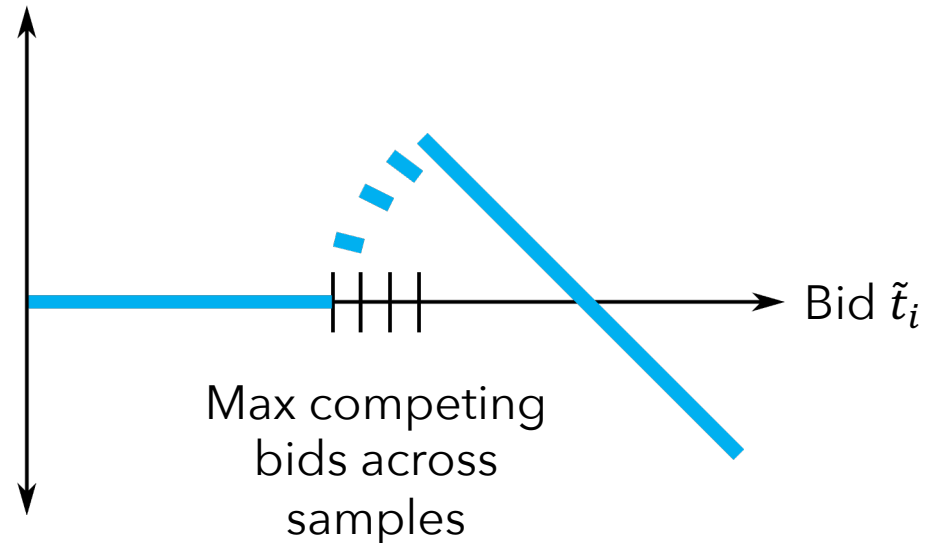
Uniform grid

Challenge:

Utility functions are volatile, especially on average over samples

First-price auction average

$$\text{utility } \frac{1}{N} \sum_{j=1}^N u(t_i, \cdot, \mathbf{t}_{-i}^{(j)})$$

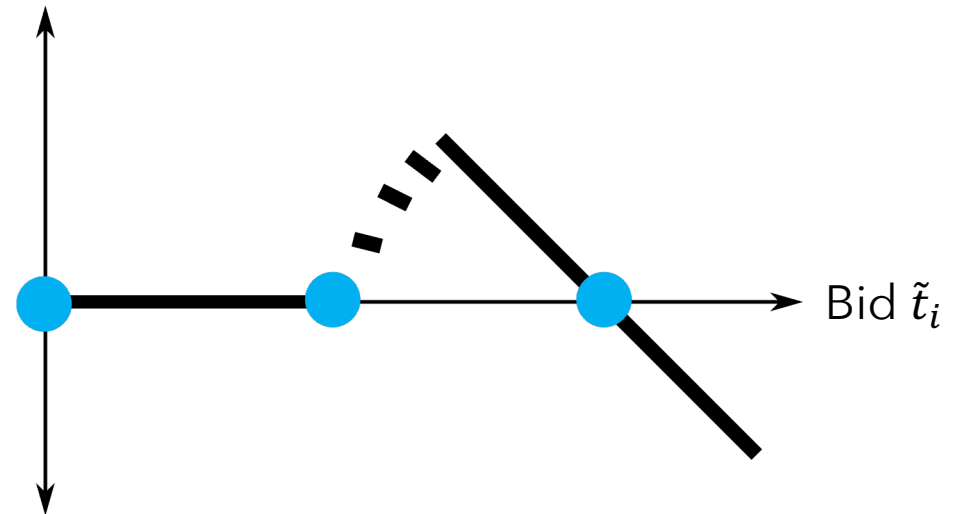


Uniform grid

Coarse discretization can lead to poor utility estimation

First-price auction average

$$\text{utility } \frac{1}{N} \sum_{j=1}^N u(t_i, \cdot, \mathbf{t}_{-i}^{(j)})$$



**When is the distribution “nice”
enough to use a grid?**

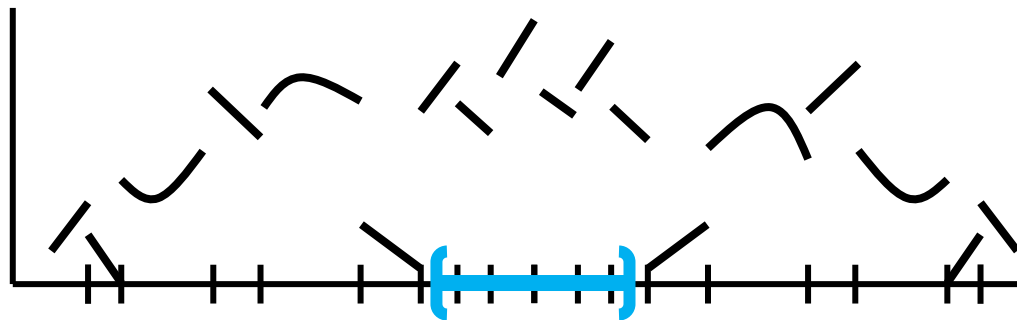
Dispersion

Functions $f_1, \dots, f_N: \mathbb{R}^M \rightarrow \mathbb{R}$ are **(w, k)-dispersed** if:

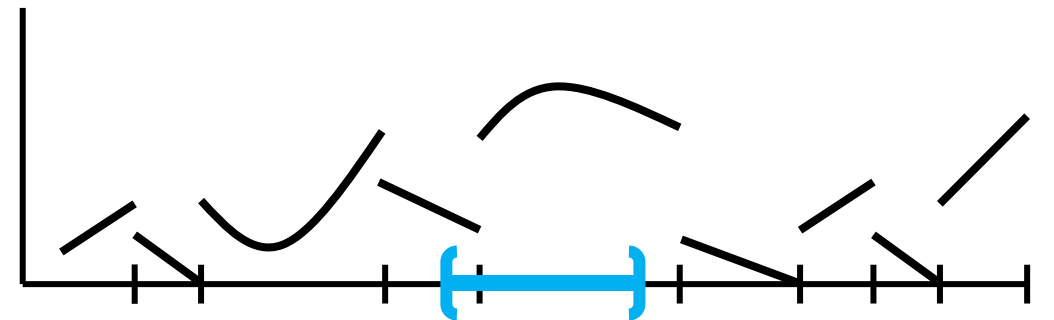
Every w -ball contains discontinuities of $\leq k$ functions

[Balcan, Dick, **v.**, FOCS'18]

Plot $\sum f_i$:



Not dispersed



Dispersed

Dispersed utility functions

Definition. $\mathbf{t}_{-i}^{(1)}, \dots, \mathbf{t}_{-i}^{(N)}$ induce **L -Lipschitz (w, k) -dispersion** if:

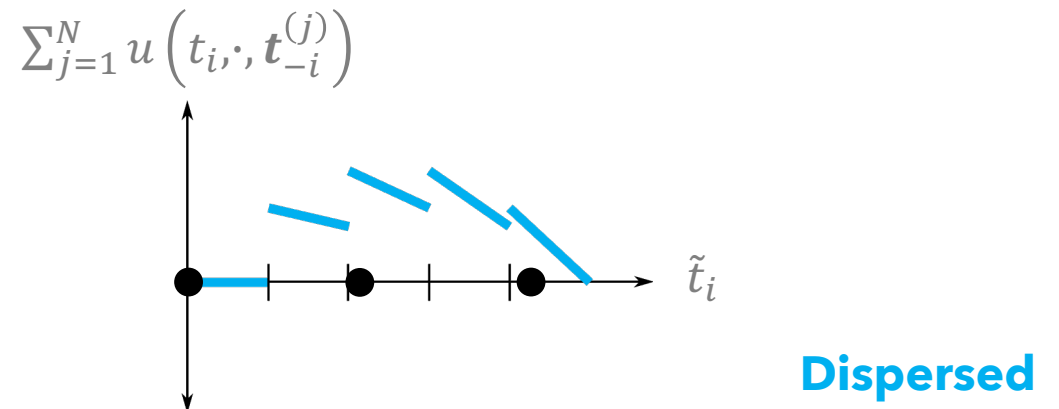
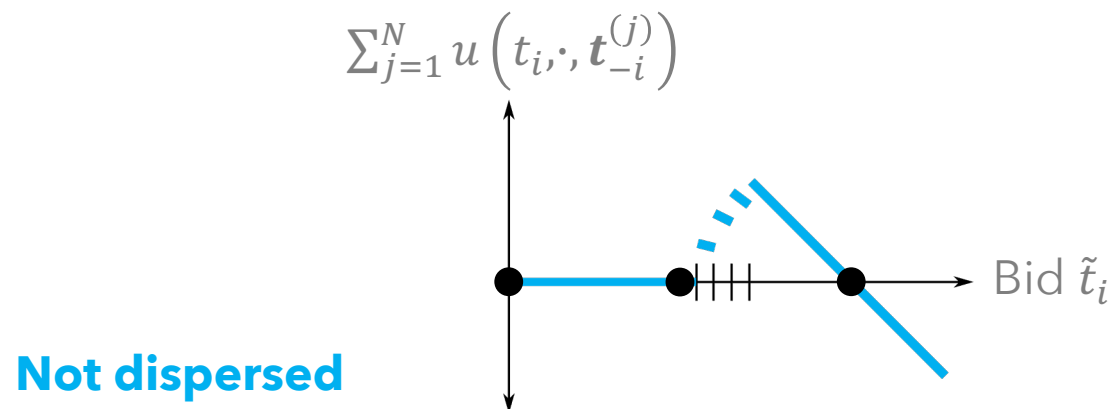
1. For any t_i , $\underline{u(t_i, \cdot, \mathbf{t}_{-i}^{(1)})}, \dots, \underline{u(t_i, \cdot, \mathbf{t}_{-i}^{(N)})}$ are:

Utility as a function of the bid \tilde{t}_i

Dispersed utility functions

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Piecewise L -Lipschitz and (w, k) -dispersed



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2. For any \tilde{t}_i , $u\left(\cdot, \tilde{t}_i, \mathbf{t}_{-i}^{(1)}\right), \dots, u\left(\cdot, \tilde{t}_i, \mathbf{t}_{-i}^{(N)}\right)$ are:

Utility as a function of the value t_i

Dispersed utility functions

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2. For any \tilde{t}_i , $u(\cdot, \tilde{t}_i, \mathbf{t}_{-i}^{(1)}), \dots, u(\cdot, \tilde{t}_i, \mathbf{t}_{-i}^{(N)})$ are:
Piecewise L -Lipschitz and (w, k) -dispersed

- Prove that WHP, for all **infinitely-many** function sequences:
dispersion holds for “good” values of w and k
 - Show discontinuities are shared across function sequences

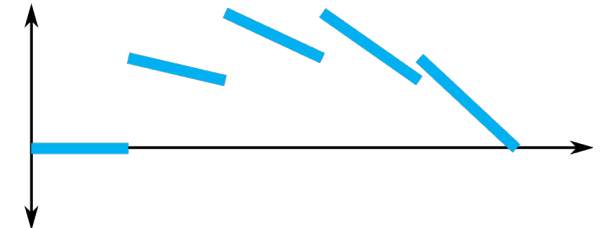
Guarantees

Our estimate $\hat{\gamma}$:

Maximum utility agent i can gain by misreporting her type,
on average over samples $\mathbf{t}_{-i}^{(1)}, \dots, \mathbf{t}_{-i}^{(N)} \sim \mathcal{D}_{-i}$,
if true & reported types from uniform grid \mathcal{G}

Theorem:

If WHP, for all i , $\mathbf{t}_{-i}^{(1)}, \dots, \mathbf{t}_{-i}^{(N)}$ induce L -Lipschitz (w, k) -dispersion



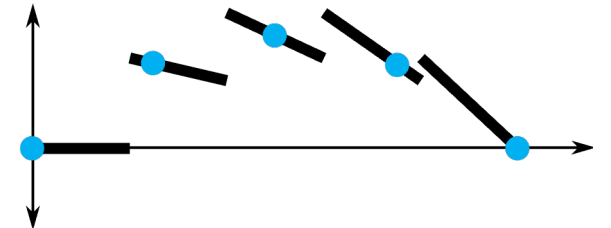
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Theorem:

If WHP, for all i , $\mathbf{t}_{-i}^{(1)}, \dots, \mathbf{t}_{-i}^{(N)}$ induce L -Lipschitz (w, k) -dispersion
 \Rightarrow Can estimate using w -grid



Guarantees

Theorem:

If WHP, for all i , $\mathbf{t}_{-i}^{(1)}, \dots, \mathbf{t}_{-i}^{(N)}$ induce L -Lipschitz (w, k) -dispersion
 \Rightarrow Can estimate using w -grid

Estimation error: WHP, $|\hat{\gamma} - \gamma| = \tilde{O} \left(Lw + \frac{k}{N} + \sqrt{\frac{d}{N}} \right)$

d = standard ML measure of utility functions' intrinsic complexity

Guarantees

Theorem:

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Proof idea:

- If snap types to grid, average utility only changes by $\leq Lw + \frac{k}{N}$

Guarantees

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Proof idea:

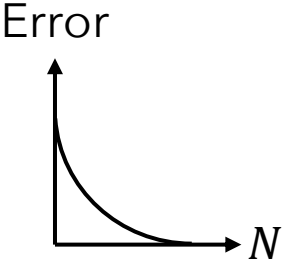
- If snap types to grid, average utility only changes by $\leq Lw + \frac{k}{N}$
- $\sqrt{\frac{d}{N}}$ additional error incurred from sampling

Guarantees

Theorem:

If WHP, for all i , $\mathbf{t}_{-i}^{(1)}, \dots, \mathbf{t}_{-i}^{(N)}$ induce L -Lipschitz (w, k) -dispersion
 \Rightarrow Can estimate using w -grid

Estimation error: WHP, $|\hat{\gamma} - \gamma| = \tilde{O} \left(Lw + \frac{k}{N} + \sqrt{\frac{d}{N}} \right)$

When $w = O\left(\frac{1}{\sqrt{N}}\right)$, $k = O(\sqrt{N})$: 

We prove these (w, k) values hold when distribution is **nice**

Applications

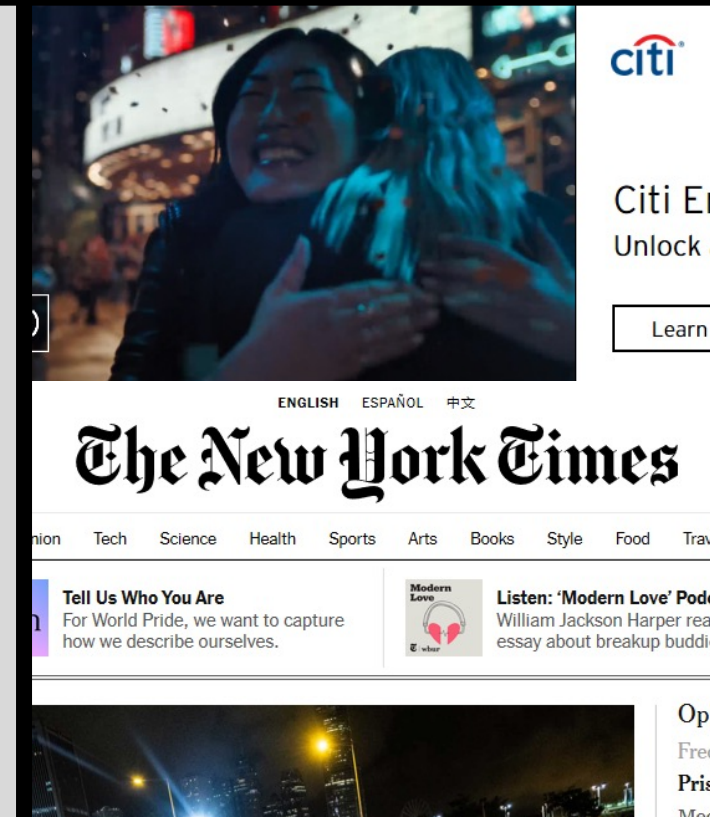
When does dispersion hold?

$[0, \kappa]$ = range of density functions defining agents' type distributions

First-price auction

$$\text{Error: } |\hat{\gamma} - \gamma| = \tilde{O} \left(\frac{(\#\text{agents}) + \kappa^{-1}}{\sqrt{(\#\text{samples})}} \right)$$

Also analyze **combinatorial** first-price auctions



The image shows a portion of a website. At the top right, there is a Citi logo and a promotional banner for Citi Entertainment with the text 'Citi Entertainment' and 'Unlock a...' and a 'Learn more' button. Below this is the masthead for 'The New York Times' with language options 'ENGLISH', 'ESPAÑOL', and '中文'. A navigation bar lists categories: 'Opinion', 'Tech', 'Science', 'Health', 'Sports', 'Arts', 'Books', 'Style', 'Food', and 'Travel'. Below the navigation bar, there are two promotional tiles. The first is titled 'Tell Us Who You Are' with the text 'For World Pride, we want to capture how we describe ourselves.' The second is for a 'Modern Love' podcast, titled 'Listen: 'Modern Love' Podcast' and 'William Jackson Harper reads an essay about breakup buddies'.

Applications

When does dispersion hold?

$[0, \kappa]$ = range of density functions defining agents' type distributions

Generalized second-price auction

$$\text{Error: } |\hat{\gamma} - \gamma| = \tilde{O} \left(\frac{(\#\text{agents})^{3/2} + \kappa^{-1}}{\sqrt{(\#\text{samples})}} \right)$$

A screenshot of a search engine results page for the query "flowers". The search bar at the top shows "flowers" and a magnifying glass icon. Below the search bar, there are navigation links for "All", "Images", "Maps", "Shopping", "News", "More", "Settings", and "Tools". The search results indicate "About 6,950,000,000 results (0.81 seconds)". The first result is an advertisement for "1800Flowers.com | 1-800-FLOWERS.COM® | Same Day Delivery Available", with a link to "www.1800flowers.com/". The second result is an advertisement for "fromyouflowers.com/" with a 4.5-star rating and a link to "www.fromyouflowers.com/". The third result is an advertisement for "FTD® Official Site | Fresh Flowers For Any Occasion | FTD.com" with a link to "www.ftd.com/FlowerDelivery/FTDFlowers". At the bottom of the screenshot, there is a map showing the locations of "Paper Petals of Pittsburgh", "Leone's Florist", and "Whole Foods Market" in the Friendship neighborhood.

Applications

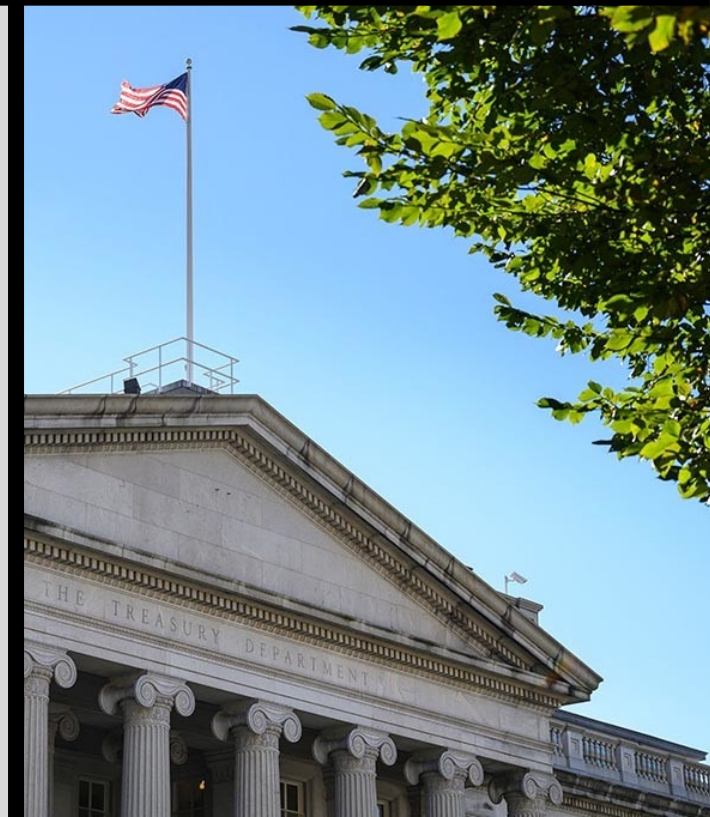
When does dispersion hold?

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Discriminatory and uniform price auctions

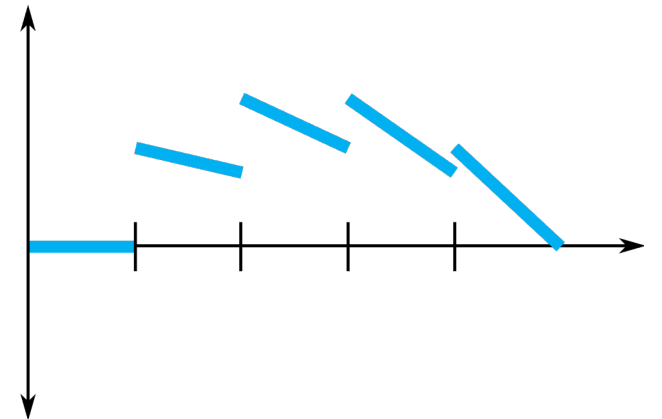
Generalization of first-price auction to multi-unit settings

$$\text{Error: } |\hat{\gamma} - \gamma| = \tilde{O} \left(\frac{(\#\text{agents})(\#\text{units})^2 + \kappa^{-1}}{\sqrt{(\#\text{samples})}} \right)$$



Conclusion

- Provide techniques for estimating how far mechanism is from IC
- Introduce empirical variant of approximate IC
- Bound estimate's error using *dispersion*
- Guarantees for:
 - First-price (combinatorial) auction
 - Generalized second-price auction
 - Discriminatory auction
 - Uniform price auction
 - Second-price auction under spiteful agents



Future directions

What if samples strategically manipulated?

Also applies to literature on revenue maximization via machine learning

Likhodedov, Sandholm, AAAI'04, '05; Elkind, SODA'07; Cole, Roughgarden, STOC'14; Mohri, Medina, ICML'14; Huang, Mansour, Roughgarden, EC'15; Morgenstern, Roughgarden, NeurIPS'15, COLT'16; Devanur, Huang, Psomas, STOC'16; Balcan, Sandholm, [V.](#), NeurIPS'16; Gonczarowski, Nisan, STOC'17; Cai, Daskalakis, FOCS'17; Balcan, Sandholm, [V.](#), EC'18; ...

What about when report space not real-valued?

E.g., school choice mechanisms: report preference order over schools

Estimating Approximate Incentive Compatibility

Maria-Florina Balcan, Tuomas Sandholm,
and **Ellen Vitercik**

Berkeley (EECS) → Stanford (MS&E and CS)

Working paper, preliminary version in
Conference on Economics and Computation (EC)