

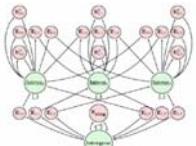
# Computational Genomics

10-810/02-710, Spring 2009

## Model-based Comparative Genomics

Eric Xing

Lecture 14, March 2, 2009



Reading: class assignment

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# Uses of evolutionary theory



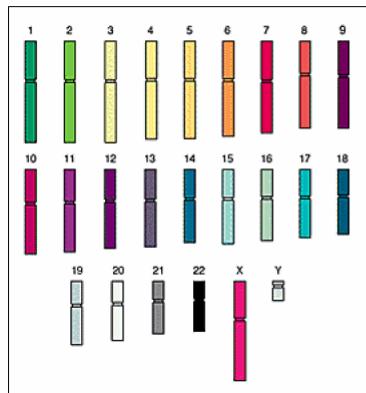
- Comparative genomics (this lecture)
  - Cladistics: figuring out closely related species, proteins, sequences
    - Drug design and testing
    - Building chimera : mixing genetic codes of species, genetic technology
    - Sequence prediction in related unsequenced species : use in sequencing, primer design, etc
  - Phylogenetic footprinting
    - Functional constraints on a genomic region inversely proportional to evolutionary rate, from neutral theory
    - Look at two concrete examples : transcription factor binding site (motif) and gene prediction
- Population genetics (module 4)
  - Population structure
  - Understanding evolutionary driving force underlying genome variation

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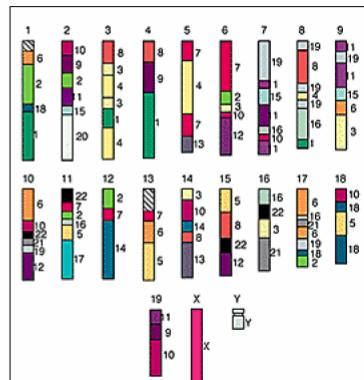
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# Comparative Genomics

Human



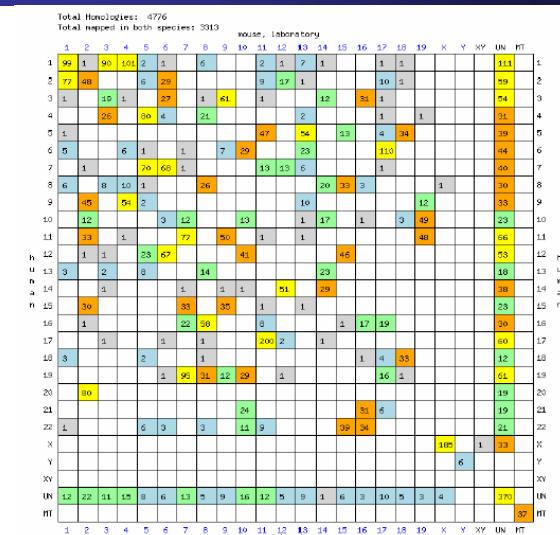
Mouse



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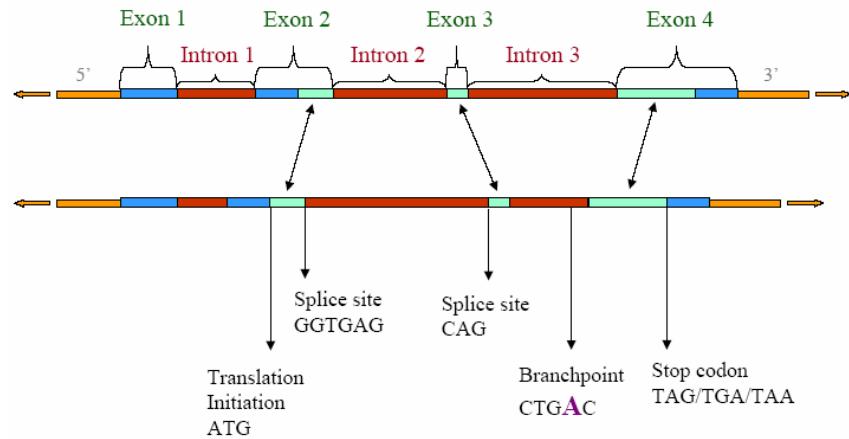
# A pairwise comparison between human and mouse genome



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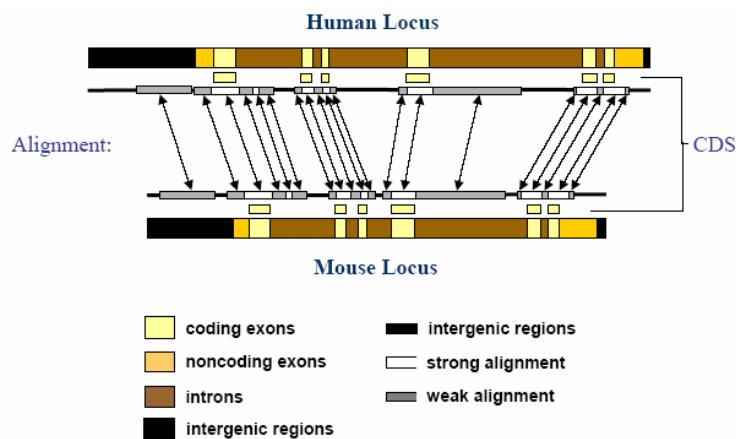
## Aligning One Locus



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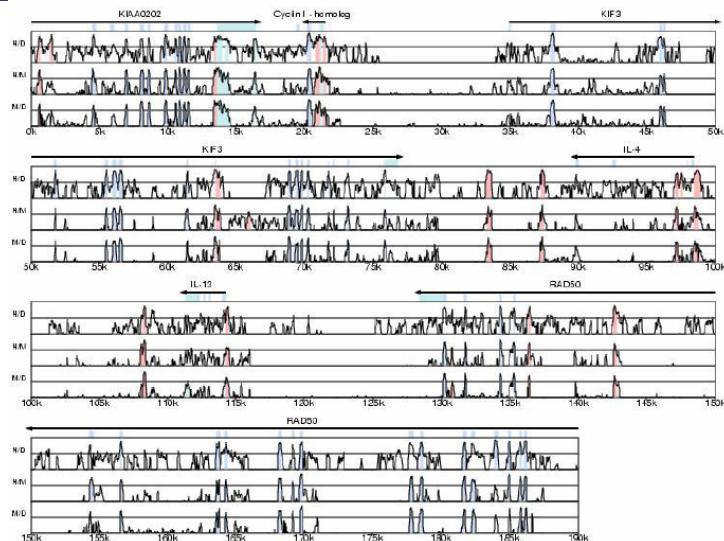
## Example: a human/mouse ortholog



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## Three Pairwise Alignments

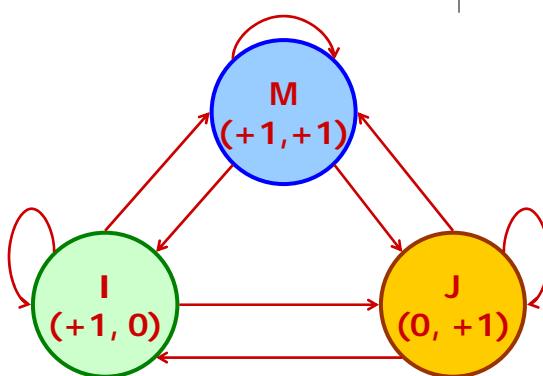


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## Paired HMM

Alignments correspond 1-to-1 with sequences of states M, I, J



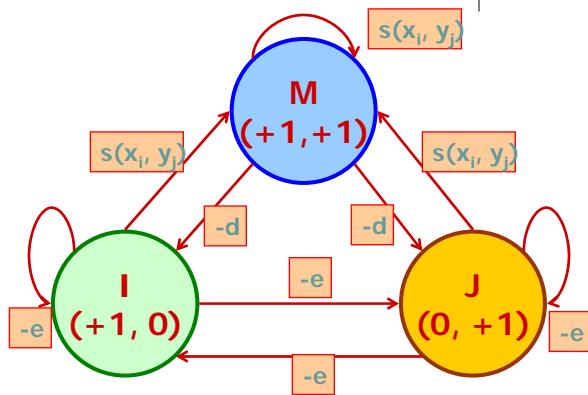
-AGGCTATCACCTGACCTCCAGGCCGA--TGCCC---  
TAG-CTATCAC--GACCGC-GGTCGATTGCCGACC  
IMMJMMMMMMMJUJMHHHHHHHHHIIIMHHHHHHIII

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## Let's score the transitions

Alignments correspond 1-to-1 with sequences of states M, I, J

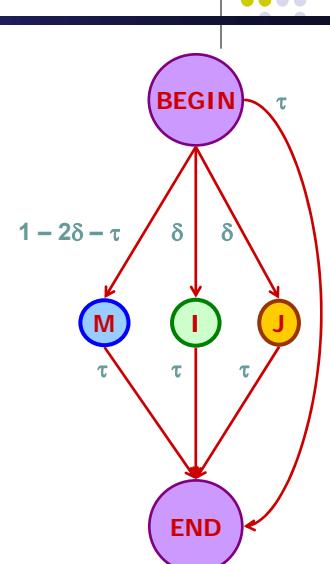
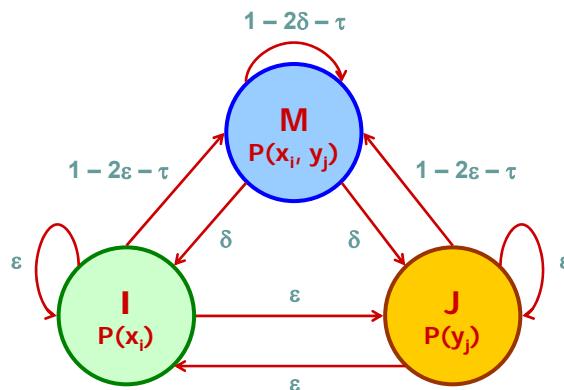


-AGGCTATCACCTGACCTCCAGGCCGA--TGCCC---  
 TAG-CTATCAC--GACCGC-GGTCGATTTGCCGACC  
**I**MMJMMMMMMJJMMMMMMJMMMMMMIIMMMMMIII

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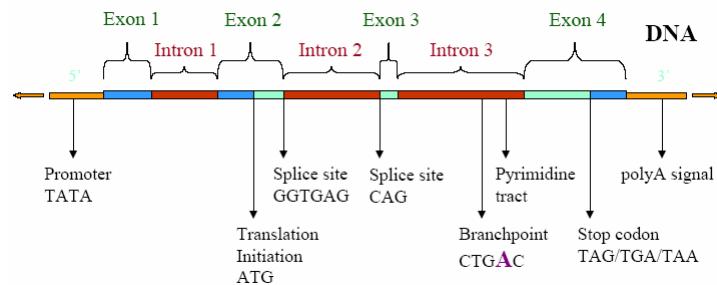
## A Pair HMM for alignments



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## Gene Finding

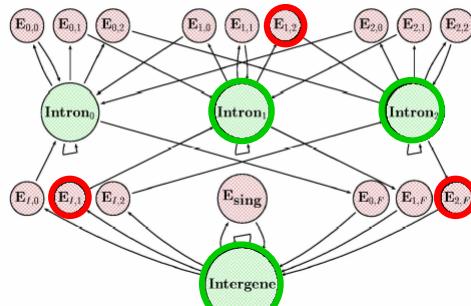


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## Recall generalized HMM gene finder

TAAT ATGTCACGG GTATTGAG CATTGTACACGGG GTATTGAG CATGTAA TGAA

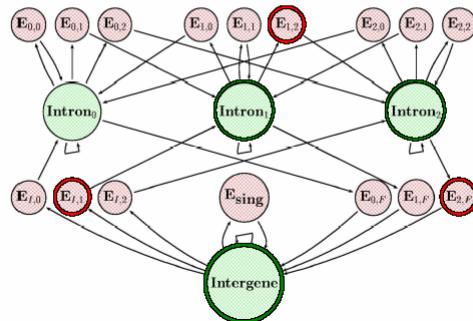


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## Generalized Pair-HMM gene finder

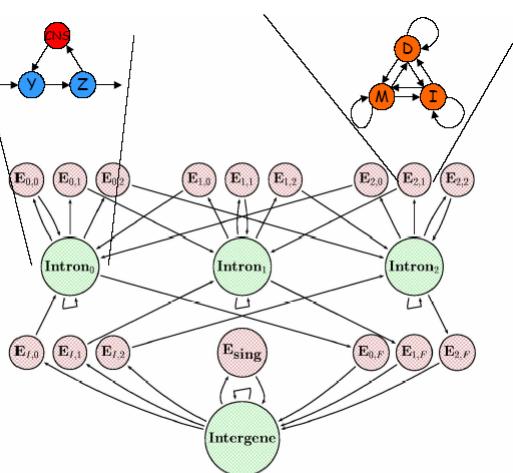
TAAT ATGTCCACGG GTATTGAG CATTGTACACGGG GTATTGAG CATGTAA TGAA  
 CTG ATGTACACTG GTTGGTCCTCAG CTTTGTACGGG GTG CATGTAA T6TC



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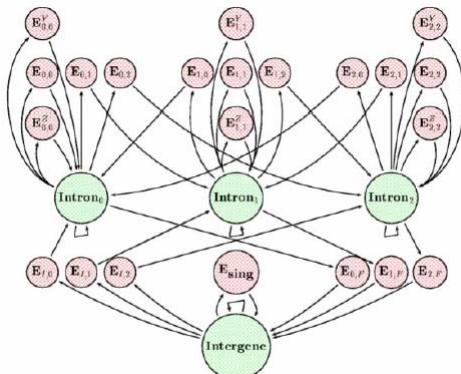
## Hierarchical state transition in pHMM : knowledge of structure



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## Allowing for inserted exons: knowledge of structure



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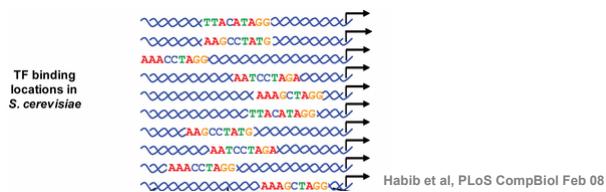
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## Motif finding: recap



### Recap:

- Functional regions in sequences often occur as small, noisy, repeating subsequences
  - In DNA : transcription factor binding sites, transcription start sites, splicing signals, etc
  - In proteins : transmembrane domains, phosphorylation sites, signal peptides. etc
- Subsequence similarity and functional significance go hand in hand



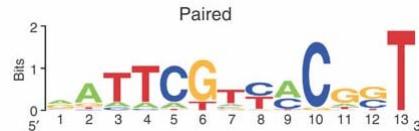
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## Problem formulation



- Models : Consensus, RegEx, Weight Matrix



- **Supervised motif detection** : Given a set of sequences, and a PWM  $w$  of length  $k$ , find the maximum likelihood set of  $k$ -mers which correspond to the WMM.
  - Given TF binding specificities, can we find the TF binding sites ?
- **De novo (unsupervised) motif detection** : Given a set of sequences, find the most overrepresented set of  $k$ -mers and the corresponding WMM  $w$ 
  - Given a set of genes with similar expression (putatively co-regulated), can we find the TF binding sites common to them and the specificity of the corresponding TF?

## Multi-species data pooling



- Simply pool together regulatory regions in related species
  - Bacterial DNA motifs, McGuire *et al* **Gen Res** 2000 & Gelfand *et al* **Nuc Ac Res** 2000
  - Hunchback TFBS in Drosophila species demonstration :

**CACCACTTTTATGCCGAGTTAAT** D. melanogaster

**GGTTTTTCGATTCAATCGGTATA** D. yakuba

**AGTTAGCGTTTACCTA****TTTTTAC** D. persimilis

**GCATTTATC****CTCTTTTATAAGCTT** D. mojavensis

- What could be problematic with this approach ?

## Multi-species data pooling



- Biases analyses towards motifs in a bunch of closely related species – no explicit phylogenetic information used
- No distinction between paralogs and orthologs
- Variation in number of binding sites in orthologous CRMs much less than in CRMs of coregulated genes in same species
  - Signals in one species may be drowned out by cross species signals, or vice versa

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## Orthologous sequence analysis



- What if the sequences are orthologous ?

D. melanogaster	CTTACGTA <span style="color: orange;">TTTTAGTT</span> ATCGAGTTATCTTCTGCTTGCTATCTCGCGC
D. yakuba	T--TACGTA <span style="color: orange;">TTTTAGTT</span> ATCGAGTTATCTTCTGCTTGCTATCTCGCGC
D. persimilis	GTTCACGTA <span style="color: orange;">TTTTAGTT</span> ATCGAGTTATCTTCTGCTT-----TCTCGC
D. mojavensis	CTTTACGTA <span style="color: orange;">TTTGAGTT</span> ATCAACTTGT--TTTGCTT--TGCTTTCGC

- Functional regions like TFBSS are more conserved than background
- Phylogenetic dependencies between orthologs may be modelled to get more accurate scores for  $P(\text{data} \mid \text{model})$
- Paralogous sequence analysis also possible

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## Chronology : from one to many

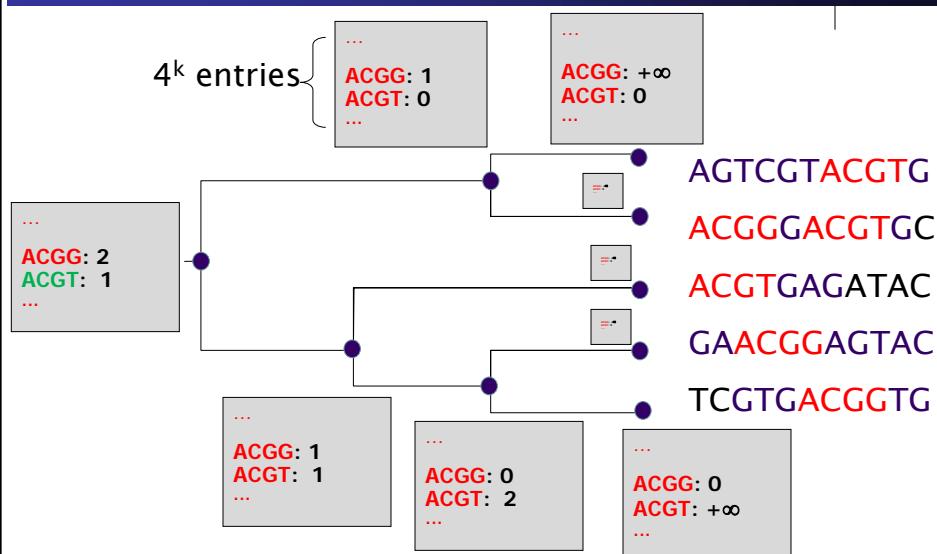
Method	Single species	Multispecies
Combinatorial	Waterman (1985)	FootPrinter (2002)
LRT-like score threshold	Staden-PWM (1989)	rVista (2002) PhyloCon (2003)
Explicit mixture models	MEME (1994)	EMnEM (2004) PhyME (2004) CSMET (2008)
Gibbs Sampling	GMS (1993/1995) BioProspector (2001) AlignACE (2000)	Motif Sampler+ (2000) CompareProspector (2004) PhyloGibbs (2005)
HMM+	Cister (2001) HMDM (2002) LOGOS (2003) BayCis (2008)	PhyloHMM (2004 –gene / 2008 – motif) PhyME (2004) MORPH (2008) CSMET (2008)
Ensemble models	EMD (2006)	-

- First usage of term **phylogenetic footprinting**  
: Tagle et al, J Mol Biol 1988 : Regulatory regions of paralogous gamma and epsilon globin genes in Galago
- Google scholar hits for “phylogenetic footprinting”
  - 1988 – 1990 : 11 hits
  - 1991 – 2000 : 141 hits
  - 2001 – 2009 : 1850 hits
- Gibbs Sampling particularly easily adapted to incorporate phylogenetic footprinting

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## FootPrinter: going combinatorial



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## Footprinter



- For each node from the leaf up
  - Fill up a parsimony table  $W$  for each k-mer
- If the node  $n$  is a leaf
  - If the word is present in corr. string,  $W[k]^{(n)} = 0$  else  $W[k]^{(n)} = +\infty$
- Else
  - $W[k]^{(n)} = \sum_{v \in \text{child}(n)} \min_{t \in \text{k-mer}} (W[t]^{(v)} + d(t, k))$
- Choose most parsimonious k-mer(s) over the tree and alignment from table at root
- Time complexity =  $O(n 4^{2k})$  for  $n$  species for fixed topology

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## Motif Sampler + footprinting



- A simple idea that works reasonably well
- Wasserman et al, 2000
  - Given an input multiple alignment  $A$
  - Compute a score for conservation across the alignment
  - Filter out all regions of the alignment with a score below a threshold  $t$
  - Perform Gibbs Motif Sampling on the remaining alignment  $A'$

D. melanogaster

CTTACGTATTTTAGTTATCGATTTTATTTTCTGCTTGCATCTCGCGC

D. yakuba

T--TACGTATTTTAGTTATCGATTTTATTTTCTGCTTGCATCTCGCGC

D. persimilis

GTAAACGTATTTTAGTTATCGATTTTAGTTTTCGCTT-----TCTCGC

D. mojavensis

CTTACGTATTTGAGTTATCAATTTTGGTTTTTGCTT--TGCTTTCGC

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## Motif Sampler + footprinting



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- D. melanogaster  
D. yakuba  
D. persimilis  
D. mojavensis

CTTACGTATTTAGTTATCGA TTTTATTTCTGCTTGCATCTCGCGC  
T--TACGTATTTAGTTATCGA TTTTATTTCTGCTTGCATCTCGCGC  
GTTTACGTATTTAGTTATCGA TTTTAGTTTCGCTT-----TCTCGC  
CTTACGTATTTAGTTATCAA TTTGGTTTTGCTT--TGCTTTTCGC

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## Motif Sampler + footprinting



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- D. melanogaster  
D. yakuba  
D. persimilis  
D. mojavensis

CTTACGTATTTAGTTATCGA TTTTATTTCTGCTTGCATCTCGCGC  
T--TACGTATTTAGTTATCGA TTTTATTTCTGCTTGCATCTCGCGC  
GTTTACGTATTTAGTTATCGA TTTTAGTTTCGCTT-----TCTCGC  
CTTACGTATTTAGTTATCAA TTTGGTTTTGCTT--TGCTTTTCGC

- Motifs sampled according to:

$$\frac{\prod_{k=a}^{a+w} pm_{k-a, R_k}}{\prod_{k=a}^{a+w} p_0_{k, R_k}}$$

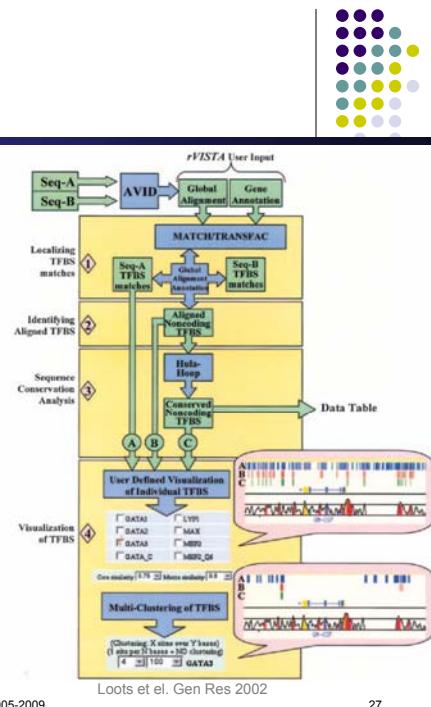
Score (likelihood) under  
PWM (motif) scenario  
Score (likelihood) under  
background scenario

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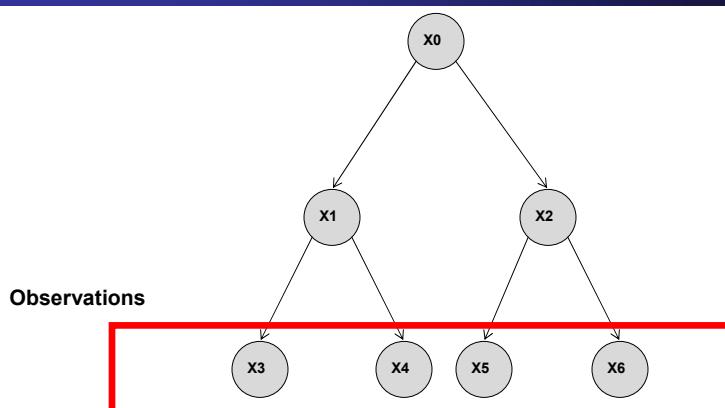
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## rVISTA

- Select motifs with a PWM score greater than a threshold
- Screens for motifs above a certain threshold for nucleotide conservation
- Two step screening a common way to capture both overrepresentation & conservation
  - Loots et al, Gen Res 2002 [rVISTA]
  - Kellis et al, Nature 2003
  - Wang & Stormo, Bioinf 2003 [PhyloCon]



## Phylogenetic model: recap

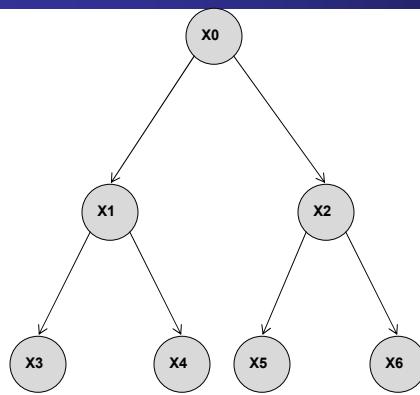


$$\begin{aligned}
 P(D|M) &= P(X_0, X_1, X_2, X_3, X_4, X_5, X_6 | \tau, \beta, \theta, \pi) \\
 &= \sum_{x_0, x_1, x_2} P(X_3|X_1; \text{tree}) P(X_4|X_1; \text{tree}) P(X_5|X_2; \text{tree}) P(X_6|X_2; \text{tree}) \\
 &\quad P(X_2|X_0; \text{tree}) P(X_1|X_0; \text{tree}) P(X_0; \text{tree})
 \end{aligned}$$

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## Phylogenetic model: recap



• **Topology** – how the observations are “tied together”:  $\tau$

• **Branch lengths** – the length for which the CTMP runs:  $\beta$

• **Parameters of CTMP** – characterizing the substitution model:  $\theta$

• **Distribution at root** - maybe stationary dist of CTMP :  $\pi$

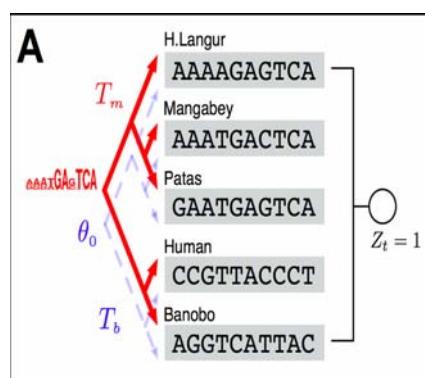
$$\begin{aligned}
 P(D|M) &= P(X_0, X_1, X_2, X_3, X_4, X_5, X_6 | \tau, \beta, \theta, \pi) \\
 &= \sum_{X_0, X_1, X_2} P(X_3|X_1; \text{tree}) P(X_4|X_1; \text{tree}) P(X_5|X_2; \text{tree}) P(X_6|X_2; \text{tree}) \\
 &\quad P(X_2|X_0; \text{tree}) P(X_1|X_0; \text{tree}) P(X_0) \text{ tree}
 \end{aligned}$$

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## EMnEM: model based approaches

- Mixture model
- Each block of k-mer could be generated from a background model with probability  $1-\pi_m$  or from a motif model with probability  $\pi_m$
- Bernoulli draw for the mixture indicator
- In the spirit of MEME



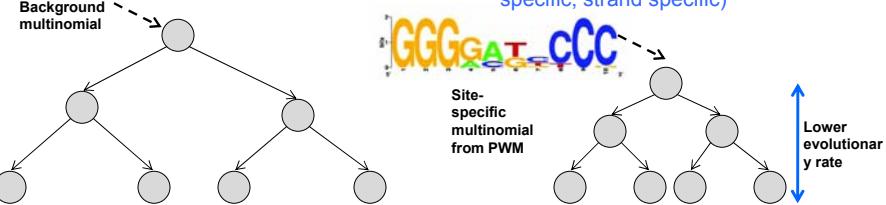
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## Function specific phylogenetic models



- Background model  $T_b$ 
  - Topology invariant unless evidence otherwise
  - Substitution matrix invariant unless evidence otherwise
  - Branch lengths longer than functional sites
  - Root distribution : background frequency
- Motif site-specific, strand-specific model  $T_{m, k, +/-}$ 
  - Topology invariant unless evidence otherwise
  - Substitution matrix invariant unless evidence otherwise
  - Branch lengths shorter than background sites
  - Root distribution : from PWM (site specific, strand specific)



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## EMnEM: Expectation maximization on mixtures of phylogenies



$$L = \prod_{i=0}^{N-w} \sum_{m_i} p(m_i) \prod_{k=i}^{i+w-1} \sum_{b=0}^3 p(X_k, Y_k | A_{kb}, m_i) p(A_{kb} | m_i)$$

**E-step :**

- Mixture parameter:**

$$\langle m_i \rangle = p(m_i | X, Y) = \frac{p(m_i) p(X, Y | m_i)}{p(X, Y)}$$

$$p(X, Y | m_i) = \prod_{k=i}^{i+w-1} \sum_{b=0}^3 p(X_k, Y_k | A_{kb}, m_i) p(A_{kb} | m_i)$$

$$p(X, Y) = \sum_{m_i} p(X, Y | m_i) p(m_i)$$
- Ancestral nucleotide:**

$$\langle A_{ib} \rangle = p(A_{ib} | X_i, Y_i) = \sum_{m_i} p(A_{ib} | X_i, Y_i, m_i) p(m_i) = \sum_{m_i} \frac{p(A_{ib}) p(X_i, Y_i | A_{ib}, m_i)}{p(X_i, Y_i | m_i)} p(m_i)$$
- M-step :**

$$\langle \ln L_c \rangle = \sum_{i=0}^{N-w} \sum_{m_i} \langle m_i \rangle \left[ \ln \pi_m + \sum_{k=i}^{i+w-1} \sum_{b=0}^3 \langle A_{kb} \rangle (\ln p(X_k, Y_k | A_{kb}, m_i) + \ln f_{mkb}) \right]$$

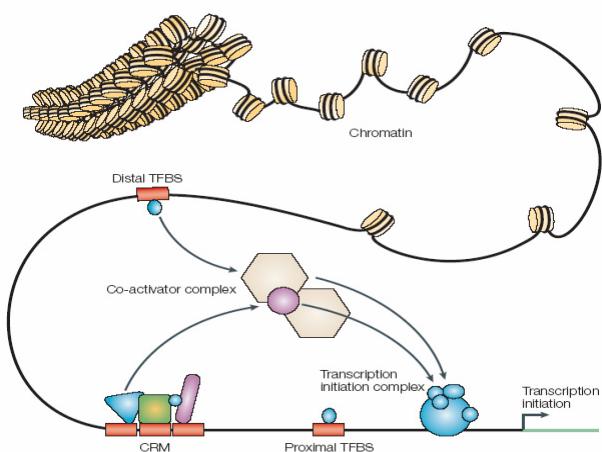
$$\frac{\partial \langle \ln L_c \rangle}{\partial \pi_m} = 0, \frac{\partial \langle \ln L_c \rangle}{\partial f_{mkb}} = 0 \text{ and } \frac{\partial \langle \ln L_c \rangle}{\partial \langle A_{mb} \rangle} = 0$$

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## CRM: putting the pieces together

- HMMs !

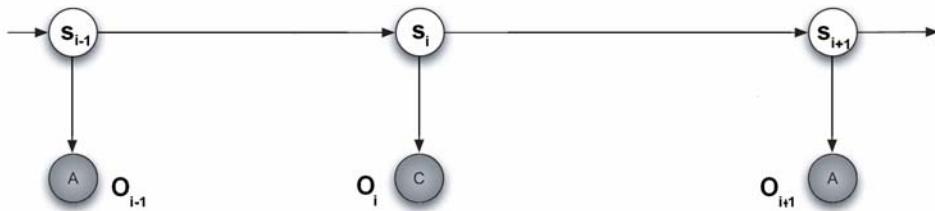


Courtesy: Simone Scalabria

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## A vanilla HMM ...



ACATTGCCATACCAATCCTTAATT...

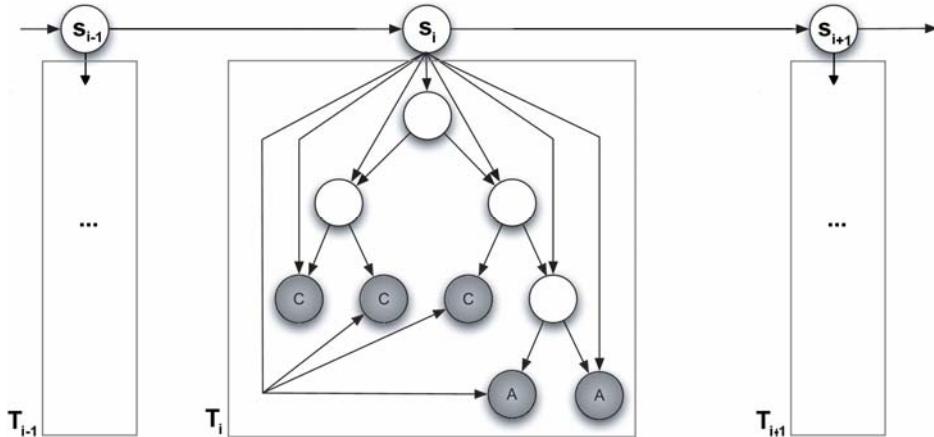
- Emits a symbol at every discrete step
- A run of the HMM outputs a sequence
- PhyloHMM outputs a vector of characters
- A run of the PhyloHMM outputs a multiple sequence alignment

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## Phylo-HMM

- The emission vector  $\mathbf{O}_i$  is shaded in gray

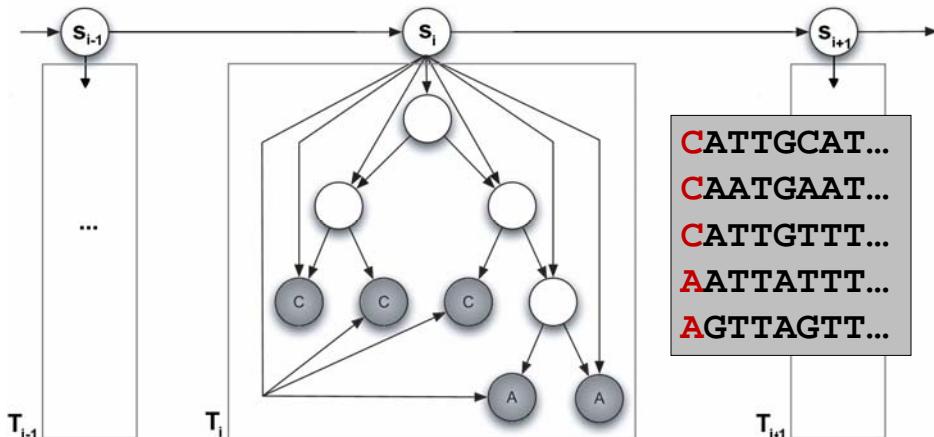


Courtesy: McAuliffe

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## Phylo-HMM

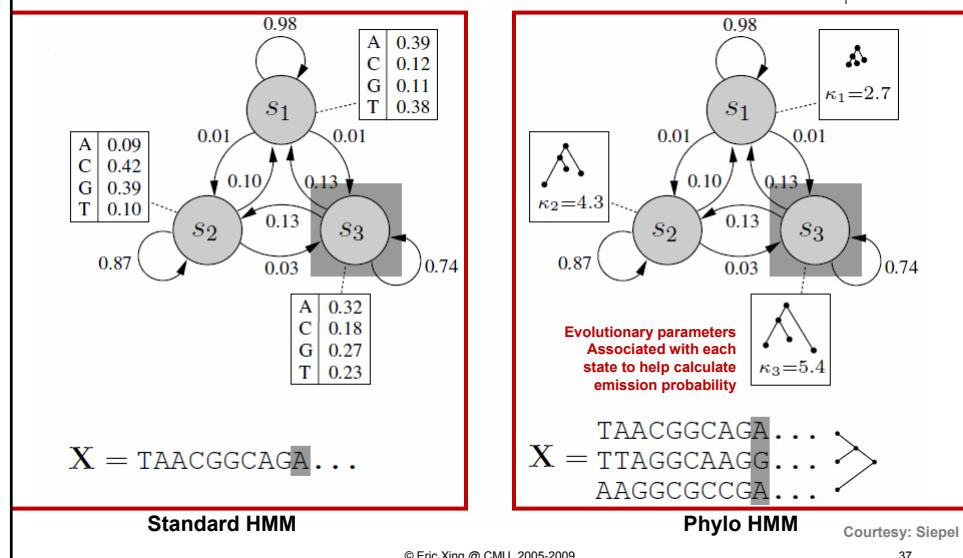
- Emits a vector at each step, generates alignment in a run



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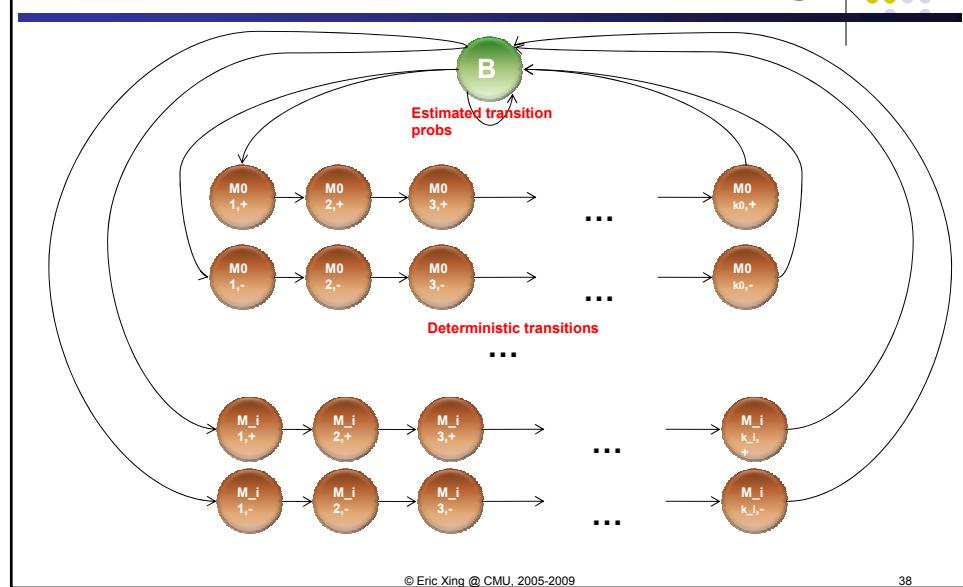
## State space comparisons



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## HMM state space for motif finding



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## More realism, more parameters

Estimated transition probs

Deterministic transitions

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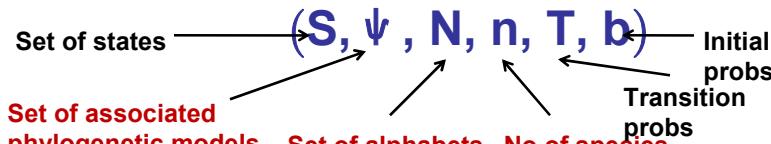
## Phylo-HMM

- A normal HMM, except the emission probabilities are a **multinomial distribution over the space of  $[ATGC]^n$** ,  $n$  being the number of sequences in the alignment
- $4^n$  emission probabilities can be pre computed
- But usually calculated on the fly using Felsenstein's Pruning Algorithm - a special case of the GM Belief Propagation Algorithm on trees
  - Siepel & Haussler, RECOMB 2004, for gene finding
  - Ray et al. PLoS CompBio 2008, adapted for motif finding

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## Analogy with HMM



- Emission probability =  $P(O_i | S_i = s)$   
 $= P(O_i | \text{phylogenetic model}_s)$   
 $= P(\text{Alignment column}_i | \psi_s)$   
 $\Rightarrow$  Calculate using the Pruning Algorithm
- Apply standard Viterbi (maximize joint) or posterior decoding on the Forward-Backward matrix
- Baum-Welch algorithm (E-M) for unsupervised settings
- Exactly analogous to single species motif finding

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## Missing motifs

melanogaster	ccgggatc	—	gcag	tttttacat	gcatac	tttttacg	—	acc	—	tcgtccgttt
sechellia	ccgggatc	—	gctgtttttacat	gcatac	tttttacg	—	acc	—	tcgtccgttt	
simulans	ccgggatc	—	gctgtttttacat	gcatac	tttttacg	—	acc	—	tcgtccgttt	
yakuba	ccgggatc	—	gctgtttttacat	gcatac	tttttacg	—	acc	—	tccgccccgtt	
erecta	ccgggatc	—	gcccgtttttacat	gcatac	tttttacg	—	acc	—	tccgtccgttt	
ananasae	ccgggttc	—	gctgtttttacat	gcatac	ttttatgtgtc	—	acc	—	cgcgtccgttt	
pseudoobscura	ccgggttccct	—	caggcactttttac	gcatac	aaaggatgtttatg	—	gcgtccgcgtccgtgttt	—		
persimilis	ccgggttccct	—	caggcgtttttac	gcatac	aaaggatgtttatg	—	gcgtccgcgcgtgttt	—		
melanogaster	tttattcat	—	cggcgacctt	gaatggccgtttt	gatggccgtgggtgg	—	ttacct	—		
sechellia	tttattcat	—	ggcgacctt	gaatggccgtttat	gtgtgtgggtgg	—	ttacct	—		
simulans	tttattcat	—	ggcgacctt	gaatggccgtttat	gtgtgtgggtgg	—	ttacct	—		
yakuba	tttattcat	—	cggcgacctt	gaatggccgtttgggtgg	cagtgggtgg	—	ttacct	—		
erecta	tttattcat	—	cggcgacctt	gaatggccgtttgggtgg	cagtgggtgg	—	ttacct	—		
ananasae	tttattcat	—	tagcgacctt	gaatggccgtttgggtgg	cagtggat	—	aaagcgggttgcctacatgtt	—		
pseudoobscura	tttattcat	—	tagcgacctt	gaatggccgtttgggtgg	cagtggat	—	cagtgggtcg	—	gtacct	
persimilis	tttattcat	—	tagcgacctt	gaatggccgtttgggtgg	cagtggat	—	cagtgggtcg	—	gtacct	

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## Functional turnover

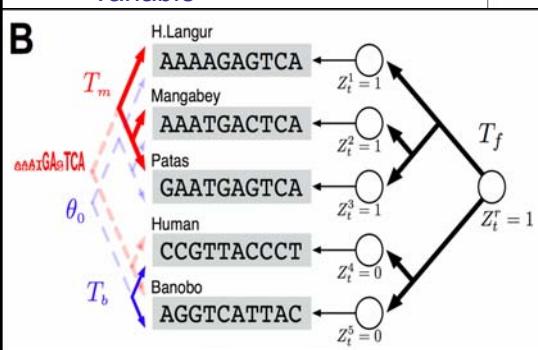
	kr - 6	kr - 5	kr - 4	kr - 3	kr - 2	kr - 1
melanogaster	ATAACCCAAT	TTAACCGTT	ACC-CGGTTGC	GAAGGGATTAG	ACTGGTTAT	TTAACCCGTT
yakuba	.....	.....	.....	C.	.....	.....
erecta	.....	.....	.....	C.	.....	.....
pseudoobscura	.....	.....	AA. -	.....	A. TC. ....	C. G
	bcd - 5	bcd - 4	bcd - 3	bcd - 2	bcd - 1	
melanogaster	GTAAATCCG	GAGATTATT	TATAATCGC	GGGATTAGC	GAAGGGATTAG	
yakuba	.....	C. ....	.GC. C. G	.....	...C. ....	
erecta	.....	C. ....	GT....	.....	...C. ....	
pseudoobscura	.....	A. ....	N/A	A. ....	..... A	
	hb-3	hb-2	hb-1	gt-3		
melanogaster	CATAAAA-ACA	TTATTTTTT	CGATTTTTT	CGAGATTATTAGTCATTG	CAGTTGC	
yakuba	..... -	..... G	..... C.	..... C.	..... A.	
erecta	..... -	C. ....	N/A	..... C.	..... C.	
pseudoobscura	..... C.	.....	N/A	..... C.	..... C. -	
	gt-2		gt-1			
melanogaster	GACTTTATTGCAGCATCTTG	—AACATCGTC-GCAGTTGGTAACAC	GAAAGTCATAAAA-ACACATAATA			
yakuba	..... C.	..... —	..... G. -	.....	.....	
erecta	.....	CAGC	..... G. -	.....	.....	
pseudoobscura	..... T.	.....	AA. T. G. A.	..... T	..... C.	

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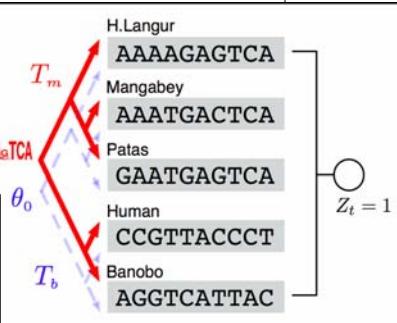
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## CSMET : Phylogenetic mixtures of phylogenies

- Mixture models for evolutionary model selection (EMnEM) →
- Bernoulli draw for mixture variable



**A**



- What if the mixture variables are phylogenetically related ?
- Output of a “functional” phylogeny

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# CSMET

- CSMET-HMM :
  - An HMM with emission vector [A,T,G,C]\*
  - Each vector is the output of a generative process involving a mixture of trees
  - Mixture indicator variables themselves generated by a phylogeny
  - Similar scheme to PhyloHMM, except for calculating emission probs
- Schematic of generation and ML inference
- CSMET-HMM : **Corr character sets**

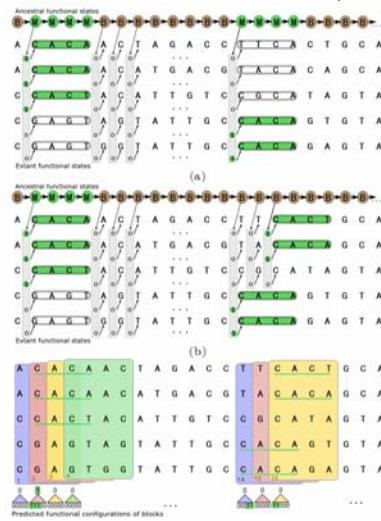
(S, TN, TF, N, F, n, T, b)

Set of nucleotide phylogenetic models corr to each annotation

Set of functional phylogenetic models corr to each state in S

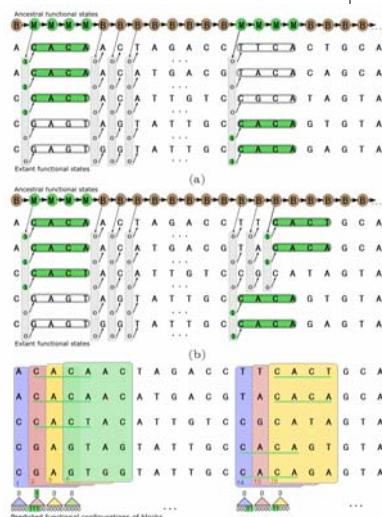
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# CSMET

- To calculate emission probabilities:
  - Calculate likelihoods of nucleotide data for each subtree of the nucleotide phylogeny
  - Calculate likelihood of functional indicators for the functional phylogeny
  - Putting the likelihoods together using conditional independences
  - Marginalize out hidden variables
- The rest would be analogous to an HMM !

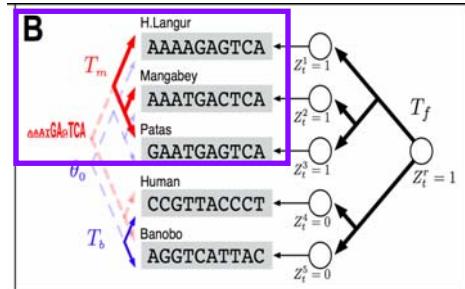


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## Likelihoods on partial phylogenies



- Marginalize out observed nucleotides present in parts of the phylogeny we are not interested in
- Turns out to be equivalent to calculating the likelihood of the data on the subtree !



$$P(A'_l | T'^{(l)}) = \sum_{A''_l} P(A'_l, A''_l | T^{(l)}) = \sum_{A''_l} \sum_{V_{1:K'}} P(V_{1:K'} = v_{1:K'}, V = A''_l, V' = A'_l)$$

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## CSMET: toolkit for calculations



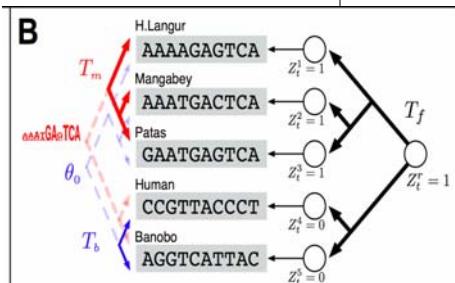
- Calculating likelihoods on the nucleotide phylogeny and functional phylogeny

**Nucleotide phylogeny :**  
F84 model – simplest arbitrary stationary distribution

$$Q_N = \begin{pmatrix} * & (1 + \kappa/\pi_Y)\pi_C & \kappa\pi_A & \kappa\pi_G \\ (1 + \kappa/\pi_Y)\pi_T & * & \kappa\pi_A & \kappa\pi_G \\ \kappa\pi_T & \kappa\pi_C & * & (1 + \kappa/\pi_R)\pi_A \\ \kappa\pi_T & \kappa\pi_C & (1 + \kappa/\pi_R)\pi_A & * \end{pmatrix}$$

**Functional phylogeny**  
Jukes Cantor model

$$P_F = \begin{pmatrix} \frac{1}{2} + \frac{1}{2}e^{-2\beta} & \frac{1}{2} - \frac{1}{2}e^{-2\beta} \\ \frac{1}{2} - \frac{1}{2}e^{-2\beta} & \frac{1}{2} + \frac{1}{2}e^{-2\beta} \end{pmatrix}$$



**Likelihoods on partial phylogenies**

$$P(A'_l | T'^{(l)}) = \sum_{A''_l} P(A'_l, A''_l | T^{(l)}) = \sum_{A''_l} \sum_{V_{1:K'}} P(V_{1:K'} = v_{1:K'}, V = A''_l, V' = A'_l)$$

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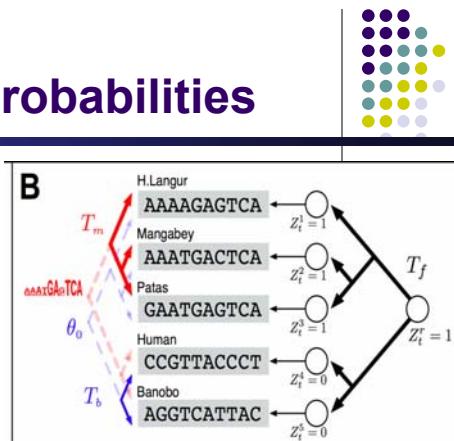
## CSMET : emission probabilities

- Emission prob : Prob of block surrounding particular aligned site
- Again, analogous to an HMM, with one twist :  $Z_i$  s not observed

### Joint Probability for an instantiated block

$$P(\mathbf{A}_t, \mathbf{z}_t, \mathbf{z}'_t) = P(\mathbf{A}_t | Z_t = \mathbf{z}_t, T_m, T_b) P(Z_t = \mathbf{z}_t | Z'_t = \mathbf{z}'_t, T_f)$$

$$P(Z'_t = \mathbf{z}'_t) = P(\mathbf{A}'_t | T'_m) P(\mathbf{A}''_t | T'_b) P(\mathbf{z}'_t | \mathbf{z}'_t, T_a) P(\mathbf{z}'_t).$$



### Conditional probability for the block

$$P(\mathbf{A}_t | Z_t = \mathbf{z}_t, T_m, T_b) = P(\mathbf{A}'_t | T'_m) P(\mathbf{A}''_t | T'_b) =$$

$$\prod_{l=1}^L P(\mathcal{A}'_l(t) | T'_m(l)) P(\mathcal{A}''_l(t) | T'_b).$$

### Emission probability for the block (marginalized)

$$P(\mathbf{A}_t | \mathbf{z}'_t) = \sum_{\mathbf{z}_t} P(\mathbf{A}_t, \mathbf{z}_t | \mathbf{z}'_t) = \sum_{\mathbf{z}_t} P(\mathbf{A}'_t(\mathbf{z}_t) | T'_m(\mathbf{z}_t))$$

$$P(\mathbf{A}''_t(\mathbf{z}_t) | T'_b(\mathbf{z}_t)) P(\mathbf{z}_t | T_a, \mathbf{z}'_t),$$

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## Chronology : aspects of footprinting

- Footprinting + Gibbs Sampling
  - **Motif Sampler+** : 2 species alignment
  - **CompareProspector** : Pairwise alignment
  - **PhyloGibbs** : Multiple alignment
- Footprinting + HMM
  - **PhyloHMM** : Emission of HMM generated by a CTMP phylogenetic tree, no tolerance for functional turnover
  - **PhyME**
  - **CSMET** : Emission of HMM generated by a mixture of CTMP phylogenetic trees, explicit tolerance for functional turnover
- **Footprinting + alignment**
  - **OrthoMEME**
  - **MORPH**

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## Can we do even better ?

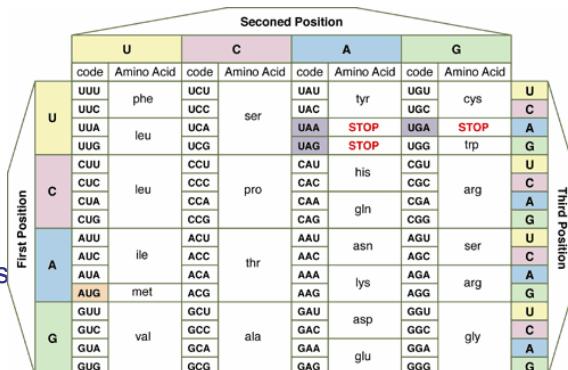
- Footprinting improves with
  - More knowledge about the functional component we are searching for : what to look for in a single species
  - More knowledge about how it evolves : what to look for in related species
- We know a lot about both aspects for protein coding regions, or genes
- Initial footprinting algorithms on genes and proteins

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## Evolution of codons

- Genes evolve at a level of higher granularity
  - Nucleotide
  - Codon
- HMM states corresponding to codons
- How to choose priors for transition probabilities ?



Courtesy: Bioephemera.com

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# Incorporating evolutionary processes

- Selection
- Transition probabilities can reflect
  - Synonymous transitions more frequent than non synonymous ones
  - How much more frequent ?
  - Selection parameters estimated from data

		Second Position						
		U	C	A	G			
		code	Amino Acid	code	Amino Acid	code	Amino Acid	
U	UUU	phe	UCU	ser	UAU	tyr	UGU	cys
	UUC		UCC		UAC		UGC	
	UUA	leu	UCA		UAA	STOP	UGA	STOP
	UUG		UCG		UAG	STOP	UGG	trp
C	CUU		CCU	pro	CAU	his	CGU	
	CUC	leu	CCC		CAC		CGC	
	CUA		CCA		CAA	gln	CGA	arg
	CUG		CCG		CAG		CGG	
A	AUU		ACU	thr	AAU	asn	AGU	ser
	AUC	ile	ACC		AAC		AGC	
	AUA		ACA		AAA	lys	AGA	arg
	AUG	met	ACG		AAG		AGG	
G	GUU		GCU	ala	GAU	asp	GGU	
	GUU		GCC		GAC		GCC	
	GUU	val	GCA		GAA	glu	GGA	gly
	GUG		GCG		GAG		GGG	

Courtesy: Bioephemera.com

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# Summary

- Use genomic representation of functional component
- Use evolutionary models of functional component
- Can be used for non-sequence data too :
  - Gene regulatory network
  - Expression levels : microarray data

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