15-820-a
Assignment 2
Computation Tree Logics

Due Feb. 26, 2003

1 CTL*

Show

$$\mathbf{AGF}p \quad \equiv \quad \mathbf{AGAF}p$$

2 CTL Operators

Show

$$\mathbf{A}[f\ \mathbf{U}\ g] \quad \equiv \quad \neg\,\mathbf{E}[\neg g\ \mathbf{U}\ \neg f \wedge \neg g] \wedge \neg\,\mathbf{EG}\,\neg g$$

3 Büchi Automata

A *non-deterministic Büchi automaton* (NBA) B is a five-tuple

$$B = (A, Q, Q_0, N, F)$$

where A is the alphabet, Q a finite set of states, $Q_0 \subseteq Q$ the set of initial states, and $F \subseteq Q$ the set of accepting states, and $N : Q \times A \longrightarrow 2^Q$ is the next state function.

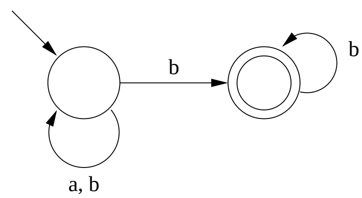
Büchi automata take infinite sequences over A as input. Let $x : \mathbb{N}_0 \longrightarrow A$ be an input sequence. A *run* ρ of B on x is a mapping from $\mathbb{N}_0$ into the set of states Q such that for all $i \in \mathbb{N}_0$:

$$\begin{aligned} \rho(0) &\in Q_0 \\ \rho(i+1) &\in N(\rho(i), x(i)) \end{aligned}$$

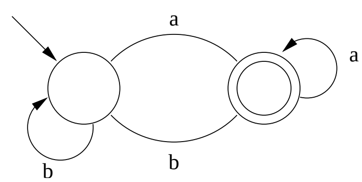
An accepting run of B is a run of B such that there are infinitley many i for which $p(i) \in F$, i.e., accepting states must appear infinitley often.

The language of a Büchi automaton B is the set of sequences x such that there exists an accepting run of B on x.

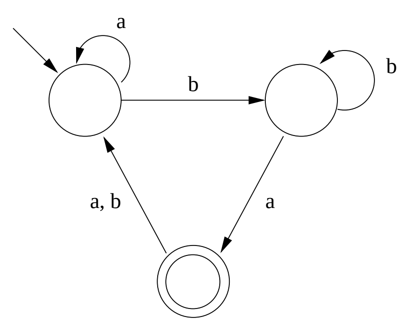
Example: The following NBA accepts the language that corresponds to ”eventually always b”.



The following NBA accepts the language that corresponds to ”infinitley often a”.



The following NBA accepts the language that corresponds to ”infinitley many a’s and b’s”.



A *deterministic Büchi automaton* (DBA) is defined like an NBA, but with a single initial state q_0 and a next state function $N : Q \times A \longrightarrow Q$ that maps one current state and one element of the alphabet into exactly one next state.

Show that the languages accepted by DBAs are not equal to the languages accepted by NBAs.