Feature Selection for Approximate Offline RL

Emma Brunskill

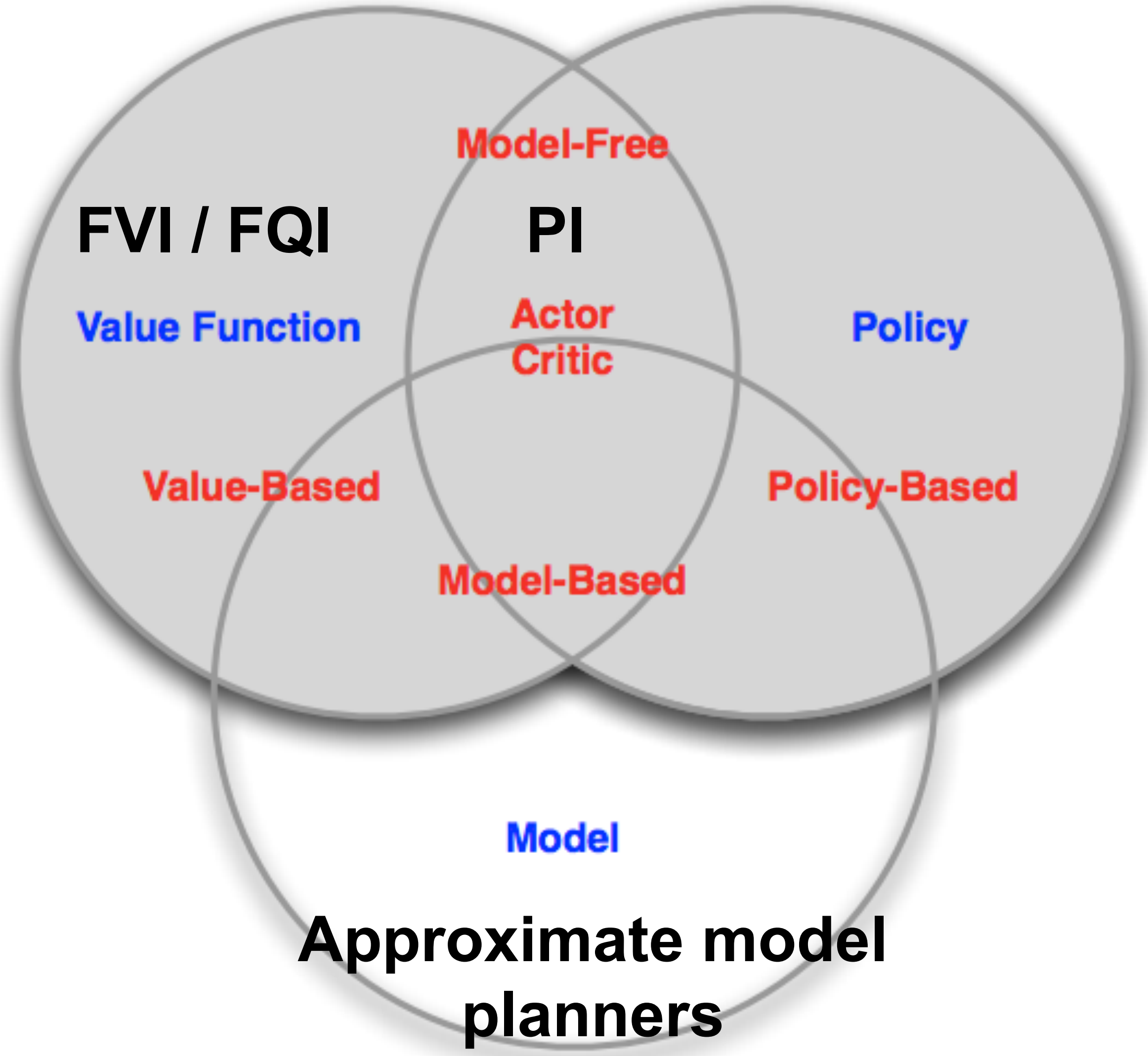

Image from David Silver

Spoken Dialogue System Example
(Pietquin, Geist, Chandramohan & Frezza-Buet)

- Form-filling, task oriented information system for restaurants
- Goal: determine the value of 3 slots
  - Restaurant location
  - Restaurant cuisine
  - Restaurant price range
- Information state of slot represents confidence in the value (from 0 to 1) $\longrightarrow$ State space is 3 dim continuous vactor
- Action space:
  - Ask-A-Slot (one for each slot), ExplicitConfirm-Slot (one for each slot), Implicit-Confirm-And-Ask-A-Slot (6 actions, in combination of 2 slots) and Close-Dialogue action.
-

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  - 351 = 3^3 x 13 Radial basis functions
  - 3 Gaussian kernels for each state dimension
  - 13 actions

What if We Have Very Little Data? What is the Danger?

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Feature-Based Approximate RL

- Where do features come from?
- Does it matter?
  - Yes!
  - Impacts computation
  - Impacts performance
    - Changes feature class, representational power
    - Changes finite sample (finite dataset) performance (can lead to overfitting, changes estimation error)

Overview of Selecting Features for Feature-Based Approximate RL

1. Feature selection

   Input: Big feature set

   Output: Subset of original features

2. Feature compression/projection

   Input: Big feature set

   Output: Projected (dimensionality reduc) features

3. Feature construction

   Input: Small feature set

   Output: Superset of original feature set

Feature Selection

- Input: Big feature set
- Output: Subset of features
- Techniques build strongly on supervised learning regularization
- L2 norm (Ridge regularization)
  - $\min_w ||Y - Xw||_2 + b\ ||w||_2$
- L1 norm (Lasso)
  - $\min_w ||Y - Xw||_2 + b\ ||w||_1$
- da

Feature Selection for Approximate RL

Objective of Fitting Q/V	L2 Regularization (Ridge)	L1 Regularization (LASSO)	Orthogonal Matching Pursuit
Fixed Point (LSTD)	X	LARS-TD (Kolter & Ng 2009), Johns et al. (2010)	Painter-Wakefiled & Parr (2009)
Fitted V/Q Iteration	X	LASSO on FQI	Value pursuit iteration
Bellman Residual Minimization	X	Loth et al (2007)	Painter-Wakefiled & Parr (2009)

Comparisons across AVI (approximate value iteration) & API (approximate policy iteration) are rare

Feature Dimensionality Reduction

Take a set of features, and project down to a lower dimensional basis

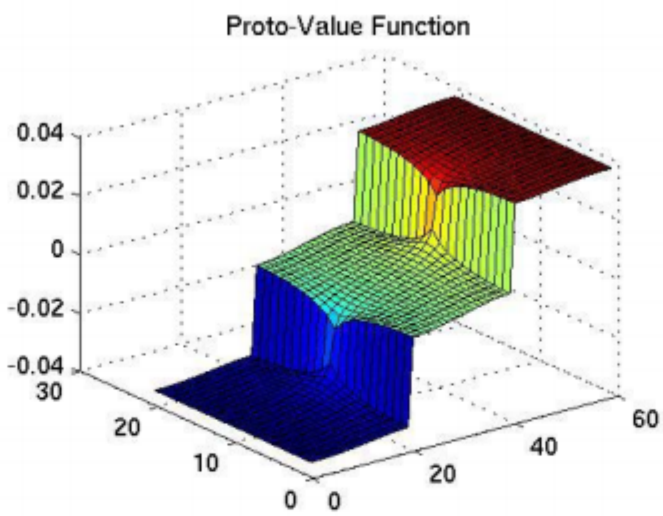
Can use any form of dimensionality reduction (Principle component analysis, …)

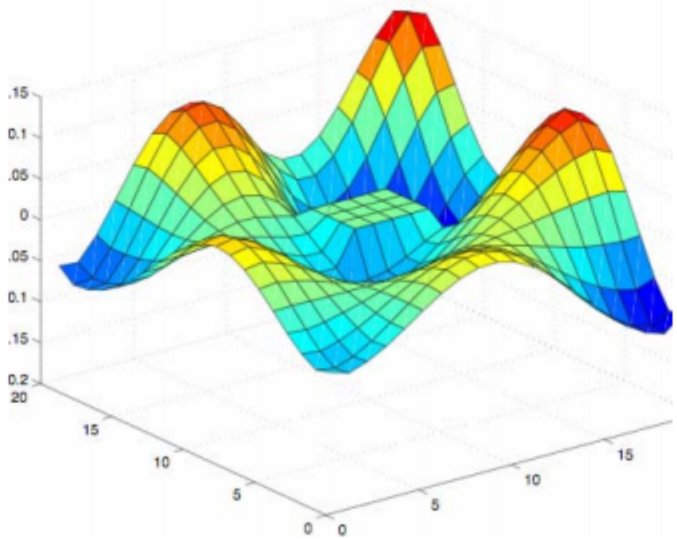
Feature Construction

- Protovalue function construction (Mahadevan & colleagues)
- Bellman Error Basis Function (BEBF) (Parr et al. 2007)
- Incremental Feature Dependency Discovery (Geramifard & colleagues)

Proto-Value Functions

(Mahadevan: AAAI 2005, ICML 2005, UAI 2005)

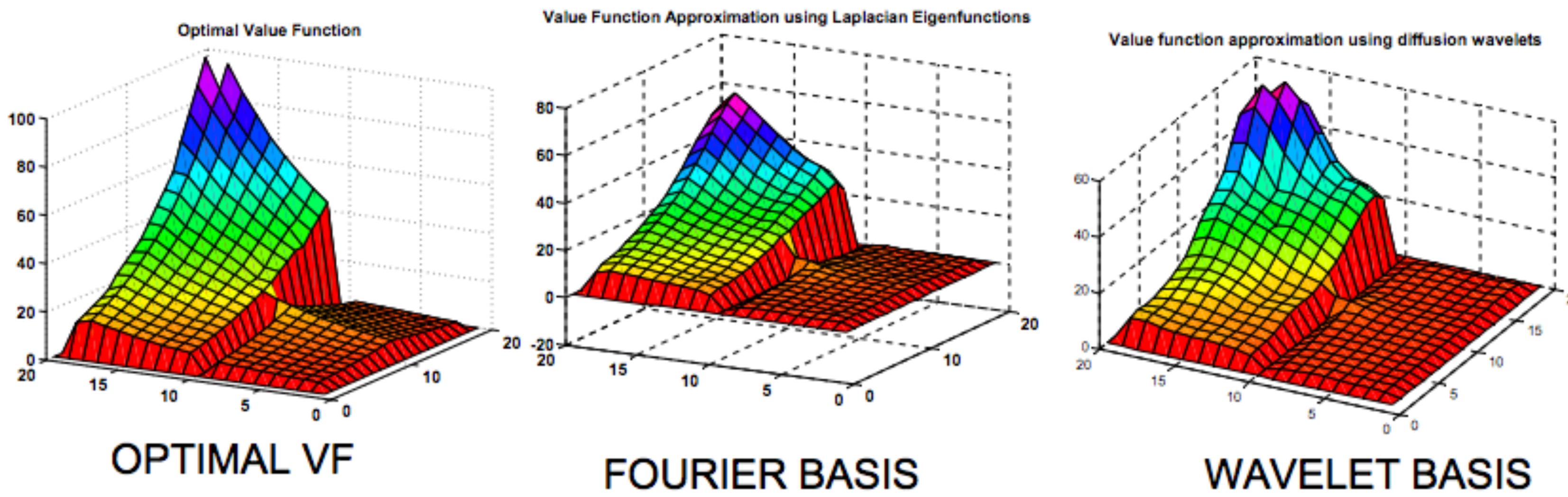




Proto-value functions are reward-independent global (or local) basis functions, customized to a state (action) space

Slide from Mahadevan

Value Function Approximation using Fourier and Wavelet Bases

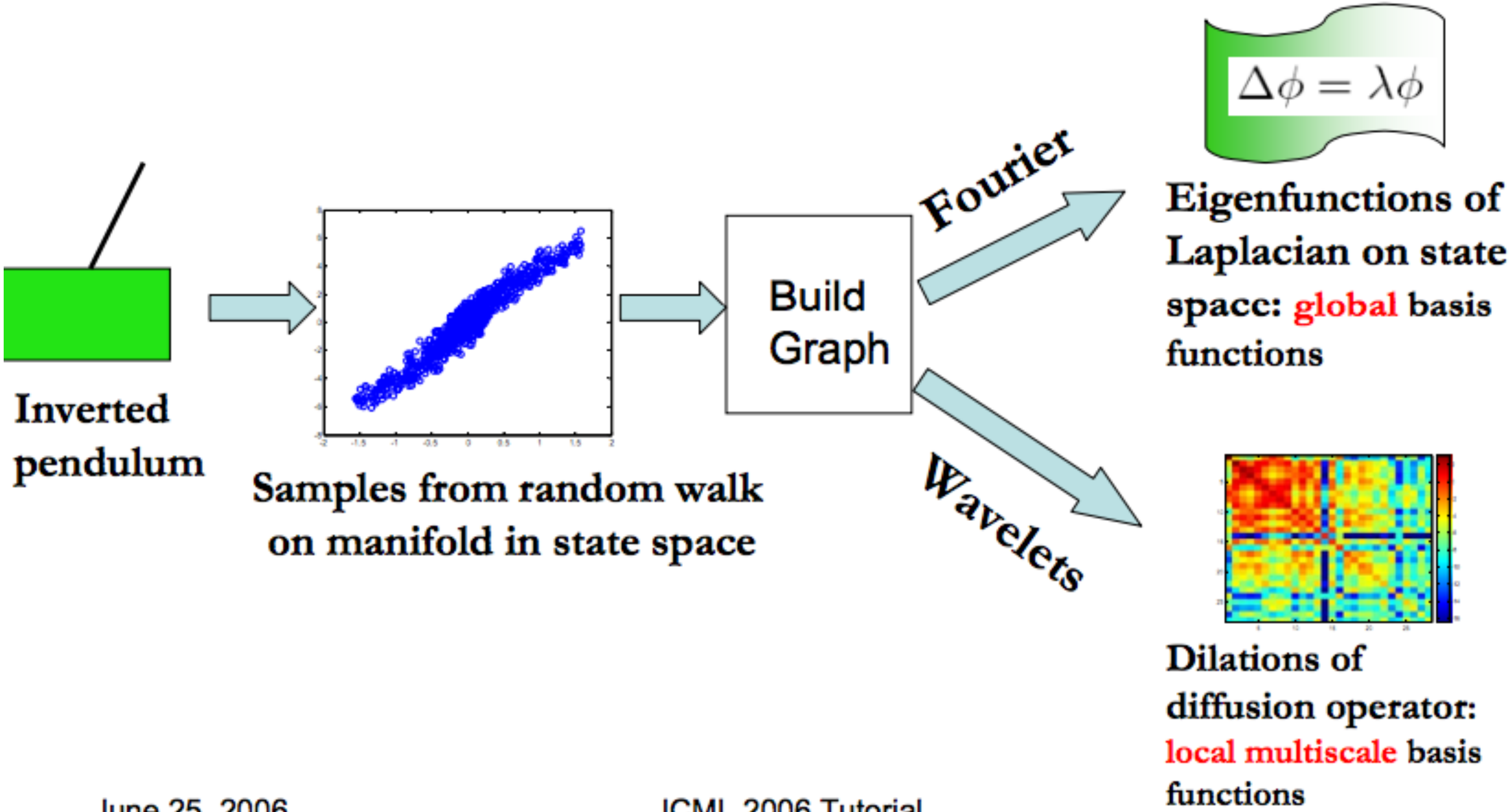


These bases are automatically learned from a set of transitions (s,a,s')

Slide from Mahadevan

Overview for Protovalue Function Basis Invention

(Mahadevan, AAAI,ICML,UAI 2005; Mahadevan & Maggioni, NIPS 2005; Maggioni and Mahadevan, ICML 2006)



June 25, 2006 ICML 2006 Tutorial

Slide from Mahadevan

Overview of Selecting Features for Feature-Based Approximate RL

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   Input: Big feature set

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3. Feature construction

   Input: Small feature set

   Output: Superset of original feature set

Evaluation of Methods for Designing Features (for RL)

1. Empirical quality of resulting solution

   Mean square error relative to true value function

   Output: Subset of original features

2. Computational complexity

   As a function of features, data set size, ...

   Output: Projected (dimensionality reduc) features

3. Formal guarantees on performance

   Is the method stable? (Converge to a fixed set of features)

   If a small set of features is sufficient to represent V, can find that set?

4. Sample efficiency

   How well does it use the available data to find good features?

Rest of Today

1. **Feature selection**

   Input: Big feature set

   Output: Subset of original features

   Idea: Greedily select features.

2. Feature compression/projection

   Input: Big feature set

   Output: Projected (dimensionality reduc) features

3. **Feature construction (may get to)**

   Input: Small feature set

   Output: Superset of original feature set

OMP Overview: On the board

OMP Empirical Comparison

- LARS-TD: LSTD + L1 regularization
- LARS-BRM: BRM + L1 regularization
- OMP-TD
- OMP-BRM

Empirical Setup

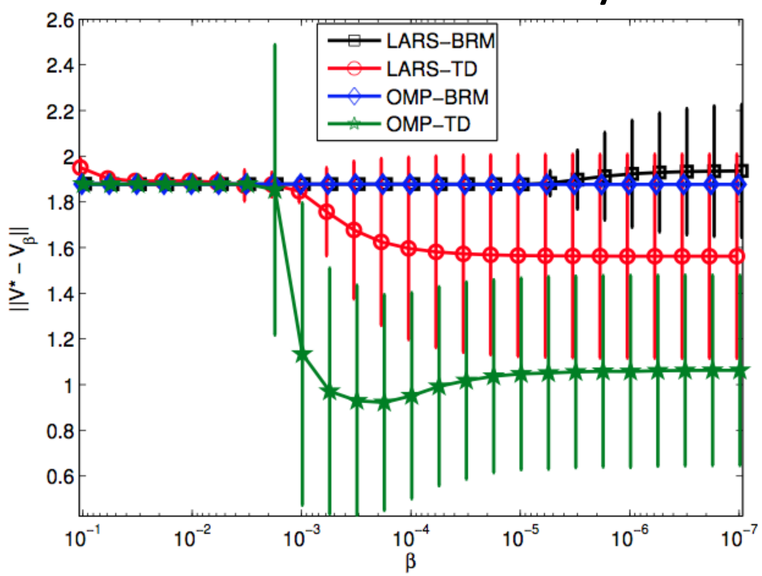
Problem	State space	Features	Samples	Trials	LARS-TD L_2?	BRM double samples?
Chain	Discrete, 50 states	208	500	1000	×	√
Pendulum	Continuous, 2d	268	200	1000	√	√
Blackjack	Discrete, 203 states	219	1600	1000	×	×
Mountain Car	Continuous, 2d	1366	5000	100	√	×
Puddleworld	Continuous, 2d	570	2000	500	×	×
Two Room	Continuous, 2d	2227	5000	1000	×	×

size of dataset used to fit V*

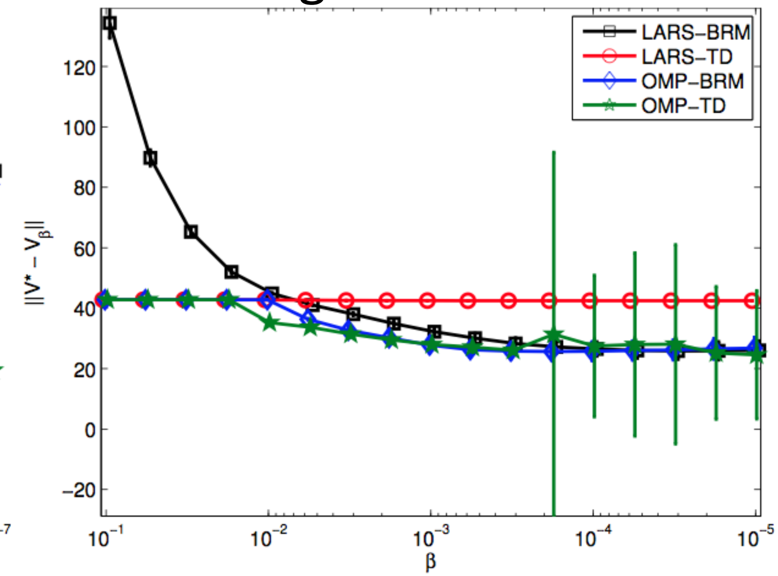
number of trials used to evaluated resulting solution/weights

Slide from Painter-Wakefield & Parr

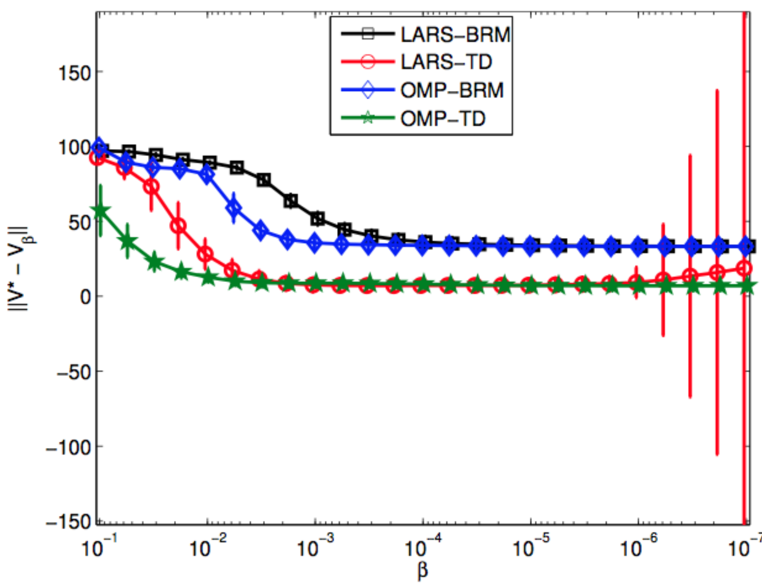
OMP Results: TD generally better than BRM, OMP Generally better than L1 Regularization



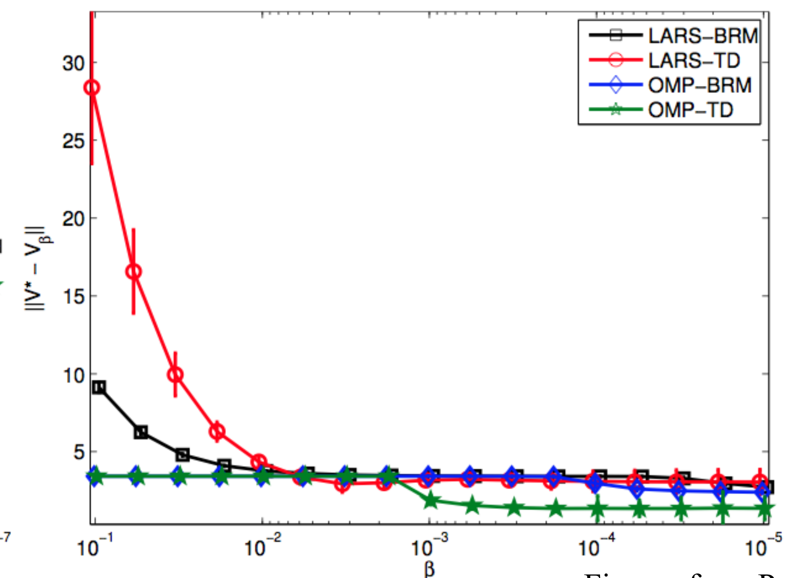
(c) Blackjack



(d) Mountain Car



(e) Puddleworld



(f) Two Room

Figures from Painter-Wakefiled & Parr

*Important Notes on Empirical Comparison

- LARS-TD
  - Sometimes added small amount of L2 regularization on top of L1 regularization
- OMP-BRM
  - Sometimes added in small amount of L2 regularization
- OMP-TD
  - Added small amount of L2 regularization when computing final solution for a given beta
  - Seemed critical to get stable performance for harder problems
  - When # samples very small, more unstable
-

OMP-BRM and OMP-TD Summary

Takes in a set of features

Greedily adds features to set

OMP-TD has better empirical performance than OMP-BRM, but OMP-BRM has stronger theoretical guarantees

OMP-BRM/TD Limitation

Scalability

Required to compute residual with all (remaining) features at every iteration

Rest of Today

1. Feature selection

   Input: Big feature set

   Output: Subset of original features

   Idea: Greedily select features.

2. Feature compression/projection
3. **Feature construction**

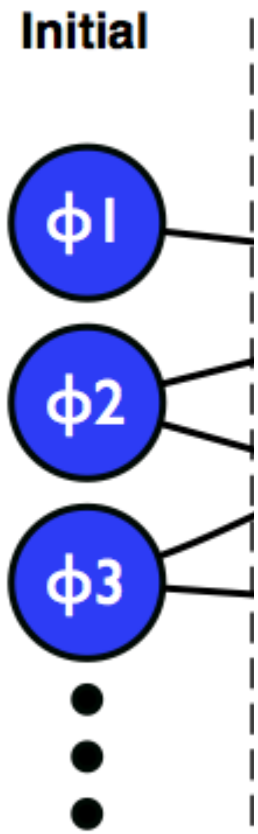
   Input: Small feature set

   Output: Superset of original feature set

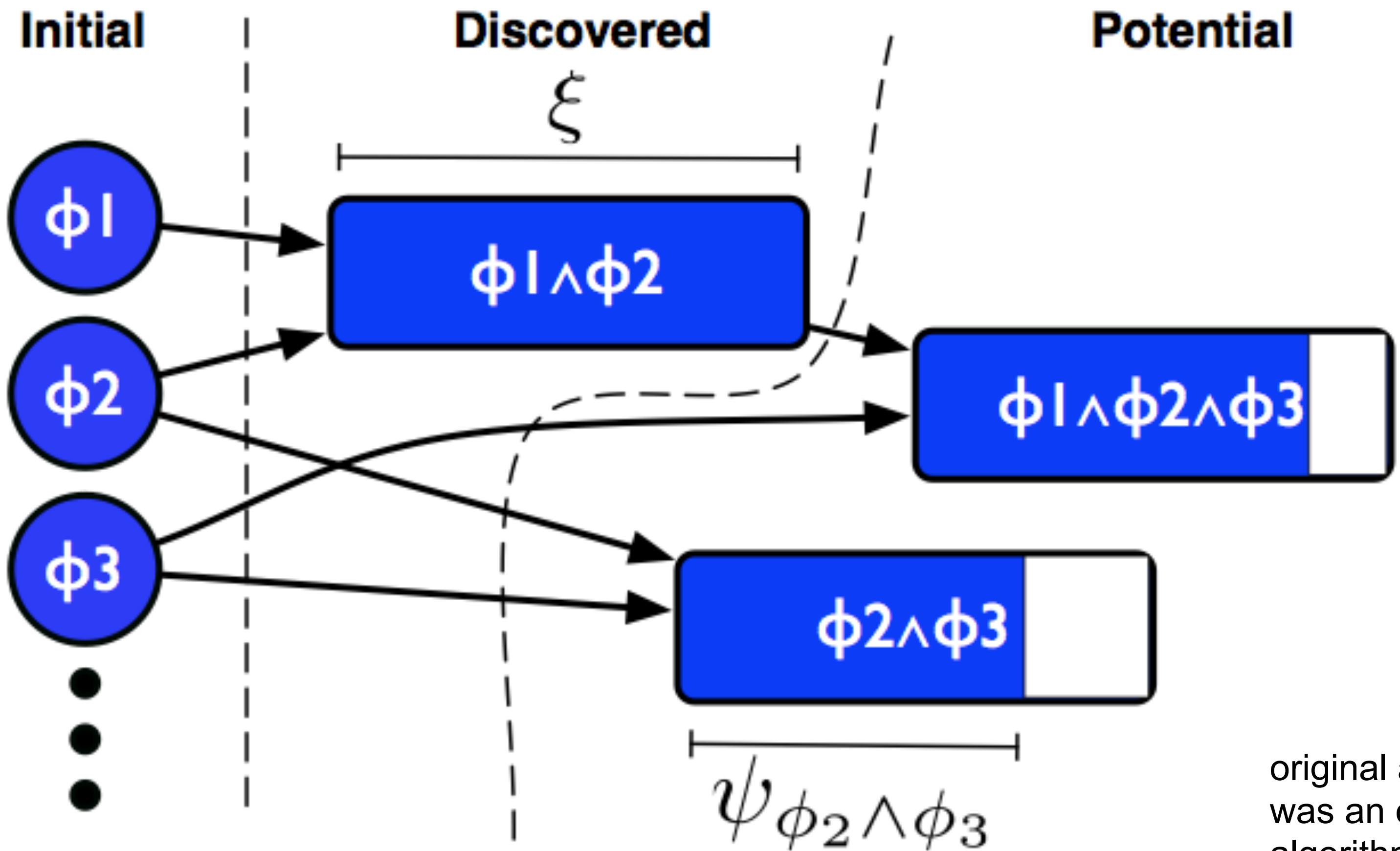
   Idea: Greedily add conjunctions of features

Alternative: Generate Features

iFDD [Geramifard et al. 2011]



iFDD [Geramifard et al. 2011]



original algorithm was an online algorithm

Batch-iFDD

- Run iFDD in Batch: Add new feature (conjunction) with highest error reduction (akin to OMP-TD).
- *Theorem*: iFDD in batch approximately finds the feature with the best guaranteed error reduction.

$$||\tilde{\mathbf{V}} - \mathbf{\Pi T}(\tilde{\mathbf{V}})||$$

$$\tilde{f}_1^* = \operatorname*{argmax}_{f \in \mathtt{pair}(\chi)} \frac{|\sum_{i \in \{1,\cdots,m\}, \phi_f(s_i)=1} \delta_i|}{\sqrt{\sum_{i \in \{1,\cdots,m\}, \phi_f(s_i)=1} 1}} \quad \text{iFDD}^{+}$$

$$\tilde{f}_2^* = \operatorname*{argmax}_{f \in \mathtt{pair}(\chi)} \sum_{i \in \{1,\ldots,m\}, \phi_f(s_i)=1} |\delta_i| \quad \text{iFDD}$$

[Geramifard et al. 2012]

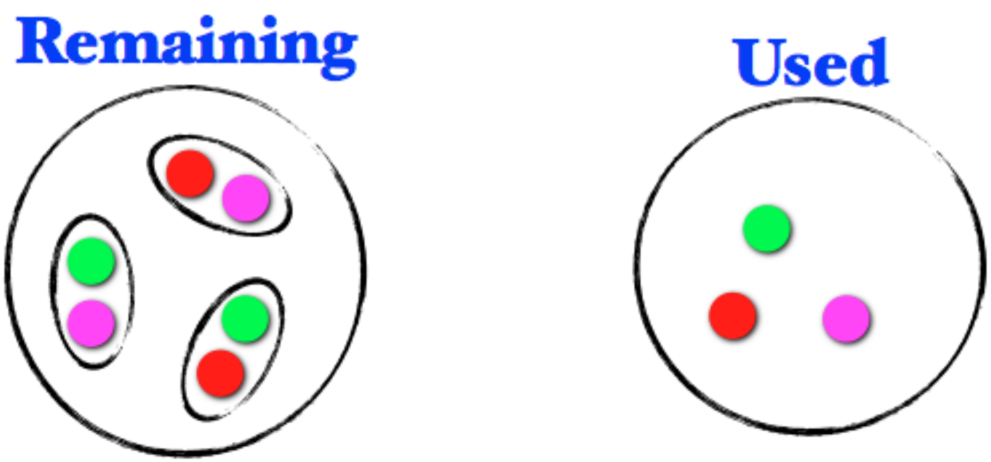
Slide from Geramifard

Batch-iFDD

- Loop

  (1) Run LSTD [Bradtke & Barto 1996]

  (2) Expand feature sets



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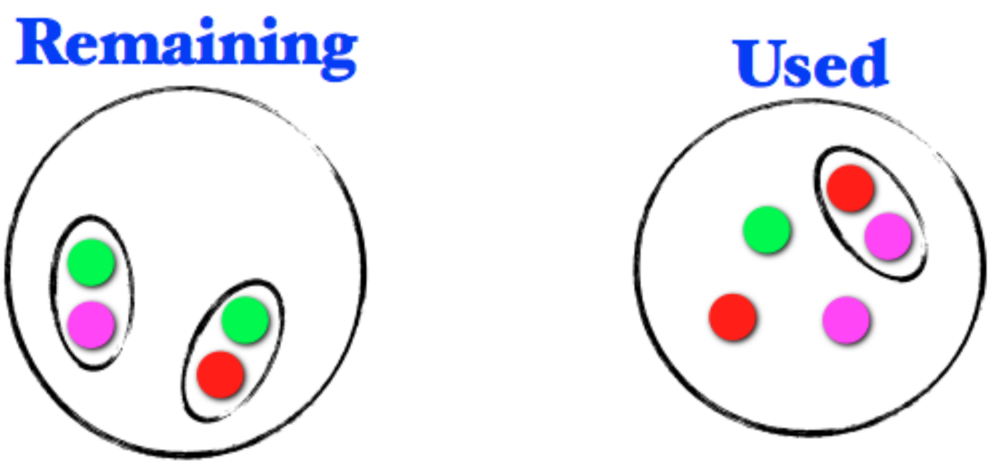

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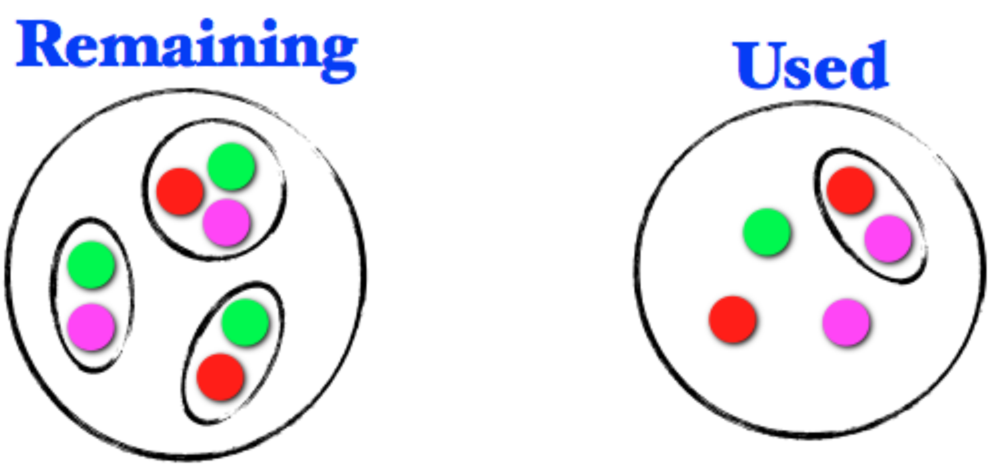
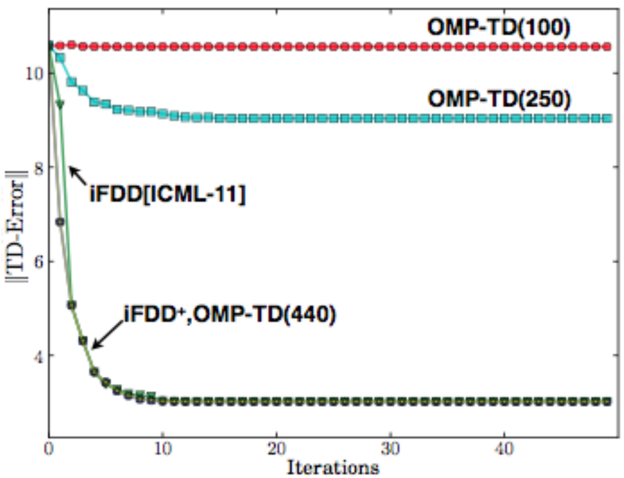
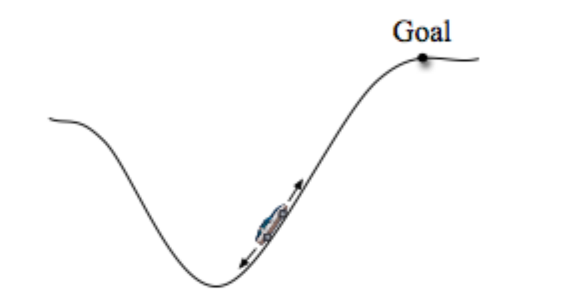
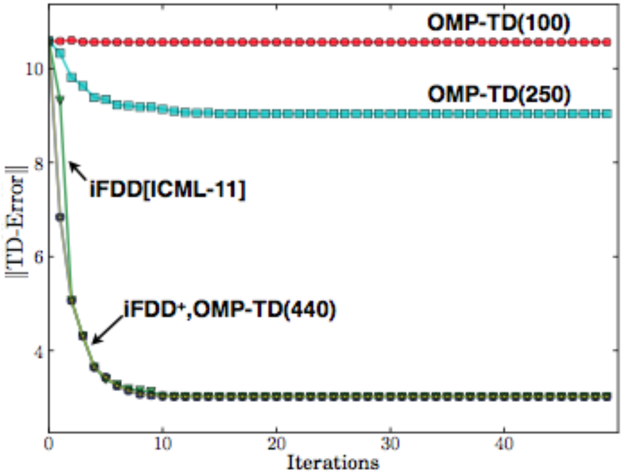
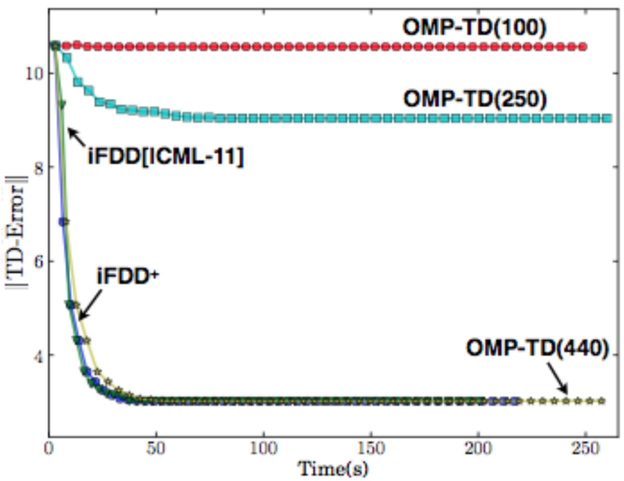



OMP-TD(100)
OMP-TD(250)
iFDD[ICML-11]
iFDD+,OMP-TD(440)
||TD-Error||
Iterations
0
10
20
30
40
4
6
8
10

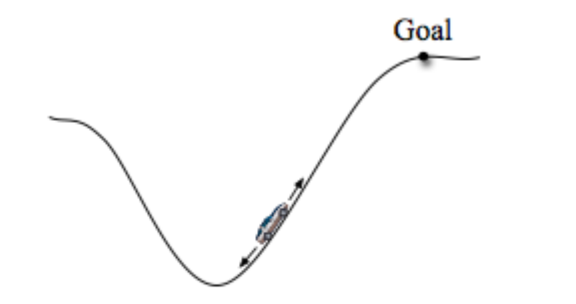

Goal

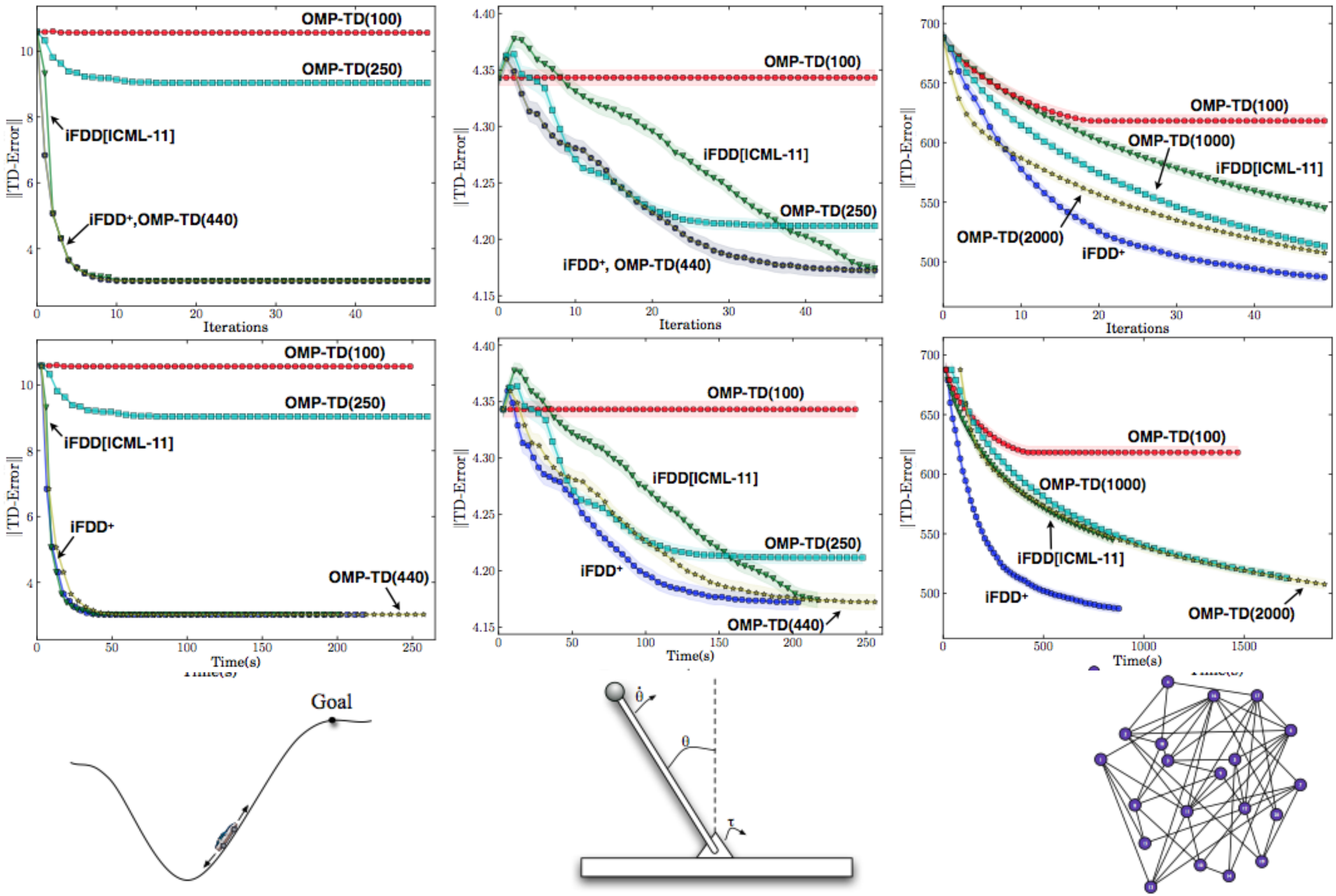


Iterations



Computation Time

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OMP-TD(250)
iFDD[ICML-11]
iFDD+,OMP-TD(440)
Iterations
||TD-Error||
iFDD+, OMP-TD(440)
OMP-TD(1000)
OMP-TD(2000)
iFDD+
OMP-TD(440)
Time(s)
Goal
θ
θ̇
τ

Feature Generation with OMP

- Batch IFDD+ sample efficient and (computationally) scalable
- Still relies on decent input set of features
- Requires input features are binary
  - Also limits type of features can create
  - OMP-TD can handle general features

Summary

1. Feature selection

   Should be able to characterize OMP-BRM & OMP-TD

   (computational complexity, strengths/limitations)

   Should be able to implement both

2. Feature compression/projection (know these exist)
3. Feature construction

   Should understand (at a high level) how Batch iFDD works

   Be able to list benefits over OMP