

A PHYLOGENETICS SURVEY

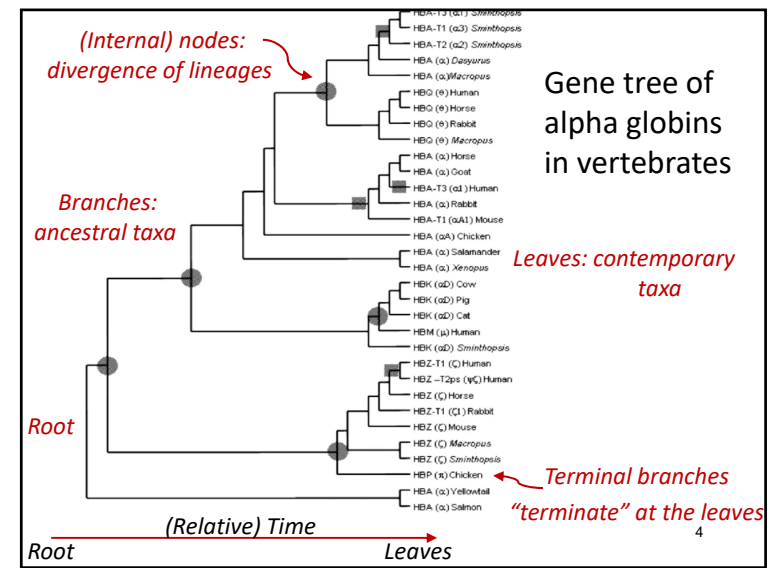
Dr. Maureen Stolzer
October 2nd, 2025

TREE TERMINOLOGY

Information encoded in trees

- Tree terminology
- Branch lengths
- Similarity, common ancestry and relatedness
- Binary and Non-binary trees
- Rooted and unrooted trees

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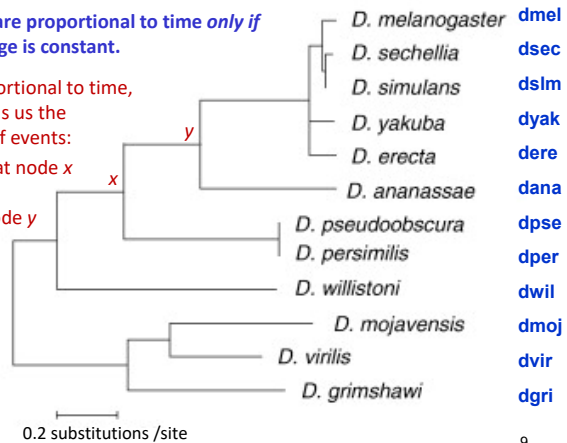


Branches are proportional to the amount of change,
typically in substitutions per site.

Branch lengths are proportional to time *only if*
the rate of change is constant.

Even if not proportional to time,
topology still tells us the
relative timing of events:

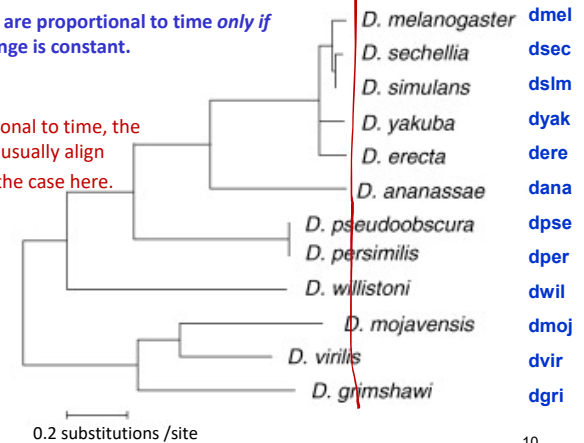
The divergence at node x
came before the
divergence at node y



Branches are proportional to the amount of change,
typically in substitutions per site.

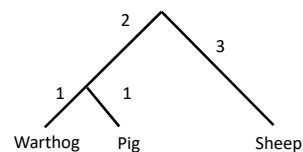
Branch lengths are proportional to time *only if*
the rate of change is constant.

When proportional to time, the
leaf nodes will usually align
which is not the case here.



Ultrametric trees

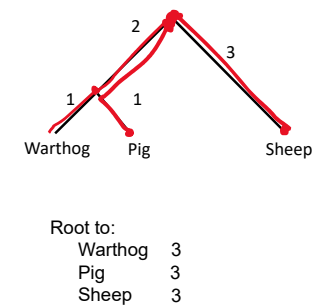
- The rate of change is the same in all lineages
- Branch lengths are proportional to time
- The distance from the root to leaf is the same for all leaves
- The root is at the midpoint between the two most distant taxa.



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Ultrametric trees

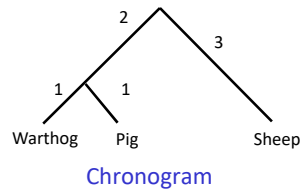
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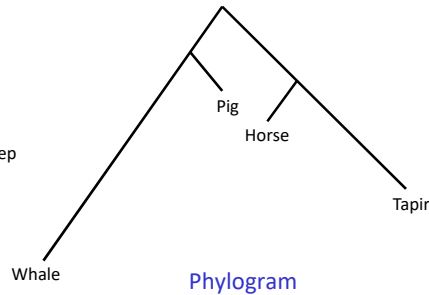
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An *ultrametric* tree has branch lengths that are proportional to time.

Ultrametric



Not ultrametric



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Information encoded in trees

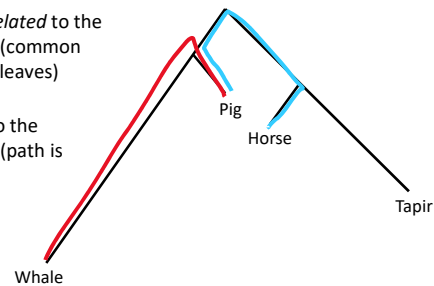
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- Rooted and unrooted trees

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Similarity versus common ancestry

More closely related taxa are not guaranteed to be more similar except in ultrametric trees, where the rate of change is proportional to time.

- The pig is *more closely related* to the **whale** than to the **horse** (common ancestor is closer to the leaves)
- The pig is *more similar* to the **horse** than to the **whale** (path is shorter)

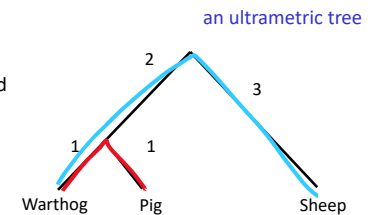


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Similarity versus common ancestry

More closely related taxa are not guaranteed to be more similar except in ultrametric trees, where the rate of change is proportional to time.

- The warthog is more closely related to the **pig** than to the **sheep**
- The warthog is more similar to the **pig** than to the **sheep**



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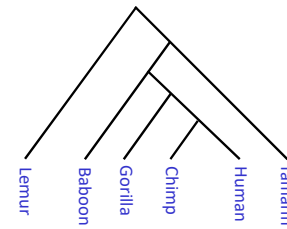
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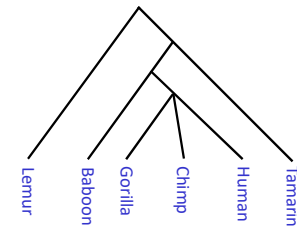
Binary versus non-binary trees

In a rooted tree, every *binary* node has 2 descendants



A tree is *binary* if every internal node is binary

In a rooted tree, a *non-binary* node or *polytomy* has 3 or more descendants



A tree is *non-binary* if one or more internal nodes are *non-binary*

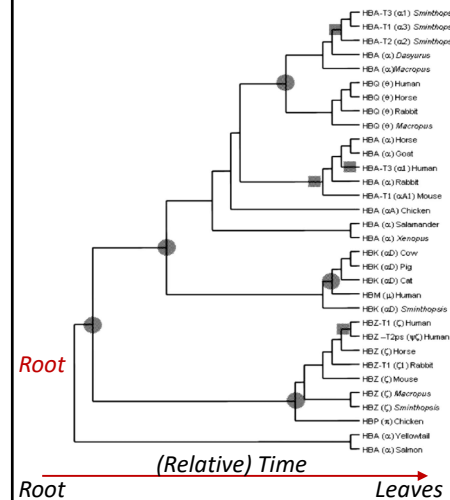
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Information encoded in trees

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Unrooted trees versus rooted trees

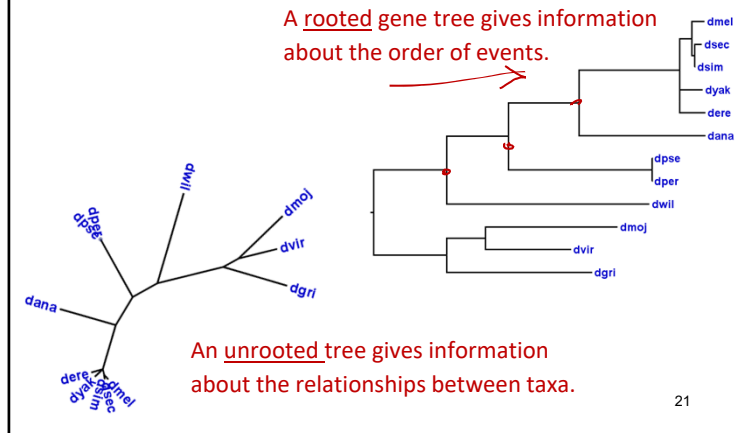


Previously, I stated that going from the root to the leaves gives us relative timing...

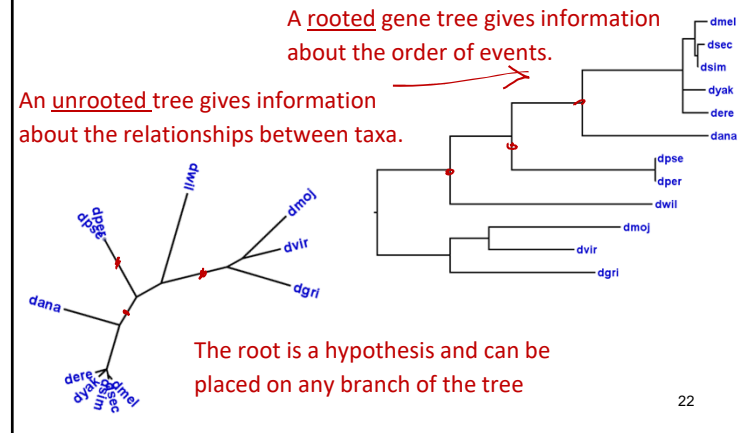
This is only for *rooted* trees

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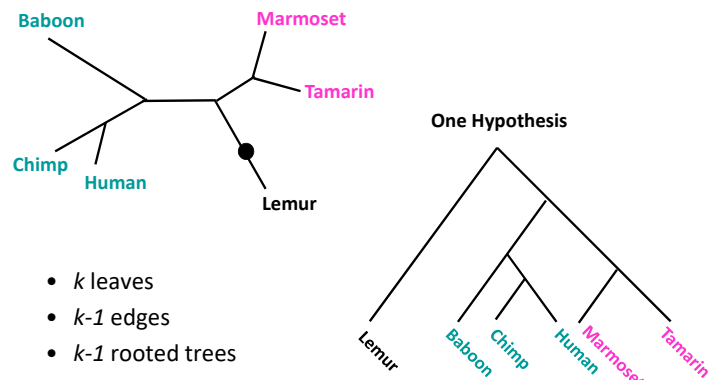
Unrooted trees versus rooted trees



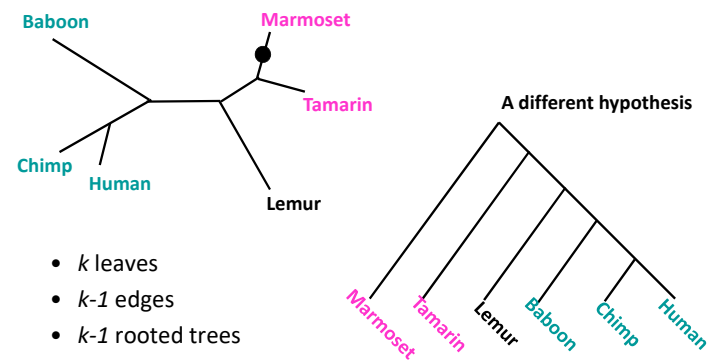
Unrooted trees versus rooted trees

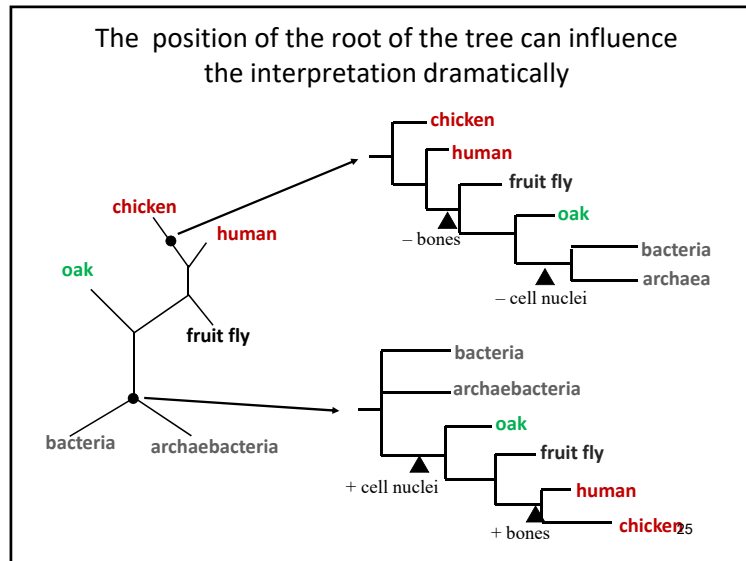


- There are as many possible roots as there are edges in the tree.
- Each of these is a different hypothesis about the order of events.



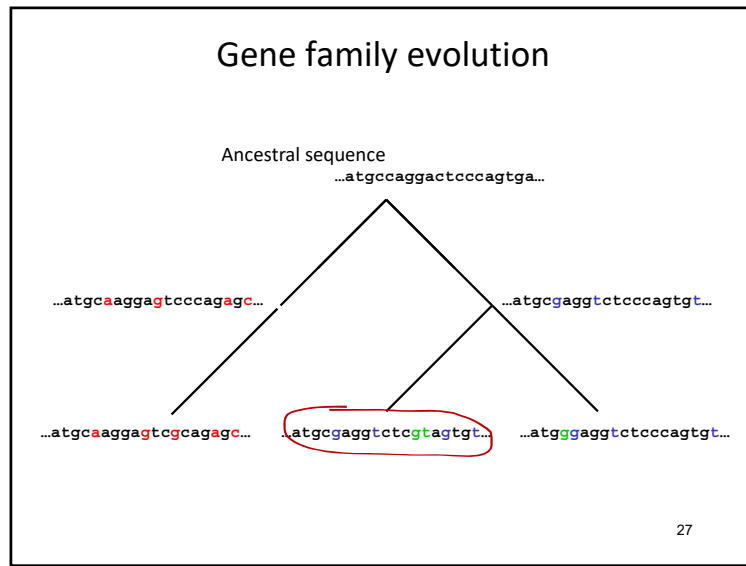
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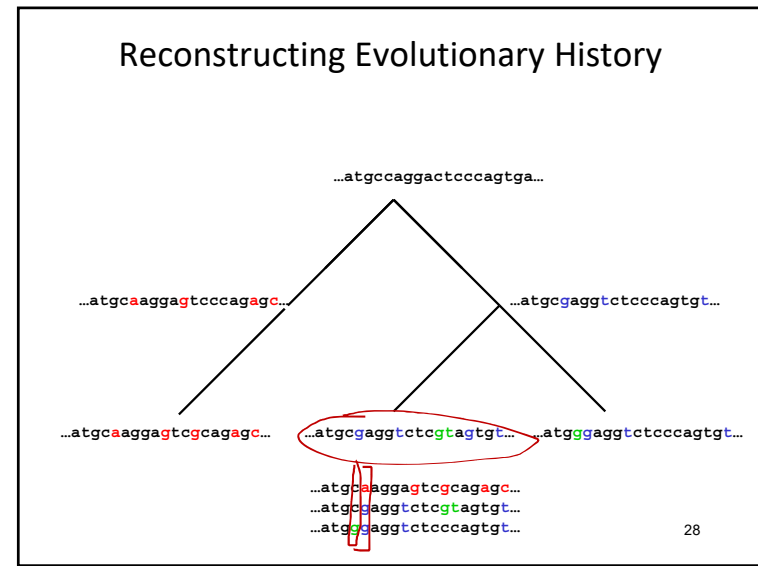


PHYLOGENY RECONSTRUCTION

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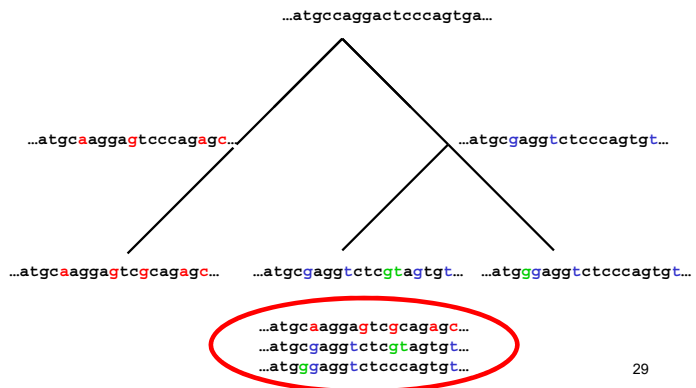


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Reconstructing Evolutionary History

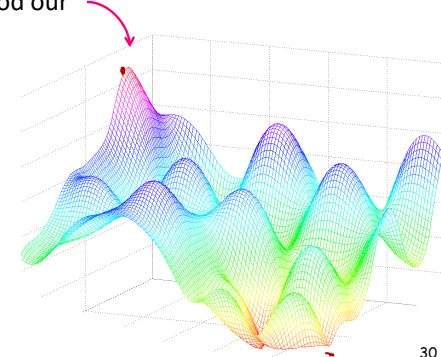


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Space of all possible trees for a given set of taxa

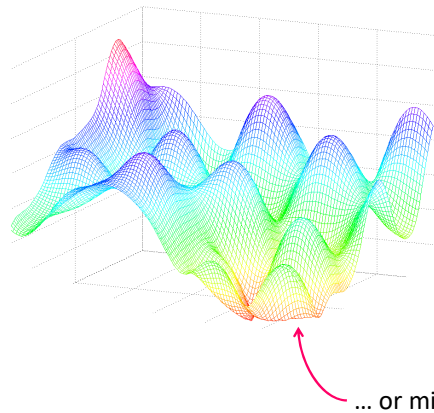
We can seek to maximize a measure of how good our solution is...

Optimality criteria



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Space of all possible trees for a given set of taxa



Optimality criteria

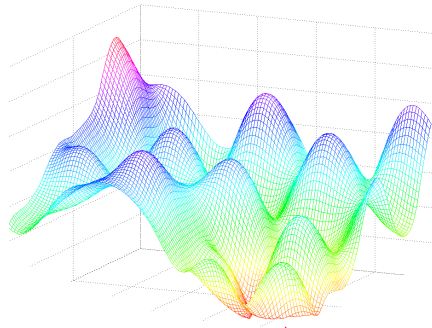
... or minimize a cost.

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TREE SPACE

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Space of all possible trees for a given set of taxa



Goal: Find the most
parsimonious tree₃₃

Tree reconstruction

1. Construct a tree that fits the data
This is only possible when the data satisfies some very restrictive conditions.
2. Score each possible tree with k leaves and select the best one

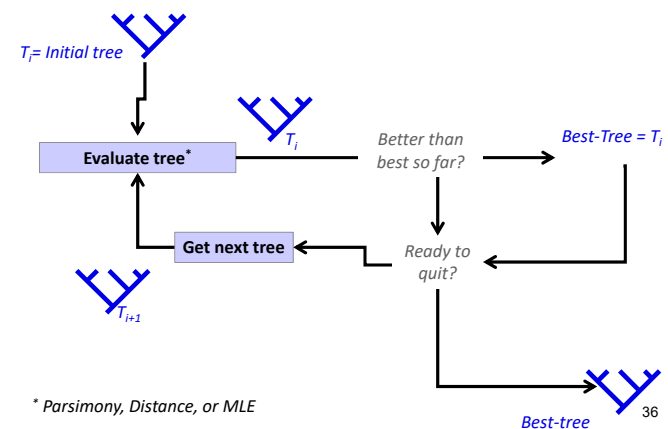
34

Tree reconstruction

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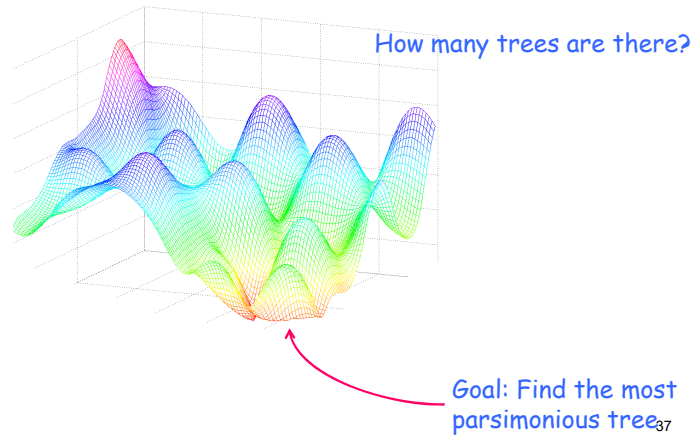
35

Phylogeny reconstruction



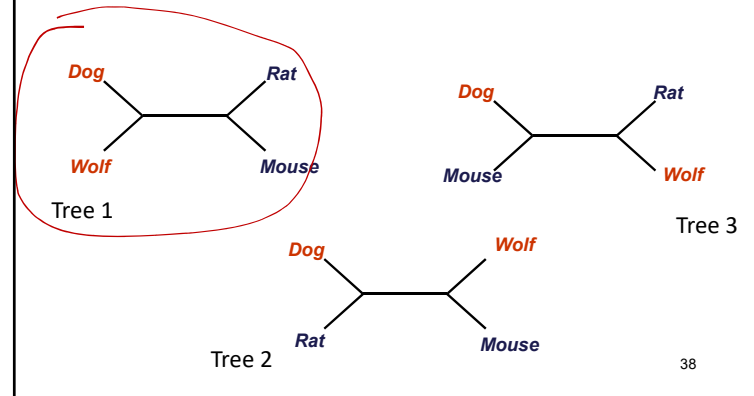
36

Space of all possible trees for a given set of taxa

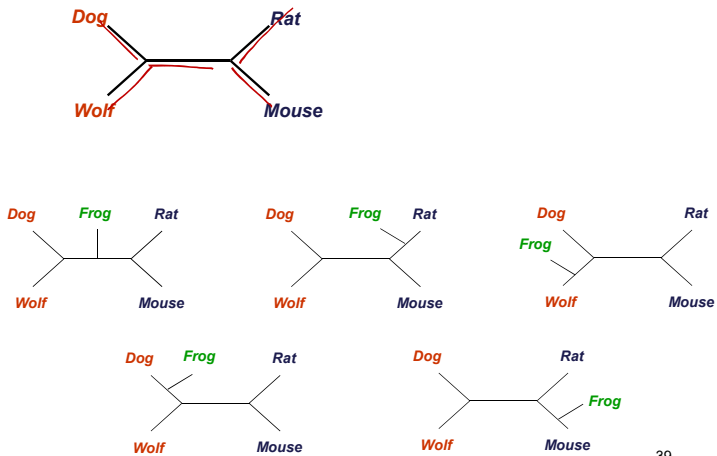


Given 4 taxa, there are three possible unrooted trees (hypotheses).

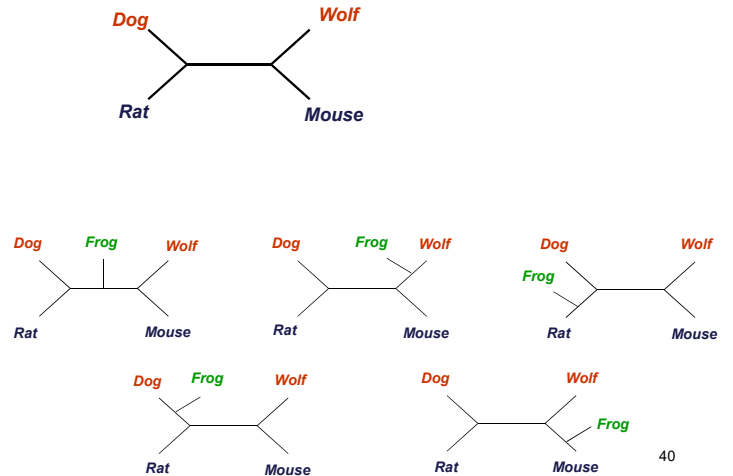
Given 5 taxa, how many unrooted trees (hypotheses) are there?



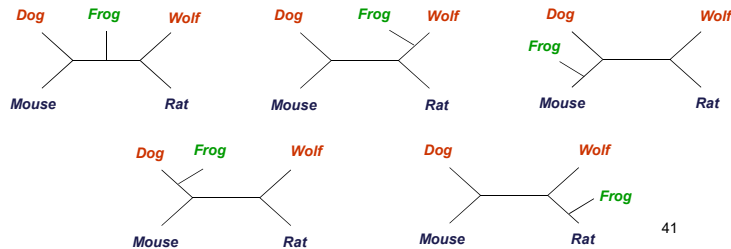
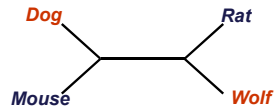
Tree 1



Tree 2



Tree 3



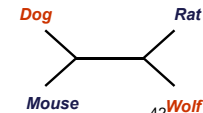
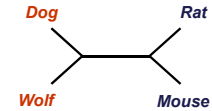
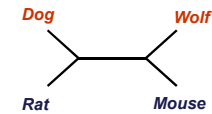
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Given 5 taxa, how many unrooted trees (hypotheses) are there?

There are 3 unrooted trees with 4 leaves

Each tree has 5 edges; i.e., 5 places where a new taxon can be attached.

3 trees x 5 edges → 15 trees with 5 leaves



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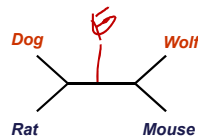
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15t x 7e



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In general, how many hypotheses must we consider for k leaf taxa?

- A tree with k leaves, has $2k - 3$ edges
- For a given tree with k leaves, we can add a new taxon to any edge to get a different tree with $k + 1$ leaves.
- That gives us $2k - 3$ new trees for *each* tree with k leaves

Ergo, the number of hypotheses we need to consider gets big very, very quickly, as the number of taxa increases

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The number of trees gets big fast

Number of taxa	Number of unrooted binary trees
3	1
4	3
5	15
6	105
10	2,027,025
20	2.2×10^{20}
50	2.8×10^{74}
500	1×10^{1074}

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How big is that?

Number of taxa	Number of unrooted binary trees
...	...
20	2.2×10^{20}
50	2.8×10^{74}
<u>500</u>	<u>1×10^{1074}</u>

Age of the universe (seconds):

 4.42×10^{17}

Diameter of the universe (cm):

 2.6×10^{28}

Number of stars in the universe:

 1×10^{22}

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In general, how many hypotheses must we consider for k leaf taxa?

The number of hypotheses we need to consider gets big very, very quickly, as the number of taxa increases.

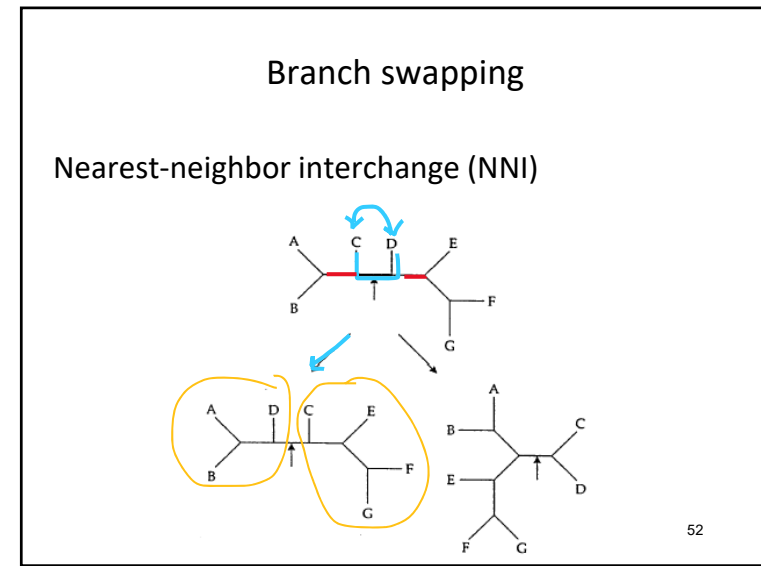
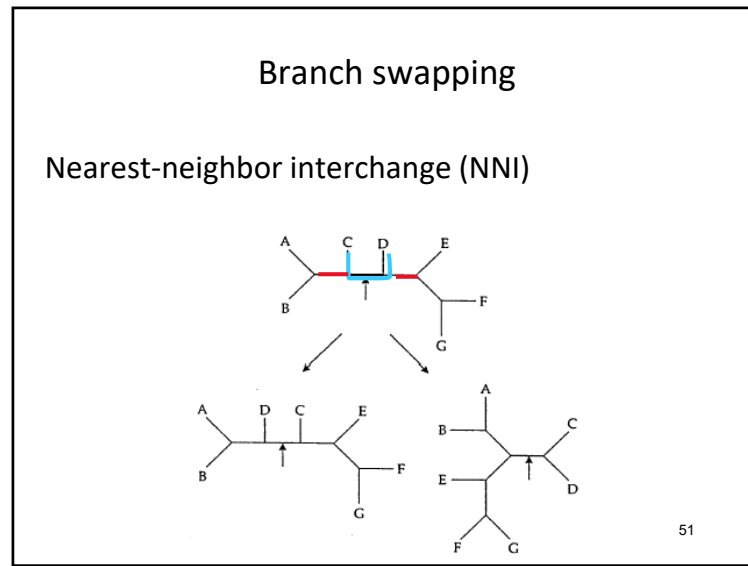
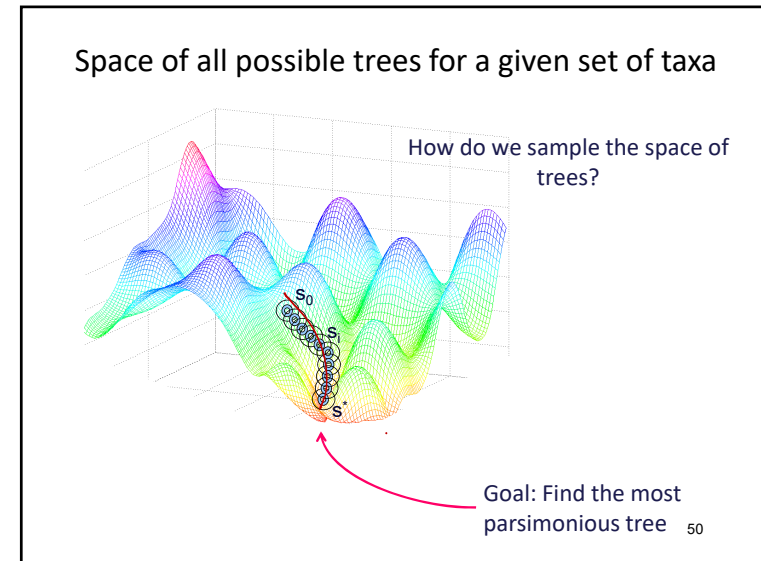
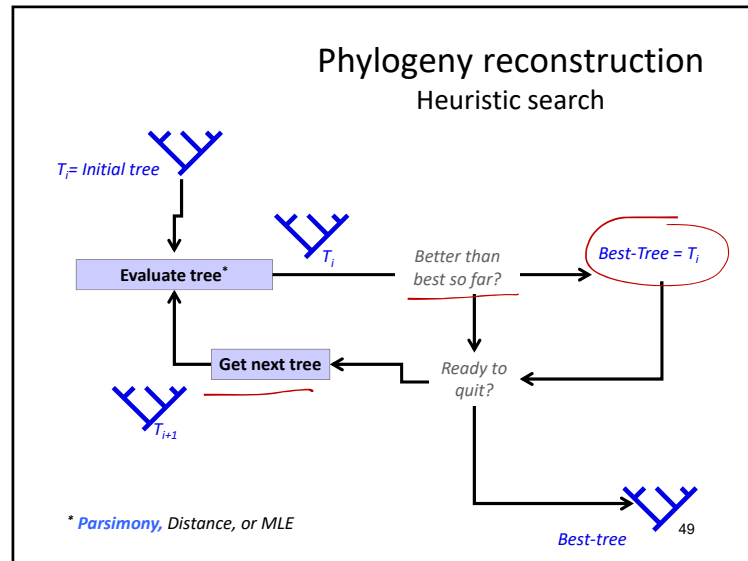
- If k is small (e.g., $k \leq 10$), we can consider all hypotheses.
- Otherwise ($k > 10$), sample a subset of all possible hypotheses.

“Heuristic search”

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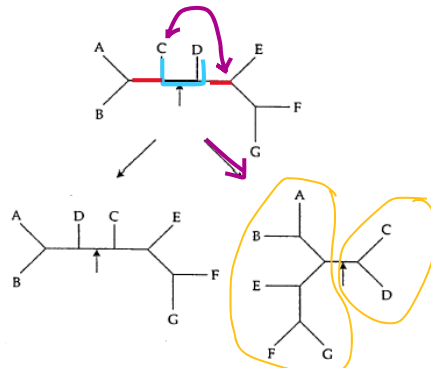
SEARCHING TREE SPACE

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Branch swapping

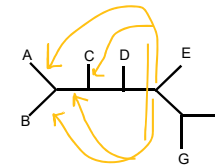
Nearest-neighbor interchange (NNI)



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Branch swapping

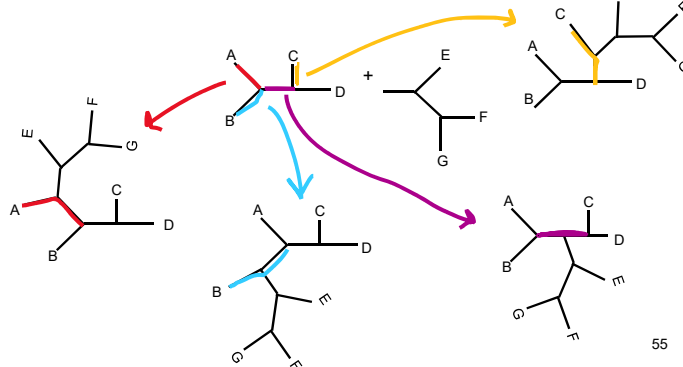
Subtree prune and regraft (SPR)



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Branch swapping

Subtree prune and regraft (SPR)



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Phylogeny reconstruction

- Design a **mathematical function** for scoring evolutionary trees such that
 - Good trees get good scores
 - Bad trees get bad scores
- Find the *optimal* tree; the tree with the best score
- Potential problems:
 - The optimal tree can be hard to find – NP Hard
 - A good numerical score does not always translate to a good evolutionary hypothesis.

**Optimality
criterion**

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Optimality criteria

Maximum parsimony

Tree that requires the fewest changes to explain the data is the best tree.

Maximum likelihood estimation

Tree that maximizes the likelihood of the data

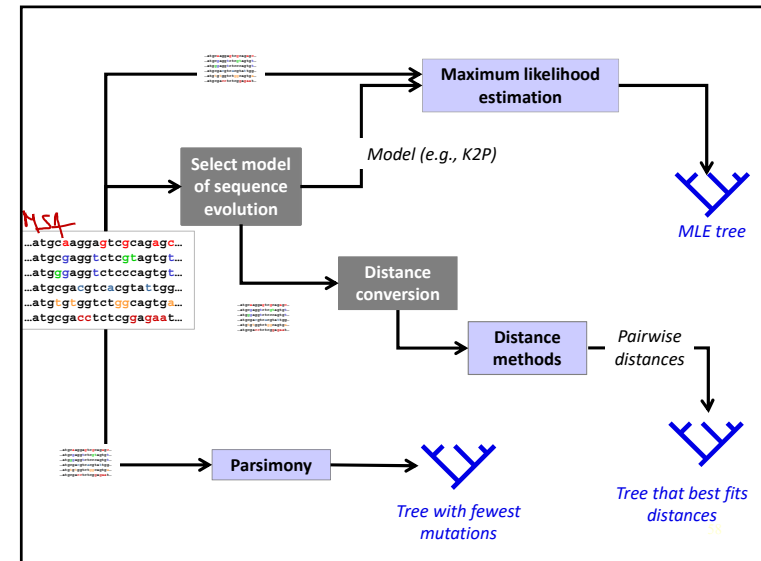
Distance-based evaluation

Tree that best fits observed distances between taxa

Character data

Distance data

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PARSIMONY

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The *parsimony* criterion

- In regular speech, “parsimonious” means excessively thrifty or sparing, stingy, or frugal
- Phylogenetics: the most parsimonious tree is the tree that requires the fewest substitutions to explain the data.

Evolutionary change on a tree

- Given
 - a tree
 - a set of characters that are variable for these taxa,
 - a character state matrix for the leaf taxa
- infer
 - the character states of each ancestral node and
 - the state changes along each branch
- such the *number of changes required is minimal*

Parsimony

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An example:

Given

- 4 taxa
- data describing each taxon

find

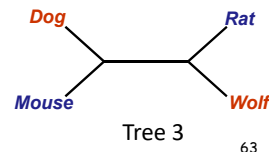
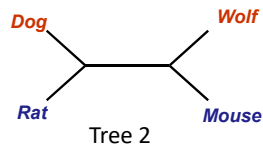
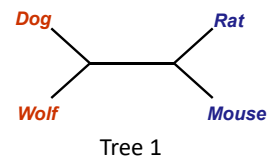
the most parsimonious tree with 4 leaves.

Dog:	ACC
Wolf:	ATC
Rat:	CCC
Mouse:	CTG

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Which tree is most parsimonious?

Dog: ACC
Wolf: ATC
Rat: CCC
Mouse: CTG



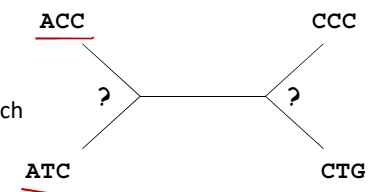
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Evaluation using the parsimony criterion

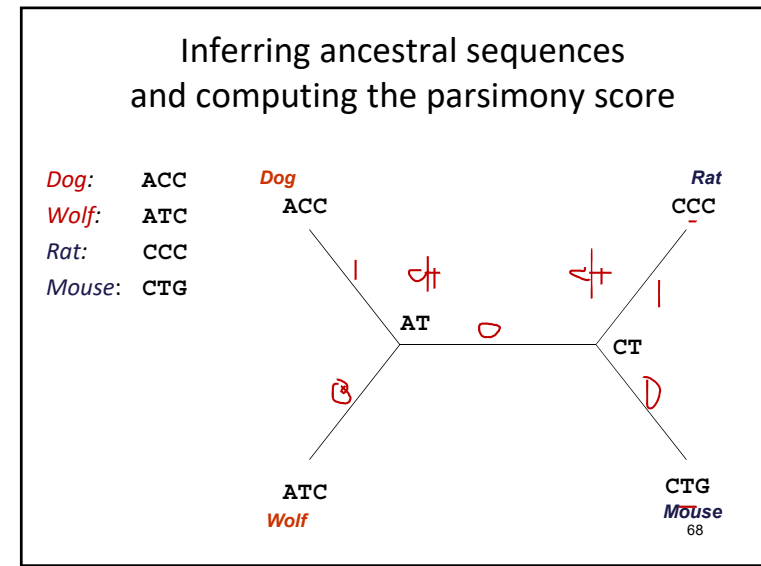
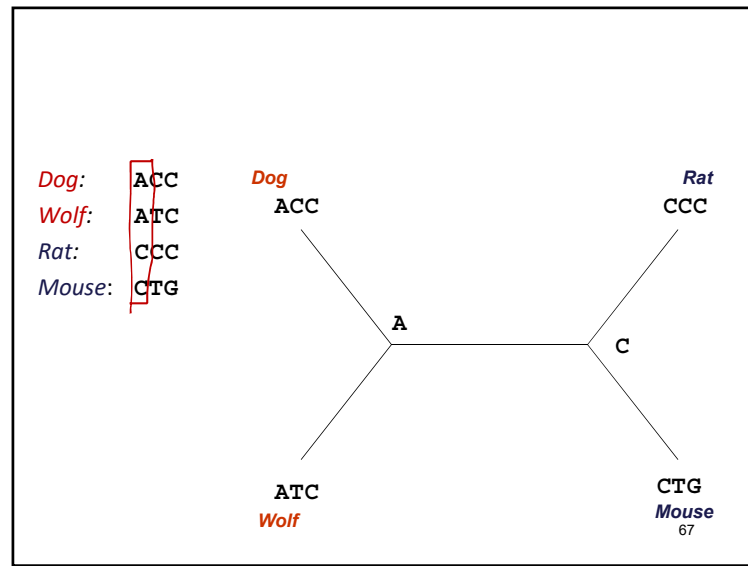
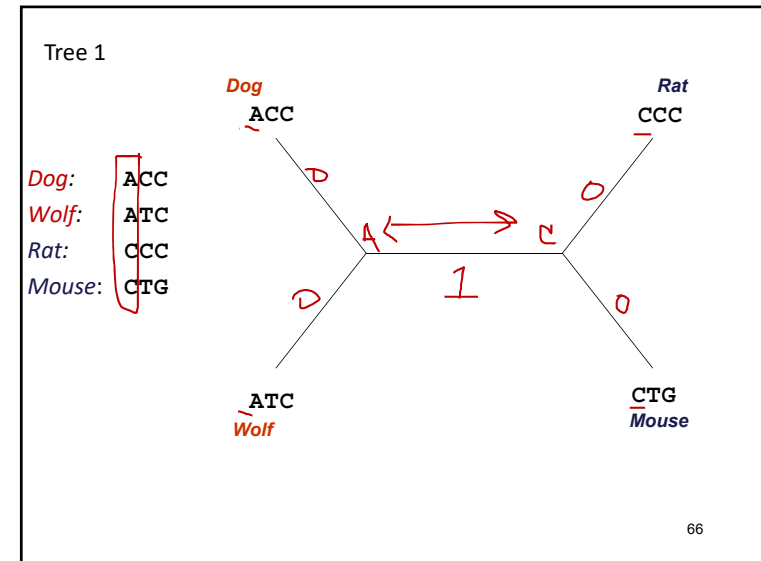
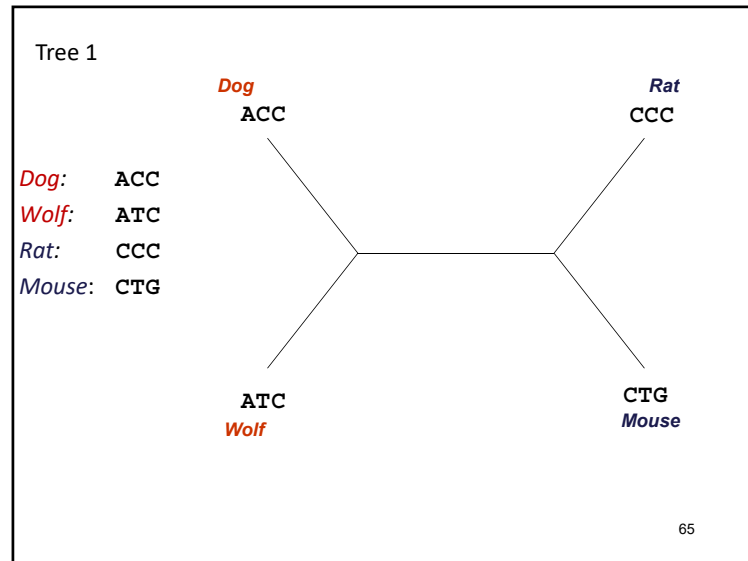
Given a leaf labeled tree

infer the internal states (ancestral sequences) that minimize the number of changes along the branches

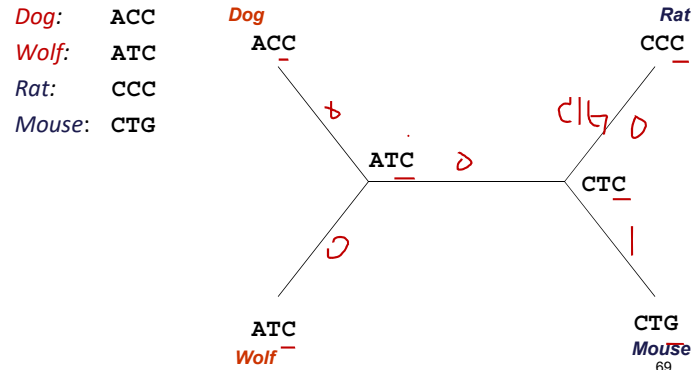
Parsimony score of this tree:
Sum of changes on each branch



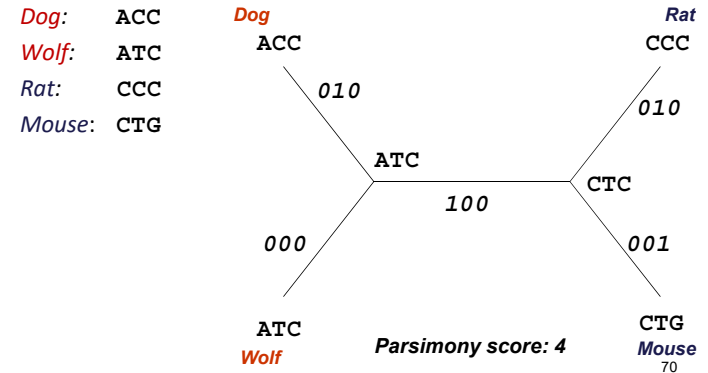
64



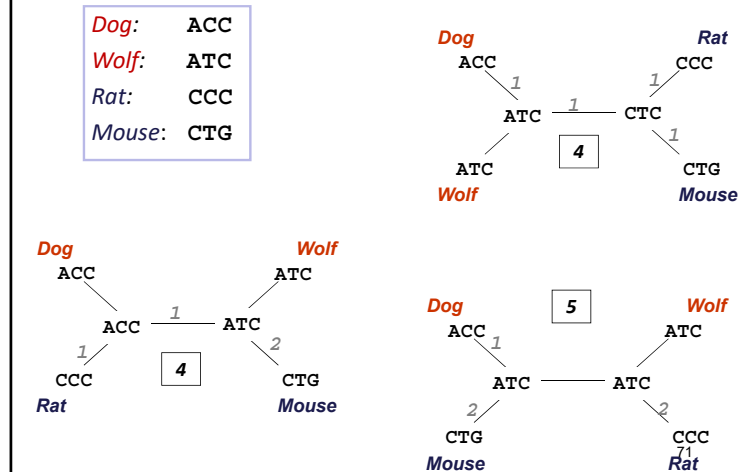
Inferring ancestral sequences and computing the parsimony score



Inferring ancestral sequences and computing the parsimony score

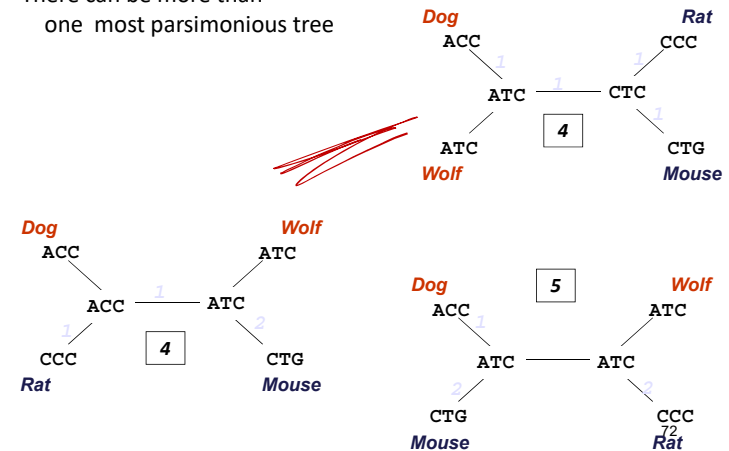


Parsimony scores of all trees with four leaves



Note:

There can be more than one most parsimonious tree



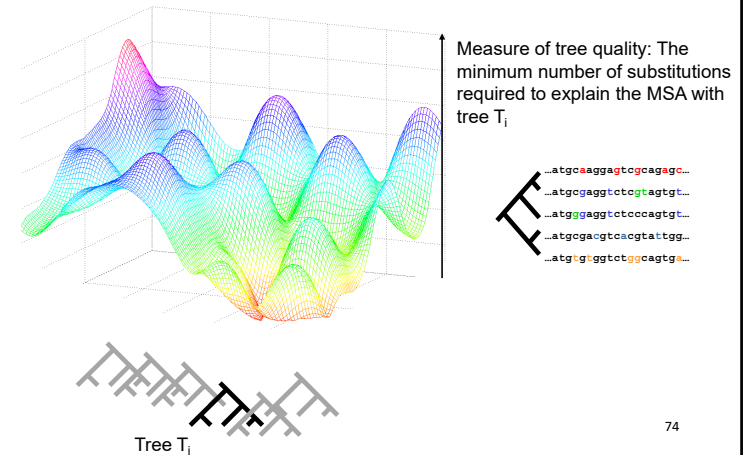
Phylogeny reconstruction

Potential problems with Maximum Parsimony:

- There can be more than one most parsimonious tree.
- Not all sites are informative
- The assumption that mutations are rare may be wrong.

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Tree reconstruction with parsimony

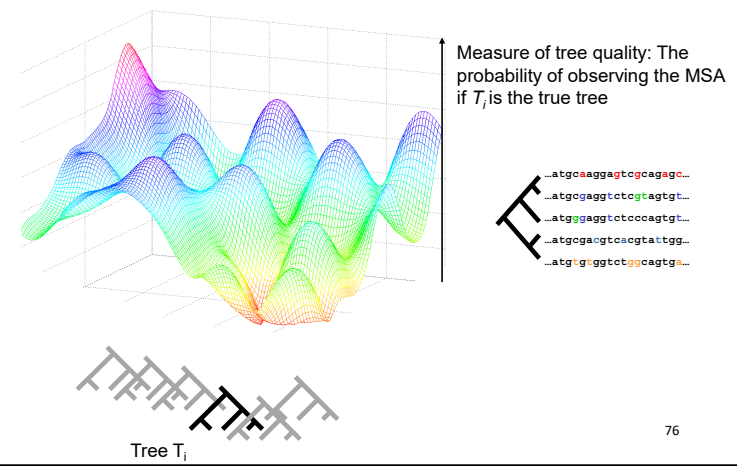


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MAXIMUM LIKELIHOOD

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Tree reconstruction with likelihood maximization



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Tree reconstruction

Given

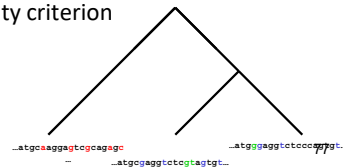
- contemporary taxa: k genes, strains, species...
- data describing each taxon

```
...atgcaaggagtcgcagagc...
...atgcgagggtctcgtagtgt...
...atgggagggtctccagtg...
```

find

the tree with k leaves that explains the data.

- Find the best tree with respect to an optimality criterion

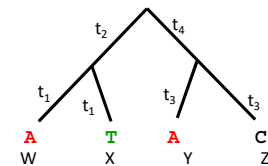


Estimating the probability of an MSA for a given tree, T_i

How to calculate the probability of the MSA given T_i :

- Calculate the probability of each column given T_i
- Multiply the probabilities over all columns.

Tree T_i : Topology and branch lengths



```
Sequence W: A A C G G A T T T G G G
Sequence X: A A C G G A T T T G G G
Sequence Y: A A C G G A T T T G G G
Sequence Z: A A C G G A T T T G G G
```

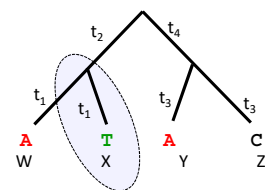
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Estimating the probability of one column for a given tree, T_i

Working up from the leaves

- Estimate contribution of each branch to the probability of the column, given the tree.
- Multiply the probabilities over all branches.

Tree T_i : Topology and branch lengths

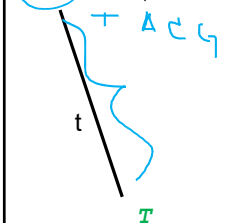


```
Sequence W: . . . A . . .
Sequence X: . . . T . . .
Sequence Y: . . . A . . .
Sequence Z: . . . C . . .
```

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Estimate contribution of one branch to the probability of the column

The ancestral base is unknown -> average over all possibilities.



$$P(\text{ancestral T}) * P(\text{T after time t has elapsed}) +$$

$$P(\text{ancestral A}) * P(\text{changed from A} \rightarrow \text{T in time t}) +$$

$$P(\text{ancestral C}) * P(\text{changed from C} \rightarrow \text{T in time t}) +$$

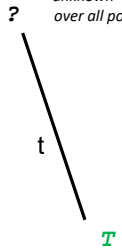
$$P(\text{ancestral G}) * P(\text{changed from G} \rightarrow \text{T in time t})$$

```
Sequence W: . . . A . . .
Sequence X: . . . T . . .
Sequence Y: . . . A . . .
Sequence Z: . . . C . . .
```

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Estimate contribution of one branch to the probability of the column

The ancestral base is unknown -> average over all possibilities.



Stated formally

$$P(A)P(t)_{AT} + P(T)P(t)_{TT} + P(C)P(t)_{CT} + P(G)P(t)_{GT}$$

$P(\text{ancestral } A) * P(\text{changed from } A \rightarrow T \text{ in time } t)$

Sequence W:

A
T
A
C

 ..
Sequence X:

A
T
A
C

 ..
Sequence Y:

A
T
A
C

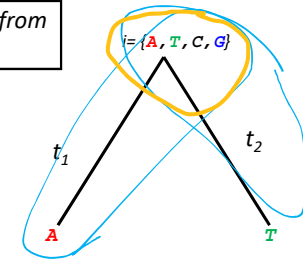
 ..
Sequence Z:

A
T
A
C

 ..

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Extending the likelihood calculation from one branch to two branches

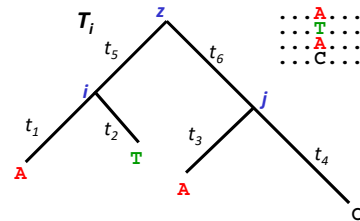


$$P(i=A)P(t_1)_{AA}P(t_2)_{AT} + P(i=T)P(t_1)_{TA}P(t_2)_{TT} \\ + P(i=C)P(t_1)_{CA}P(t_2)_{CT} + P(i=G)P(t_1)_{GA}P(t_2)_{GT}$$

This calculation accounts for the fact that the two branches are not independent: They share an ancestral nucleotide

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Extending the calculation obtain the probability of the column given the entire tree

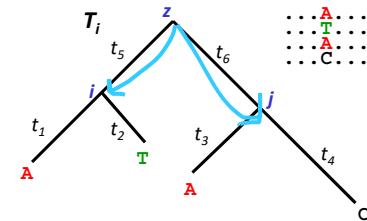


The probability of this column given T_i is

$$\sum_{i \in \{ACGT\}} \sum_{j \in \{ACGT\}} \sum_{z \in \{ACGT\}} \underbrace{p(z)p_{zi}(t_5)p_{zj}(t_6)}_{\text{root}} \cdot \underbrace{p(i)p_{iA}(t_1)p_{iT}(t_2)}_{\text{left subtree}} \cdot \underbrace{p(j)p_{jA}(t_3)p_{jC}(t_4)}_{\text{right subtree}}$$

84

Extending the calculation obtain the probability of the column given the entire tree



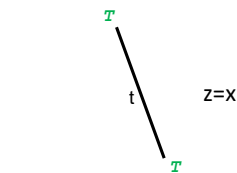
The probability of this column given T_i is

$$\sum_{i \in \{ACGT\}} \sum_{j \in \{ACGT\}} \sum_{z \in \{ACGT\}} \underbrace{p(z)p_{zi}(t_5)p_{zj}(t_6)}_{\text{root}} \cdot \underbrace{p(i)p_{iA}(t_1)p_{iT}(t_2)}_{\text{left subtree}} \cdot \underbrace{p(j)p_{jA}(t_3)p_{jC}(t_4)}_{\text{right subtree}}$$

85

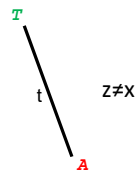
We discussed how to solve this problem with the Jukes Cantor model:

Given a site evolving according to Jukes Cantor with parameter α , what is the probability of observing z at time 0 and x at time t ?



$$p_{xx}(\alpha, t) = \frac{1}{4} + \frac{3}{4}e^{-4\alpha t}$$

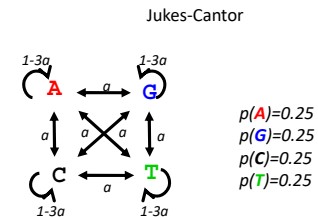
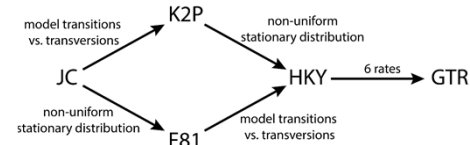
From last class



$$p_{xy}(\alpha, t) = \frac{1}{4} - \frac{1}{4}e^{-4\alpha t}$$

90

... but there are other models



To estimate the probability of an MSA given a tree requires a model of substitutions during sequence evolution

- Which evolutionary model?
 - Do all substitutions have the same rate?
- What rates?
- Do all sites have the same rate?
- Are there invariant sites that never change?

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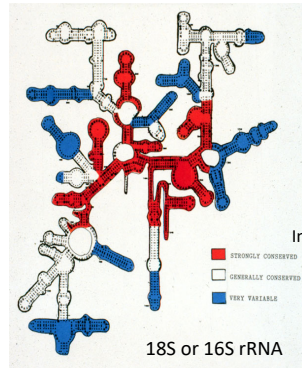
To estimate the probability of an MSA given a tree requires a model of substitutions during sequence evolution

- Which evolutionary model?
 - Do all substitutions have the same rate?
- What rates?
- Do all sites have the same rate?
- Are there invariant sites that never change?

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Site heterogeneity:

Different sites may be changing at different rates



Rates may be site specific, depending on functional and structural constraints (e.g., codon position, or alpha helix etc.)

In this example, white sites are changing faster than red sites and more slowly than blue sites.

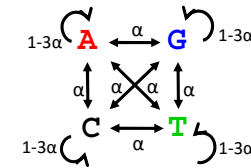
94

Modeling rate variation

It is not possible to assign a different rate to each position.

Instead, partition sites into a small number of rate categories, e.g., "slow, medium, fast, supersonic"

Use one model for the entire alignment, but different rates at different sites.

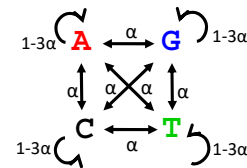


For example, all sites might be modeled with the Jukes Cantor model, but with different parameters: $\alpha_1 > \alpha_2 > \alpha_3 > \alpha_4$.

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Modeling rate variation

In addition, some sites may not be changing at all.



Invariant sites: Sites that do not change...

- Examples: very recent divergence, purifying selection, parallel substitutions
- Incorporate invariable sites in model as an extra category. For every site, calculate the likelihood that it is an invariable site.

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Discrete rates model

Given k rate classes,

- Determine the rate, r_k , for each class
- Determine the probability that site i is in class k .

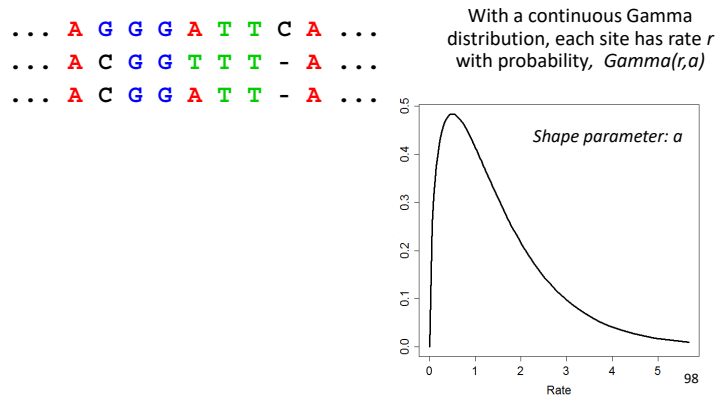
Site class	1	2	3	...	K
Probability	p_1			...	p_K
Rate	r_1			...	r_K

r_1 r_2 ... r_K
 ... A G G G A T T C A ...
 ... A C G G T T T - A ...
 ... A C G G A T T - A ...

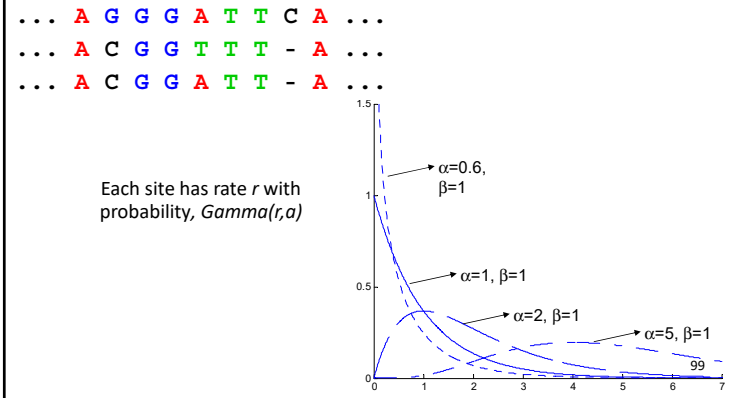
Problem: Too many parameters

97

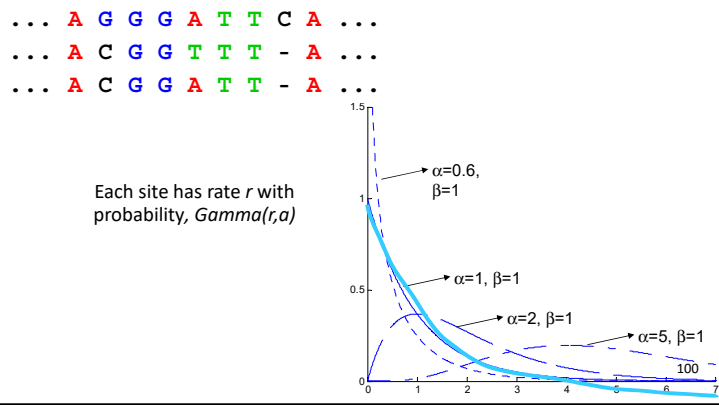
Solution: Use a Gamma distribution



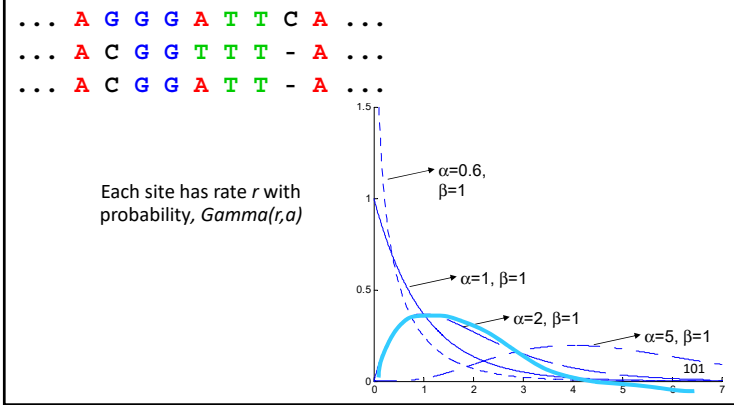
Continuous Gamma distribution: A single parameter (α) captures a broad range of distributions with different shapes.



Continuous Gamma distribution: A single parameter (α) captures a broad range of distributions with different shapes.



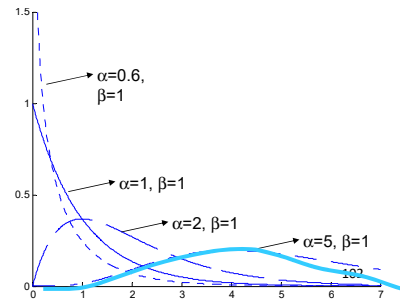
Continuous Gamma distribution: A single parameter (α) captures a broad range of distributions with different shapes.



Continuous Gamma distribution: A single parameter (α) captures a broad range of distributions with different shapes.

... A G G G A T T C A ...
 ... A C G G T T T - A ...
 ... A C G G A T T - A ...

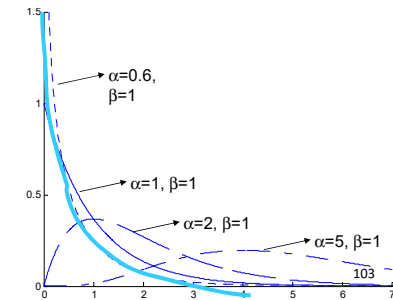
Each site has rate r with probability, $\text{Gamma}(r, a)$



Continuous Gamma distribution: A single parameter (α) captures a broad range of distributions with different shapes.

... A G G G A T T C A ...
 ... A C G G T T T - A ...
 ... A C G G A T T - A ...

Each site has rate r with probability, $\text{Gamma}(r, a)$



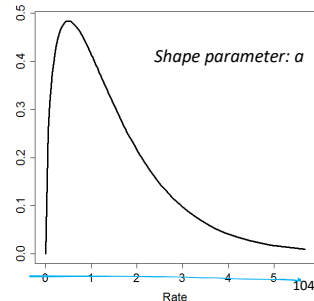
Continuous Gamma distribution

... A G G G A T T C A ...
 ... A C G G T T T - A ...
 ... A C G G A T T - A ...

Each site has rate r with probability, $\text{Gamma}(r, a)$

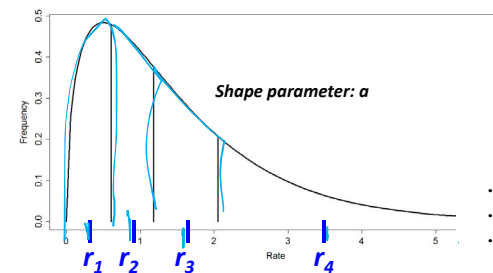
Problem:

Estimating the likelihood with a continuous function is too slow



Discrete Gamma distribution with k rate categories

- Divide area under the curve into k equal size regions
- Rates: midpoints of each region
- Only one parameter needed to describe all rate categories



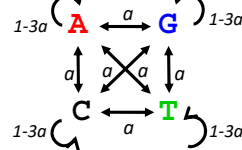
In most software, the default is $k = 4$

... A G G G A T T C ...
 ... A C G G T T T - ...
 ... A C G G A T T - ...

In summary, use models of sequence evolution to capture the following features:

Frequency of substitutions between bases/residues:

Base/residue frequencies:
 $p(A), p(C), p(G), p(T)$



Invariant sites: sites that do not change

Multiple rate categories:

- Same model, but with different values of the parameter(s)

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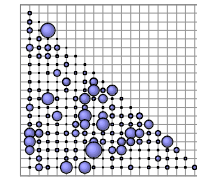
DNA substitution models

	C	G	T
A	•	•	•
C		•	•
G			•

DNA substitution model
(e.g., JC, K2P, GTr)

- Four states (A, C, G, T)
- Model specifies the relative rates of substitution for all possible pairs of nucleotides

Amino acid substitution models



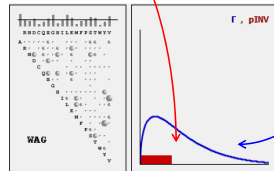
Amino acid substitution matrix
(e.g., PAM, WAG, JTT, MtREV etc)

- Twenty states (A, C, ... Y)
- Model specifies the relative rates of substitution for all possible pairs of amino acids

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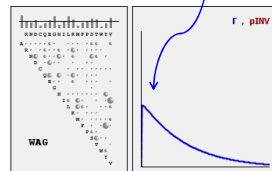
Examples

Fraction of invariant sites



WAG amino acid substitution model with a gamma model of rate heterogeneity and invariant sites

Discrete gamma distribution models four rate classes



WAG substitution model with a gamma model of rate heterogeneity only

Note that the shape parameter selected gives an increased peak at zero for the model without invariant sites (right). In this case, the lowest rate class is slower to compensate because the model does not account for invariant sites.

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DISTANCE-BASED TREE RECONSTRUCTION

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Phylogeny reconstruction outline

➤ Optimality criteria for scoring evolutionary trees

Maximum parsimony

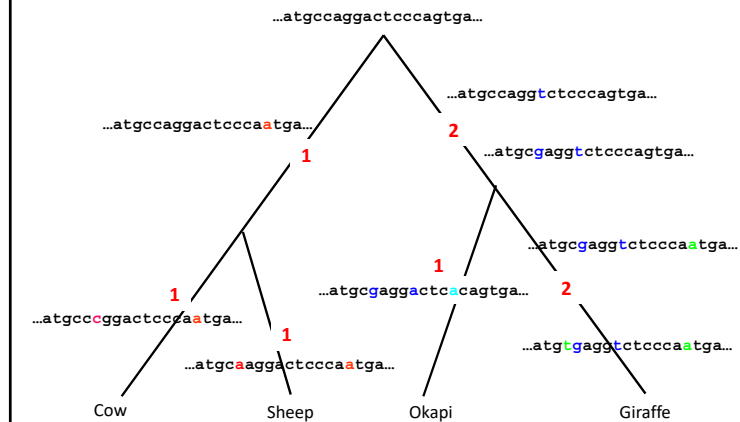
Maximum likelihood estimation

Distance-based evaluation

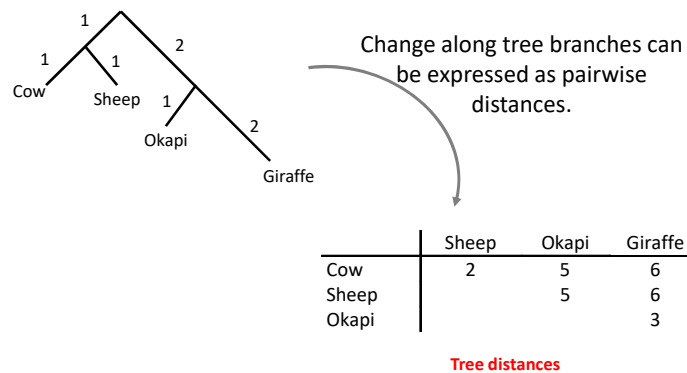
- Find the *optimal* tree; the tree with the best score
 - How many hypotheses (trees with k leaves) are there?
 - Sampling hypotheses: how to search through the space of trees with k leaves?

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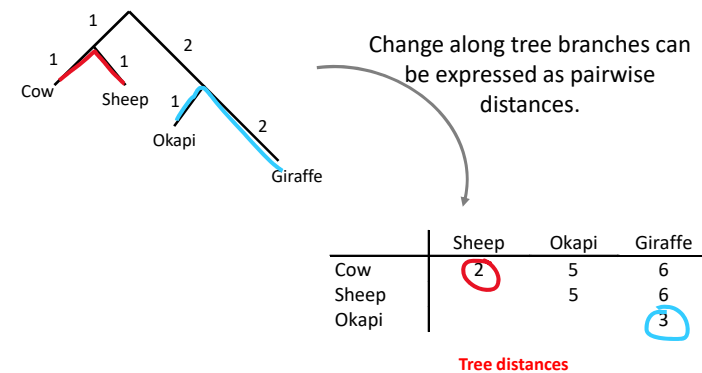
If we could observe evolution in progress
...we would know the number of changes on each branch...



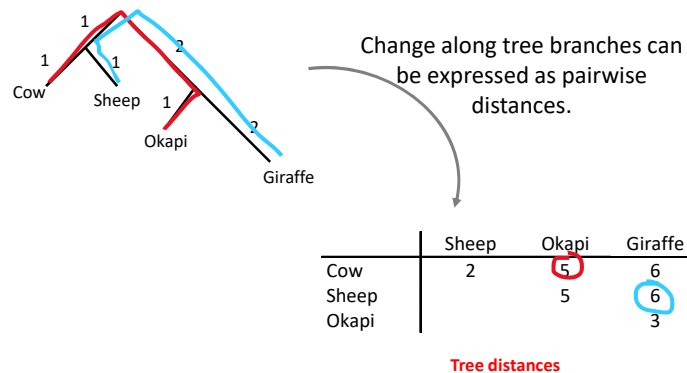
.. and the distance between each pair of taxa



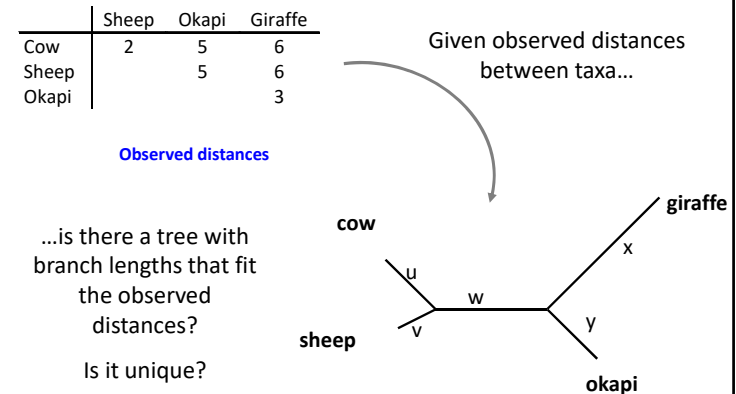
.. and the distance between each pair of taxa



.. and the distance between each pair of taxa



Can we reverse this process?



Outline

Distance-based phylogeny reconstruction

Convert multiple alignments into distances

Properties of pairwise distances between taxa

- Additive distances
- Ultrametric distances

Using pairwise distances to infer a tree: constructive methods

- UPGMA
- Neighbor Joining

If you've seen hierarchical clustering, this may seem similar

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Calculating distances from MSAs

C -	GCTTGTCCGTTACGAT			
S -	ACTTGACCGTTTCCTT			
O -	ACTTGTCCGAAACGAT			
G -	ACTTGTCTGTTACGAT			

	Sheep	Okapi	Giraffe
Cow	2	5	6
Sheep		5	6
Okapi			3

For each pair of taxa

- Use the pairwise alignment *induced by the MSA*.
- Count substitutions to obtain pairwise distances
- Correct for multiple substitutions

Correcting for multiple substitutions with Jukes-Cantor

Given an alignment of n nucleotides that differs at m positions, the expected number of substitutions since the divergence of the two sequences is given by

$$D = \frac{-3}{4} \ln \left(1 - \frac{4\mu}{3n} \right) n$$

$\dots \text{C A C A T A C G A A G A T A C G A A C G A G C} \dots$
 $\dots \text{C A G A T A G G A A G A G A C G A T C T A G C} \dots$
 n nucleotides with μ mismatches

From last lecture

For example, if we observe 200 mismatches in an alignment of 1000 nucleotides, then the number of actual substitutions is

$$\frac{-3}{4} \ln \left(1 - \frac{800}{3000} \frac{\mu}{n} \right) 3000 = 233$$

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Outline

Distance-based phylogeny reconstruction

Convert multiple alignments into distances

Properties of pairwise distances between taxa

- Additive distances
- Ultrametric distances

Using pairwise distances to infer a tree: constructive methods

- Neighbor Joining
- UPGMA

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Distance Takeaways

- Several algorithms based on various assumptions about the data.
- If the data satisfy the assumptions, then distance-based algorithms will return the correct tree.
- If the data do not deviate too far from the assumptions, these methods give reasonable approximations
- They run in polynomial time.