Clustering with Minimum Spanning Tree

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KT 4.6

Clustering: an application of MST

Clustering

You're given n items and the distance $d(u, v)$ between each of pair.

$d(u, v)$ may be an actual distance, or some abstract representation of how dissimilar two things are. (E.g. the "distance" between two species.)

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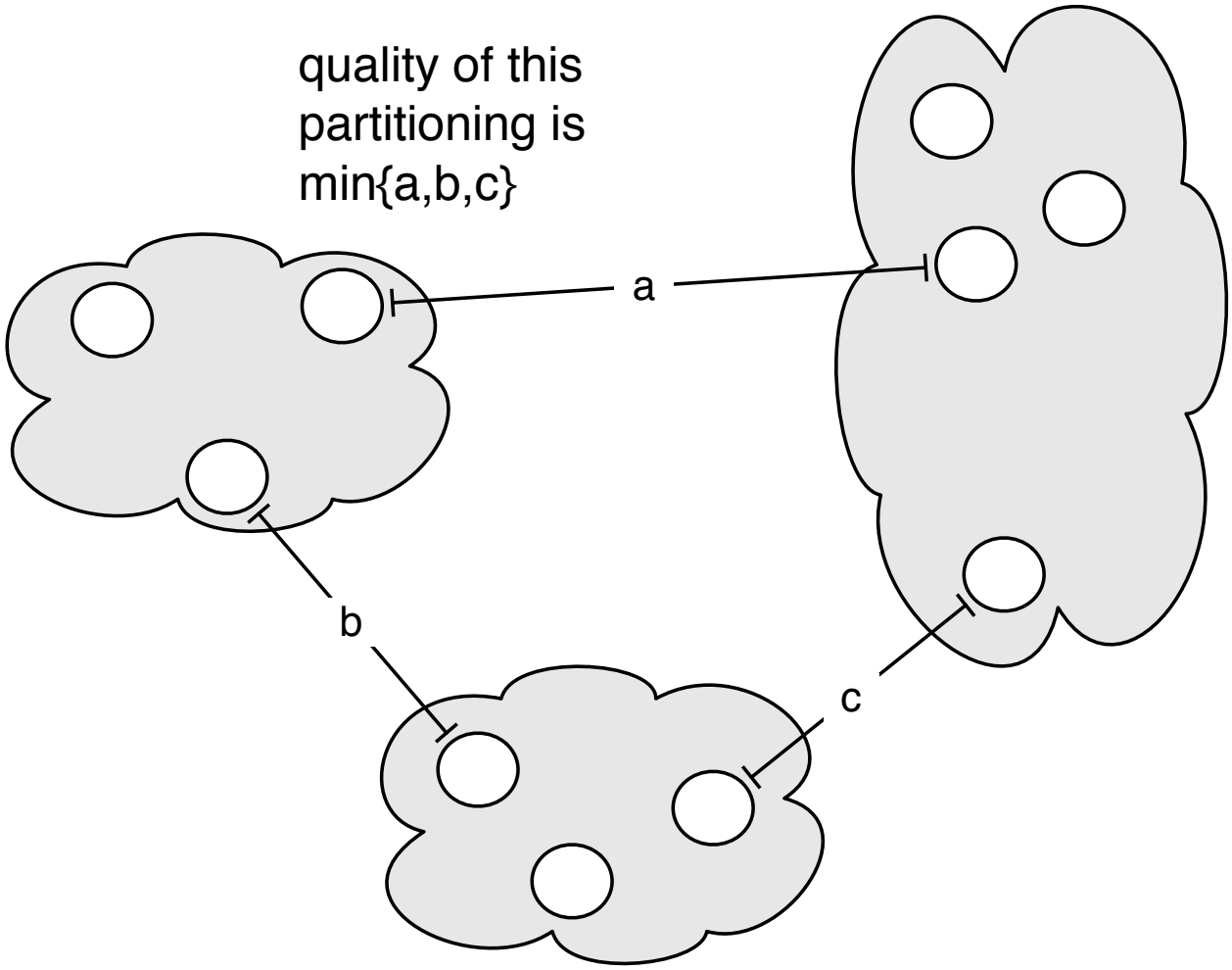

Maximum Minimum Distance

Idea:

- Maintain clusters as a set of connected components of a graph.
- Iteratively combine the clusters containing the two closest items by adding an edge between them.
- Stop when there are k clusters.

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This is exactly Kruskal's algorithm.

The "clusters" are the connected components that Kruskal's algorithm has created after a certain point.

Example of "single-linkage, agglomerative clustering."

Proof of Correctness

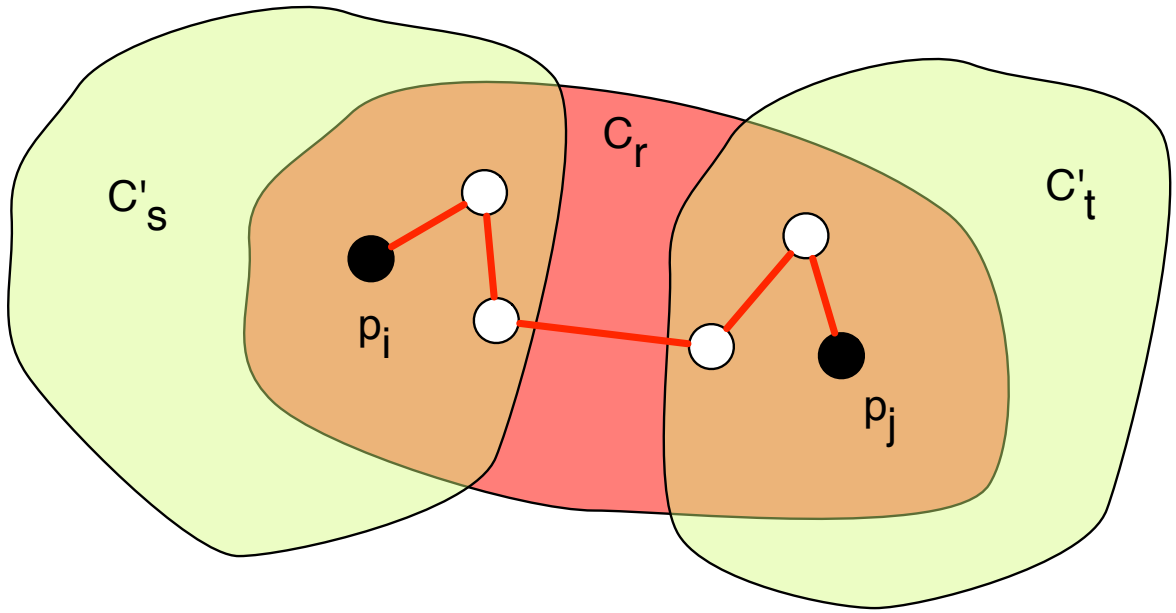
Another way too look at the algorithm: delete the $k-1$ most expensive edges from the MST.

The spacing d of the clustering C that this produces is the length of the $(k-1)^{\text{st}}$ most expensive edge.

Let C' be a different clustering. We'll show that C' must have the same or smaller separation than C.

Proof of correctness, 2

Since $C \neq C'$, there must be some pair p_i, p_j that are in the same cluster in C but different clusters in C'.



Together in $C \implies$ path P between p_i, p_j with all edges $\leq d$.

Some edge of P passes between two different clusters of C'.

Therefore, separation of $C' \leq d$.

Class So Far

6 lectures:

- Graphs, Trees
- Prim's Minimum Spanning Tree algorithm
- Heaps
- Heapsort
- 2-approximation for Euclidian traveling salesman problem
- Kruskal's MST algorithm
- Array-based union-find data structure
- Tree-based union-find data structure
- Minimum-Maximum-Distance clustering
- Python implementation of MST algorithms