

CMU SCS

Carnegie Mellon Univ.  
Dept. of Computer Science  
15-415/615 – DB Applications

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Lecture#5: *Relational calculus*

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General Overview - rel. model

- history
- concepts
- Formal query languages
  - relational algebra
  - **rel. tuple calculus**
  - rel. domain calculus

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Overview - detailed

- rel. tuple calculus
  - why?
  - details
  - examples
  - equivalence with rel. algebra
  - more examples; ‘safety’ of expressions
- rel. domain calculus + QBE

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## Motivation

- Q: weakness of rel. algebra?
- A: procedural
  - describes the steps (ie., 'how')
  - (still useful, for query optimization)

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## Solution: rel. calculus

- describes **what** we want
- two equivalent flavors: 'tuple' and 'domain' calculus
- basis for SQL and QBE, resp.
- Useful for proofs (see query optimization, later)

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## Rel. tuple calculus (RTC)

- first order logic

$$\{t \mid P(t)\}$$

'Give me tuples 't', satisfying predicate P - eg:

$$\{t \mid t \in STUDENT\}$$

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## Details

- symbols allowed:
 
$$\wedge, \vee, \neg, \Rightarrow$$

$$>, <, =, \neq, \leq, \geq,$$

$$(, ), \in$$
- quantifiers  $\forall, \exists$

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## Specifically

- Atom
 
$$t \in \text{TABLE}$$

$$t.attr \leq \text{const}$$

$$t.attr \leq s.attr'$$

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## Specifically

- Formula:
  - atom
  - if  $P1, P2$  are formulas, so are  $P1 \wedge P2; P1 \vee P2...$
  - if  $P(s)$  is a formula, so are  $\exists s(P(s))$   
 $\forall s(P(s))$

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## Specifically

- Reminders:
  - DeMorgan  $P1 \wedge P2 \equiv \neg(\neg P1 \vee \neg P2)$
  - implication:  $P1 \Rightarrow P2 \equiv \neg P1 \vee P2$
  - double negation:  $\forall s \in TABLE (P(s)) \equiv \neg \exists s \in TABLE (\neg P(s))$

**‘every human is mortal : no human is immortal’**

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## Reminder: our Mini-U db

STUDENT		
Ssn	Name	Address
123	smith	main str
234	jones	forbes ave

CLASS		
c-id	c-name	units
15-413	s.e.	2
15-412	o.s.	2

TAKES		
SSN	c-id	grade
123	15-413	A
234	15-413	B

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
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## Examples

- find all student records

$\{t \mid t \in STUDENT\}$   


output tuple of type ‘STUDENT’

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## Examples

- (selection) find student record with ssn=123

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## Examples

- (selection) find student record with ssn=123

$$\{t \mid t \in STUDENT \wedge t.ssn = 123\}$$

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## Examples

- (projection) find **name** of student with ssn=123

~~$$\{t \mid t \in STUDENT \wedge t.ssn = 123\}$$~~

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## Examples

- (projection) find name of student with  $ssn=123$

$$\{t \mid \exists s \in STUDENT(s.ssn = 123 \wedge t.name = s.name)\}$$

↗

‘t’ has only one column

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## ‘Tracing’

$$\{t \mid \exists s \in STUDENT(s.ssn = 123 \wedge t.name = s.name)\}$$

t

Name
aaaa
...
jones
...
zzzz

s

STUDENT		
Ssn	Name	Address
123	smith	main str
234	jones	forbes ave

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## Examples cont’d

- (union) get records of both PT and FT students

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## Examples cont'd

- (union) get records of both PT and FT students

$$\{t \mid t \in FT\_STUDENT \vee t \in PT\_STUDENT\}$$

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## Examples

- difference: find students that are not staff

(assuming that STUDENT and STAFF are union-compatible)

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## Examples

- difference: find students that are not staff

$$\{t \mid t \in STUDENT \wedge t \notin STAFF\}$$

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## Cartesian product

- eg., dog-breeding: MALE x FEMALE
- gives all possible couples

MALE		FEMALE	
name	x	name	
spike	⋈	lassie	=
spot		shiba	

M.name	F.name
spike	lassie
spike	shiba
spot	lassie
spot	shiba

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## Cartesian product

- find all the pairs of (male, female)

$$\{t \mid \exists m \in MALE \wedge \exists f \in FEMALE$$

$$t.m - name = m.name \wedge$$

$$t.f - name = f.name\}$$

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## 'Proof' of equivalence

- rel. algebra  $\leftrightarrow$  rel. tuple calculus

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## Overview - detailed

- rel. tuple calculus
  - why?
  - details
  - examples
  - equivalence with rel. algebra
  - **more examples**; ‘safety’ of expressions
- re. domain calculus + QBE

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## More examples

- join: find names of students taking 15-415

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## Reminder: our Mini-U db

STUDENT		
Ssn	Name	Address
123	smith	main str
234	jones	forbes ave

CLASS		
c-id	c-name	units
15-413	s.e.	2
15-412	o.s.	2

TAKES		
SSN	c-id	grade
123	15-413	A
234	15-413	B

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## More examples

- join: find names of students taking 15-415

$$\{t \mid \exists s \in STUDENT$$

$$\wedge \exists e \in TAKES (s.ssn = e.ssn \wedge$$

$$t.name = s.name \wedge$$

$$e.c - id = 15 - 415)\}$$

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## More examples

- join: find names of students taking 15-415

$$\{t \mid \exists s \in STUDENT$$

$$\wedge \exists e \in TAKES (s.ssn = e.ssn \wedge$$

$$t.name = s.name \wedge$$

$$e.c - id = 15 - 415)\}$$

join  
projection  
selection

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## More examples

- 3-way join: find names of students taking a 2-unit course

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## Reminder: our Mini-U db

STUDENT		
Ssn	Name	Address
123	smith	main str
234	jones	forbes ave

CLASS		
c-id	c-name	units
15-413	s.e.	2
15-412	o.s.	2

TAKES		
SSN	c-id	grade
123	15-413	A
234	15-413	B

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## More examples

- 3-way join: find names of students taking a 2-unit course

$$\{t \mid \exists s \in STUDENT \wedge \exists e \in TAKES$$

$$\exists c \in CLASS (s.ssn = e.ssn \wedge$$

$$e.c-id = c.c-id \wedge$$

$$t.name = s.name \wedge$$

$$c.units = 2)\}$$

join  
projection  
selection

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## More examples

- 3-way join: find names of students taking a 2-unit course - in rel. algebra??

$$\pi_{name}(\sigma_{units=2}(STUDENT \bowtie TAKES \bowtie CLASS))$$

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Even more examples:

- self -joins: find Tom's grandparent(s)

PC		
p-id	c-id	
Mary	Tom	
Peter	Mary	
John	Tom	

PC		
p-id	c-id	
Mary	Tom	
Peter	Mary	
John	Tom	

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Even more examples:

- self -joins: find Tom's grandparent(s)

$$\{t \mid \exists p \in PC \wedge \exists q \in PC$$
$$(p.c-id = q.p-id \wedge$$
$$p.p-id = t.p-id \wedge$$
$$q.c-id = "Tom")\}$$

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Hard examples: DIVISION

- find suppliers that shipped all the ABOMB parts

SHIPMENT	
s#	p#
s1	p1
s2	p1
s1	p2
s3	p1
s5	p3

÷

ABOMB	
p#	
p1	
p2	

=

BAD_S	
s#	
s1	

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## Hard examples: DIVISION

- find suppliers that shipped all the ABOMB parts

$$\{t \mid \forall p(p \in ABOMB \Rightarrow (\exists s \in SHIPMENT (t.s\# = s.s\# \wedge s.p\# = p.p\#)))\}$$

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## General pattern

- three equivalent versions:
  - 1) if it's bad, he shipped it  
 $\{t \mid \forall p(p \in ABOMB \Rightarrow (P(t)))\}$
  - 2) either it was good, or he shipped it  
 $\{t \mid \forall p(p \notin ABOMB \vee (P(t)))\}$
  - 3) there is no bad shipment that he missed  
 $\{t \mid \neg \exists p(p \in ABOMB \wedge (\neg P(t)))\}$

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## $a \Rightarrow b$ is the same as $\neg a \vee b$

		b
		T
a	T	T
	F	F
	T	T
	F	T

- If a is true, b must be true for the implication to be true. If a is true and b is false, the implication evaluates to false.
- If a is not true, we don't care about b, the expression is always true.

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## More on division

- find (SSNs of) students that take all the courses that ssn=123 does (and maybe even more)  
 find students 's' so that  
 if 123 takes a course  $\Rightarrow$  so does 's'

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## More on division

- find students that take all the courses that ssn=123 does (and maybe even more)  

$$\{o \mid \forall t((t \in TAKES \wedge t.ssn = 123) \Rightarrow \exists l \in TAKES( \\ t.l.c - id = t.c - id \wedge \\ t.l.ssn = o.ssn) \\ ))\}$$

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## Safety of expressions

- FORBIDDEN:  ~~$\{t \mid t \notin STUDENT\}$~~

It has infinite output!!

- Instead, always use  
 $\{t \mid \dots t \in SOME - TABLE\}$

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## Overview - conclusions

- rel. tuple calculus: DECLARATIVE
  - dfn
  - details
  - equivalence to rel. algebra
- **rel. domain calculus + QBE**

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## General Overview

- relational model
- Formal query languages
  - relational algebra
  - rel. tuple calculus
  - **rel. domain calculus**

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## Rel. domain calculus (RDC)

- Q: why?
- A: slightly easier than RTC, although equivalent - basis for QBE.
- idea: domain variables (w/ F.O.L.) - eg:
- ‘find STUDENT record with ssn=123’

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## Rel. Dom. Calculus

- find STUDENT record with ssn=123'

$$\{ \langle s, n, a \rangle \mid \langle s, n, a \rangle \in STUDENT \wedge s = 123 \}$$

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## Details

- Like R.T.C - symbols allowed:
  - $\wedge, \vee, \neg, \Rightarrow$
  - $>, <, =, \neq, \leq, \geq,$
  - $(, ), \in$
- quantifiers  $\forall, \exists$

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## Details

- but: domain (= column) variables, as opposed to tuple variables, eg:

$$\langle s, n, a \rangle \in STUDENT$$

ssn      name      address

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## Reminder: our Mini-U db

STUDENT		
Ssn	Name	Address
123	smith	main str
234	jones	forbes ave

CLASS		
c-id	c-name	units
15-413	s.e.	2
15-412	o.s.	2

TAKES		
SSN	c-id	grade
123	15-413	A
234	15-413	B

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## Examples

- find all student records

$$\{ \langle s, n, a \rangle \mid \langle s, n, a \rangle \in STUDENT \}$$

**RTC:**  $\{ t \mid t \in STUDENT \}$

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## Examples

- (selection) find student record with ssn=123

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## Examples

- (selection) find student record with  $ssn=123$

$$\{ \langle 123, n, a \rangle \mid \langle 123, n, a \rangle \in STUDENT \}$$

or

$$\{ \langle s, n, a \rangle \mid \langle s, n, a \rangle \in STUDENT \wedge s = 123 \}$$

**RTC:**  $\{ t \mid t \in STUDENT \wedge t.ssn = 123 \}$

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## Examples

- (projection) find name of student with  $ssn=123$

$$\{ \langle n \rangle \mid \langle 123, n, a \rangle \in STUDENT \}$$

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## Examples

- (projection) find name of student with  $ssn=123$

$$\{ \langle n \rangle \mid \exists a (\langle 123, n, a \rangle \in STUDENT) \}$$

↑  
need to 'restrict' "a"

**RTC:**  $\{ t \mid \exists s \in STUDENT (s.ssn = 123 \wedge t.name = s.name) \}$

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## Examples cont'd

- (union) get records of both PT and FT students

**RTC:**  $\{t \mid t \in FT\_STUDENT \vee t \in PT\_STUDENT\}$

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## Examples cont'd

- (union) get records of both PT and FT students

$$\{ \langle s, n, a \rangle \mid \langle s, n, a \rangle \in FT\_STUDENT \vee \langle s, n, a \rangle \in PT\_STUDENT \}$$

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## Examples

- difference: find students that are not staff

**RTC:**  $\{t \mid t \in STUDENT \wedge t \notin STAFF\}$

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## Examples

- difference: find students that are not staff

$$\{ \langle s, n, a \rangle \mid \langle s, n, a \rangle \in STUDENT \wedge \langle s, n, a \rangle \notin STAFF \}$$

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## Cartesian product

- eg., dog-breeding: MALE x FEMALE
- gives all possible couples

MALE
name
spike
spot

x

FEMALE
name
lassie
shiba

=

M.name	F.name
spike	lassie
spike	shiba
spot	lassie
spot	shiba

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## Cartesian product

- find all the pairs of (male, female) - RTC:

$$\{ t \mid \exists m \in MALE \wedge \exists f \in FEMALE \wedge t.m - name = m.name \wedge t.f - name = f.name \}$$

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## Cartesian product

- find all the pairs of (male, female) - RDC:

$$\{ \langle m, f \rangle \mid \langle m \rangle \in \text{MALE} \wedge \langle f \rangle \in \text{FEMALE} \}$$

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## 'Proof' of equivalence

- rel. algebra  $\leftrightarrow$  rel. domain calculus
- $\leftrightarrow$  rel. tuple calculus

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## Overview - detailed

- rel. domain calculus
  - why?
  - details
  - examples
  - equivalence with rel. algebra
  - **more examples**; 'safety' of expressions

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## More examples

- join: find names of students taking 15-415

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## Reminder: our Mini-U db

STUDENT		
Ssn	Name	Address
123	smith	main str
234	jones	forbes ave

CLASS		
c-id	c-name	units
15-413	s.e.	2
15-412	o.s.	2

TAKES		
SSN	c-id	grade
123	15-413	A
234	15-413	B

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## More examples

- join: find names of students taking 15-415 - in RTC

$$\{t \mid \exists s \in STUDENT$$

$$\wedge \exists e \in TAKES (s.ssn = e.ssn \wedge$$

$$t.name = s.name \wedge$$

$$e.c-id = 15-415)\}$$

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## More examples

- join: find names of students taking 15-415 - in RDC

$$\{ \langle n \rangle \mid \exists s \exists a \exists g (\langle s, n, a \rangle \in STUDENT \wedge \langle s, 15-415, g \rangle \in TAKES) \}$$

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## Sneak preview of QBE:

$$\{ \langle n \rangle \mid \exists s \exists a \exists g (\langle s, n, a \rangle \in STUDENT \wedge \langle s, 15-415, g \rangle \in TAKES) \}$$

STUDENT		
Ssn	Name	Address
_x	P.	

TAKES		
SSN	c-id	grade
_x	15-415	

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## Sneak preview of QBE:

- very user friendly
- heavily based on RDC
- very similar to MS Access interface

STUDENT		
Ssn	Name	Address
_x	P.	

TAKES		
SSN	c-id	grade
_x	15-415	

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## More examples

- 3-way join: find names of students taking a 2-unit course - in RTC:

$$\{t \mid \exists s \in STUDENT \wedge \exists e \in TAKES$$

$$\exists c \in CLASS( s.ssn = e.ssn \wedge$$

$$e.c-id = c.c-id \wedge$$

$$t.name = s.name \wedge$$

$$c.units = 2) \}$$

join

projection

selection

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## Reminder: our Mini-U db

x .P y 2

STUDENT		
Ssn	Name	Address
123	smith	main str
234	jones	forbes ave

CLASS		
c-id	c-name	units
15-413	s.e.	2
15-412	o.s.	2

TAKES		
SSN	c-id	grade
123	15-413	A
234	15-413	B

x y

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## More examples

- 3-way join: find names of students taking a 2-unit course

$$\{ \langle n \rangle \mid \dots \dots \dots$$

$$\langle s, n, a \rangle \in STUDENT \wedge$$

$$\langle s, c, g \rangle \in TAKES \wedge$$

$$\langle c, cn, 2 \rangle \in CLASS \}$$

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## More examples

- 3-way join: find names of students taking a 2-unit course

$$\{ \langle n \rangle \mid \exists s, a, c, g, cn ($$

$$\quad \langle s, n, a \rangle \in STUDENT \wedge$$

$$\quad \langle s, c, g \rangle \in TAKES \wedge$$

$$\quad \langle c, cn, 2 \rangle \in CLASS$$

$$\left. \right\}$$

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## Even more examples:

- self -joins: find Tom's grandparent(s)

PC	
p-id	c-id
Mary	Tom
Peter	Mary
John	Tom

PC	
p-id	c-id
Mary	Tom
Peter	Mary
John	Tom

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## Even more examples:

- self -joins: find Tom's grandparent(s)

$$\{ t \mid \exists p \in PC \wedge \exists q \in PC$$

$$\quad ( p.c-id = q.p-id \wedge$$

$$\quad p.p-id = t.p-id \wedge$$

$$\quad q.c-id = "Tom" ) \}$$

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## Even more examples:

- self -joins: find Tom's grandparent(s)

$$\{t \mid \exists p \in PC \wedge \exists q \in PC$$

$$(p.c-id = q.p-id \wedge$$

$$p.p-id = t.p-id \wedge$$

$$q.c-id = "Tom")\}$$

$$\{<g> \mid \exists p (<g, p> \in PC \wedge$$

$$<p, "Tom"> \in PC)\}$$

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## Even more examples:

- self -joins: find Tom's grandparent(s)

$$\{<g> \mid \exists p (<g, p> \in PC \wedge$$

$$<p, "Tom"> \in PC)\}$$

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## Hard examples: DIVISION

- find suppliers that shipped all the ABOMB parts

SHIPMENT	
s#	p#
s1	p1
s2	p1
s1	p2
s3	p1
s5	p3

÷

ABOMB	
p#	
p1	
p2	

=

BAD_S	
s#	
s1	

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## Hard examples: DIVISION

- find suppliers that shipped all the ABOMB parts

$$\{t \mid \forall p(p \in ABOMB \Rightarrow (\exists s \in SHIPMENT (t.s\# = s.s\# \wedge s.p\# = p.p\#)))\}$$

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## Hard examples: DIVISION

- find suppliers that shipped all the ABOMB parts

$$\{t \mid \forall p(p \in ABOMB \Rightarrow (\exists s \in SHIPMENT (t.s\# = s.s\# \wedge s.p\# = p.p\#)))\} \quad \{<s> \mid \forall p(<p> \in ABOMB \Rightarrow <s, p> \in SHIPMENT)\}$$

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## More on division

- find students that take all the courses that ss<sub>n</sub>=123 does (and maybe even more)

$$\{o \mid \forall t((t \in TAKES \wedge t.ssn = 123) \Rightarrow \exists t1 \in TAKES (t1.c-id = t.c-id \wedge t1.ssn = o.ssn))\}$$

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## More on division

- find students that take all the courses that ssn=123 does (and maybe even more)

$$\{ \langle s \rangle \mid \forall c (\exists g (\langle 123, c, g \rangle \in TAKES) \Rightarrow \exists g' (\langle s, c, g' \rangle \in TAKES)) \}$$

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## Safety of expressions

- similar to RTC
- FORBIDDEN:

$$\{ \langle s, n, a \rangle \mid \langle s, n, a \rangle \notin STUDENT \}$$

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## Overview - detailed

- rel. domain calculus + QBE
  - dfn
  - details
  - equivalence to rel. algebra

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## Fun Drill: Your turn ...

- Schema:
  - Movie(title, year, studioName)
  - ActsIn(movieTitle, starName)
  - Star(name, gender, birthdate, salary)

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## Your turn ...

- Queries to write in TRC:
  - Find all movies by Paramount studio
  - ... movies starring Kevin Bacon
  - Find stars who have been in a film w/Kevin Bacon
  - Stars within six degrees of Kevin Bacon\*
  - Stars connected to K. Bacon via any number of films\*\*

\* Try *two* degrees for starters      \*\* Good luck with this one!

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## Answers ...

- Find all movies by Paramount studio

$\{M \mid M \in \text{Movie} \wedge M.\text{studioName} = \text{'Paramount'}\}$

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## Answers ...

- Movies starring Kevin Bacon

$$\{M \mid M \in \text{Movie} \wedge \exists A \in \text{ActsIn} (A.\text{movieTitle} = M.\text{title} \wedge A.\text{starName} = \text{'Bacon'})\}$$

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## Answers ...

- Stars who have been in a film w/Kevin Bacon

$$\{S \mid S \in \text{Star} \wedge \exists A \in \text{ActsIn} (A.\text{starName} = S.\text{name} \wedge \exists A2 \in \text{ActsIn} (A2.\text{movieTitle} = A.\text{movieTitle} \wedge A2.\text{starName} = \text{'Bacon'}))\}$$

S: 


name	...
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A: 

movie	star
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A2: 

movie	star
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 'Bacon'

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## Answers ...

- Stars within <sup>two</sup> six degrees of Kevin Bacon

$$\{S \mid S \in \text{Star} \wedge \exists A \in \text{ActsIn} (A.\text{starName} = S.\text{name} \wedge \exists A2 \in \text{ActsIn} (A2.\text{movieTitle} = A.\text{movieTitle} \wedge \exists A3 \in \text{ActsIn} (A3.\text{starName} = A2.\text{starName} \wedge \exists A4 \in \text{ActsIn} (A4.\text{movieTitle} = A3.\text{movieTitle} \wedge A4.\text{starName} = \text{'Bacon'})))\}$$

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## Two degrees:

S: name ...

A3: movie star

A4: movie star

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## Two degrees:

S: name ...

A: movie star

A2: movie star

A3: movie star

A4: movie star

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## Answers ...

- Stars connected to K. Bacon via any number of films
- Sorry ... that was a **trick question**
  - Not expressible in relational calculus!!
- What about in relational algebra?
  - No – RA, RTC, RDC are equivalent

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## Expressive Power

- Expressive Power (Theorem due to Codd):
  - Every query that can be expressed in relational algebra can be expressed as a safe query in DRC / TRC; the converse is also true.
- *Relational Completeness:*  
 Query language (e.g., SQL) can express every query that is expressible in relational algebra/calculus.  
 (actually, SQL is more powerful, as we will see...)

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## Summary

- The relational model has rigorously defined query languages — simple and powerful.
- Relational algebra is more operational/procedural
  - useful as internal representation for query evaluation plans
- Relational calculus is **declarative**
  - users define queries in terms of what they want, not in terms of how to compute it.

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## Summary - cnt'd

- Several ways of expressing a given query
  - a *query optimizer* should choose the most efficient version.
- Algebra and safe calculus have same *expressive power*
  - leads to the notion of *relational completeness*.

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