Carnegie Mellon University
Department of Computer Science
15-826 Multimedia Databases and Data Mining
C. Faloutsos, Fall 2019

Due: **hard copy, in class, at 1:30pm, on 11/13/2019**
No tarball.

VERY IMPORTANT:

- For each question, we expect only the **hard copy** of answers and code.
  1. **Separate** your answers, on different page(s) for each question
  2. **Type** the full info on **each** page: your **name**, **Andrew ID**, **course#**, **Homework#**, **Question#** on each of the pages.

Reminders:

- *Plagiarism*: Homework is to be completed *individually*.
- *Typeset* your answers. Illegible handwriting may get zero points.
- *Late homeworks*: follow usual procedure: please email it
  - to all TAs and graders
  - with the subject line exactly `15-826 Homework Submission (HW 4)`
  - and the count of slip-days you are using.

For your information:

- Graded out of **100** points; **3** questions total
- Rough time estimate: *12-18 hours ($\approx$ 4-6 hours per question)*

Revision : 2019/12/03 23:00

Question	Points	Score
Eigen values power method	35	
SVD - Visualization	30	
Fourier and wavelets	35	
Total:	100	

Code packaging info:
As before, for your convenience, we provide a *tar-file package*, at `http://www.cs.cmu.edu/~christos/courses/826.F19/HOMEWORKS/HW4/hw4.tar.gz`. We will refer to it as the *tar-file package* from now on. It has 3 directories `/Q1, /Q2, /Q3`.

Question 1: Eigen values power method . . . . [35 points]

On separate page, with '[course-id] [hw#] [question#] [andrew-id] [your-name]'

Motivation: As we have seen in class SVD has many practical applications. A common method of computing SVD for a given matrix A is by computing the eigen values of $A^T A$ and AA^T.

If we want only the first (= dominant) eigenvalue and eigenvector, then we can use the *power iteration* method: we can multiply a random vector $\vec{r}$ with the matrix, multiple, consecutive times, and the resulting vector will be very close to the corresponding eigenvector $\vec{u}_1$ (times a large scalar).

Problem Description: Implement the power iteration method to compute the dominant eigen value and vector for a given matrix A.

(a) **[25 points]** Give the code for the `power_iteration` in `./Q1/power_iteration.py`

Solution:

(b) **[10 points]** Compute the dominant eigen value λ_1 and the corresponding eigenvector $\vec{u}_1$ for the following matrices.

$$A = \begin{bmatrix} 3 & 6 & -8 \\ 0 & 0 & 6 \\ 0 & 0 & 2 \end{bmatrix} \quad B = \begin{bmatrix} 4 & 0 & 0 & 0 \\ 0 & 3 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Solution: A: 3, (1,0,0), B: 4, (1, 0, 0, 0)

What to turn in:

- **Answers:** Hard copy of the code `./Q1/power_iteration.py` and the answers for part (b).

Question 2: SVD - Visualization .... [30 points]

On separate page, with '[course-id] [hw#] [question#] [andrew-id] [your-name]'

Motivation: Very often in our career as a data analyst, we are given a cloud of N points in M dimensions, and we have to find patterns, clusters, anomalies. If the dimensionality M is high, it is hard to plot and visualize the dataset. Here we see how to reduce the dimensionality, and how to find patterns and anomalies.

Problem Description: In this problem, we will use the Singular Value Decomposition (SVD) to explore such a cloud of points.

Consider the 6-dimensional *mystery* dataset `./Q2/mystery.dat` in *tar-file package*. The N data points lie in a lower dimensionality hyper-plane of dimensionality k - you have to guess k and project the points into a k-dimensional (hyper-)plane, using SVD. Specifically, we are told that the i-th mystery data point $\mathbf{x}_i = (x_{i,1}, \cdots, x_{i,6})$ was generated by the equations:

$$
\begin{aligned}
&x_{i,1} = a_1 * y_{i,1} + a_2 * y_{i,2} + \cdots + a_k * y_{i,k} + \epsilon_{i,1},\\
&x_{i,2} = b_1 * y_{i,1} + b_2 * y_{i,2} + \cdots + b_k * y_{i,k} + \epsilon_{i,2},\\
&\ldots\\
&x_{i,6} = f_1 * y_{i,1} + f_2 * y_{i,2} + \cdots + f_k * y_{i,k} + \epsilon_{i,6},
\end{aligned}
$$

where $\mathbf{y}_i = (y_{i,1}, ..., y_{i,k})$ is the i-th point on the k-dimensional hyper-plane ($k \leq 6$). The coefficients $a_1, \cdots, a_k, b_1, \cdots, f_k$ are constant for all the N points in the dataset, and $\epsilon_{i,j}$ indicates a small amount of noise.

Answer the following questions using SVD. We recommend MatLab.

(a) [**6 points**] Guess: What is the dimensionality k of the mystery dataset? (Do NOT use the fractal dimension - it is not the right tool to guess k.)

Solution: $k = 2$.

(b) [**2 points**] Give the singular values ($\lambda_1, \ldots$) of the matrix $X = (x_{i,j})$

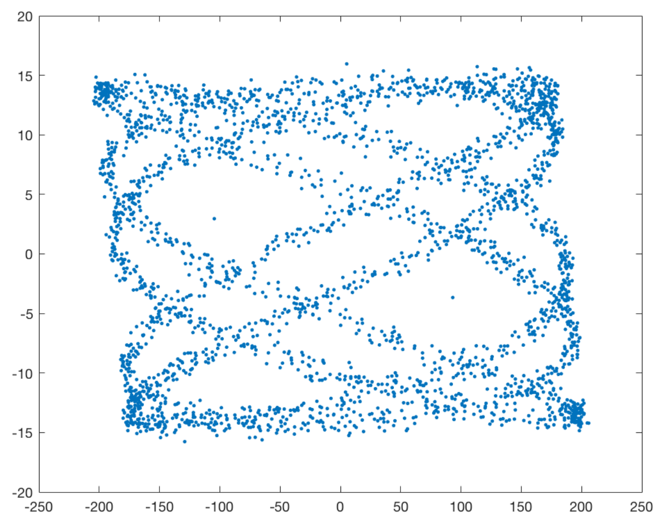
Solution: $\lambda_1 = 7294.96, \lambda_2 = 556.37, \lambda_3 = 48.25, \lambda_4 = 31.72, \lambda_5 = 31.37, \lambda_6 = 31.20$

(c) [**2 points**] Briefly justify your answer for your guess for k.

Solution: It is the effective rank of the $Nx6$ matrix $X = (x_{i,j})$ $(i = 1, \ldots N, j = 1, \ldots, 6.)$
Additionally, the first two terms dominate most of the energy causing us to guess degree 2.

(d) **[10 points]** If $k \leq 2$, give the scatter-plot (of first, vs second, principal components). If $k > 2$, give all the pair-plots, that is, the scatter-plots of all the k-choose-2 possibilities.

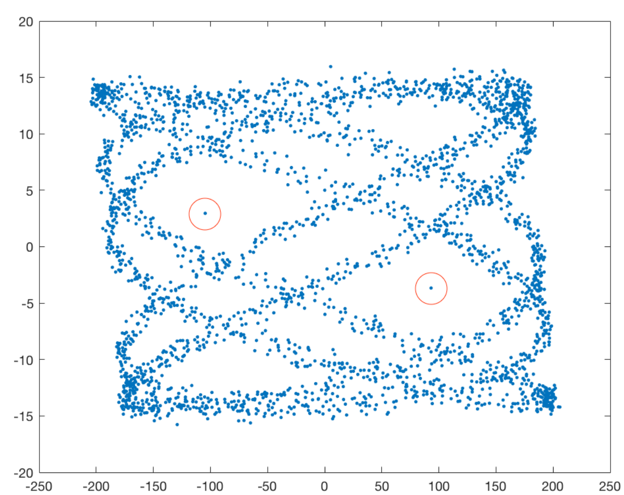
Solution:



(e) There are two outliers in the dataset. Find those two outliers by manually looking at your scatter-plot(s).

i. **[4 points]** Mark them and hand in the resulting plots

Solution:



ii. **[6 points]** report the (6-dimensional) coordinates of the two outliers.

Solution: $(6.10, 9.84, -10.6514.18, 16.25, 101.16)$,
$(-4.50, -7.89, 9.97, -12.42, -14.75, -90.26)$

What to turn in:

- **Answers:** Submit hard copy for the answers.

Question 3: Fourier and wavelets .... [35 points]

On separate page, with '[course-id] [hw#] [question#] [andrew-id] [your-name]'

Motivation: Digital Signal Processing (DSP) and specifically the Discrete Fourier (DFT) and Discrete Wavelet (DWT) transforms, are powerful tools for de-noising, anomaly detection and feature extraction in time sequences. Here we demostrate

- how they help us spot outliers, by extracting valuable features (frequencies, amplitudes), from periodic time sequences like natural sounds (flying insects), and
- how they can help us discover signals buried inside noise, like a phone conversation in a noisy street.

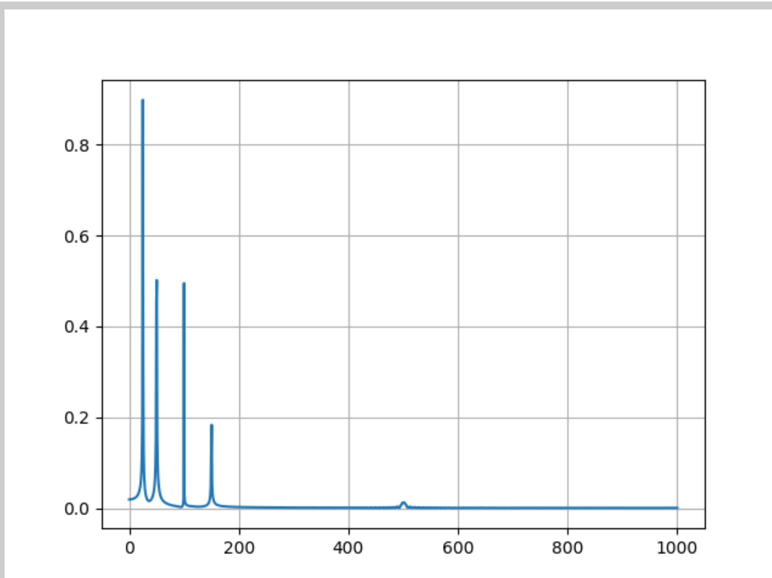
Problem Description: You will analyze the signal `./Q3/signal_with_noise.mat` using DFT (also called FFT) and wavelets to detect a high frequency injection which occurs for a short duration. The signal is a 1-d time series of 2500 samples that were sampled at a frequency of 4000 Hz, and it is a mixture of sine/cosine functions, plus the short-lived injection.

(a) DFT/FFT analysis:

i. **[10 points]** Plot the spectrum, i.e., the frequency and amplitude plot using `./Q3/wave_analysis.py`.

ii. **[5 points]** Report the main frequencies and their amplitudes you see in the plot. Can you spot the frequency of the injection? (It's OK if not).

Solution:

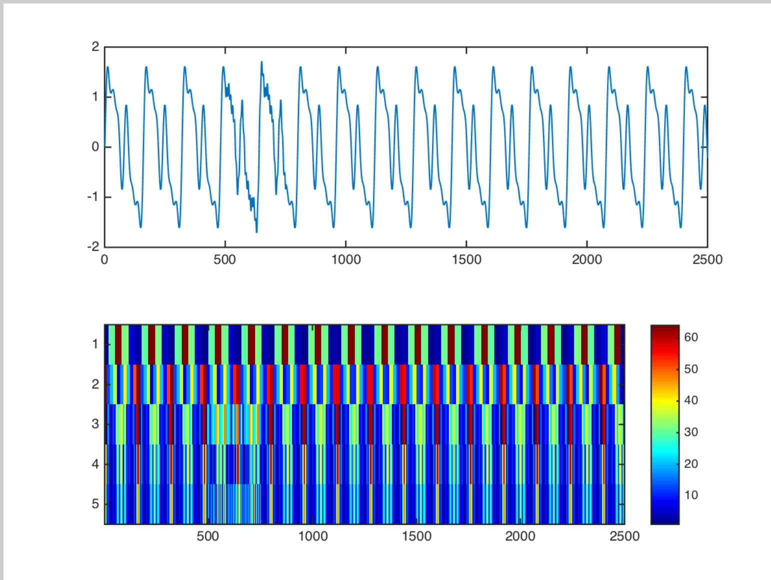


(b) Wavelet analysis: The DFT/FFT spectrum can not indicate the start-end of the high-frequency injection. It can only provide information about the overall frequencies the signal wavelet transforms also localize the frequencies in the signal in time.

i. **[10 points]** Plot the Wavelet scaleogram using `wavelet_scaleogram.m`.

ii. **[10 points]** Report the approximate start-time, end-time, and approximate frequency of the injection.

Solution:



What to turn in:

- **<u>Answers:</u>** Submit hard copy for the answers and the plots.