# Finding patterns in large, real networks

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## Thanks to

- Deepayan Chakrabarti (CMU)

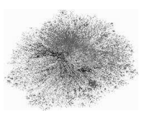
- Michalis Faloutsos (UCR)

- George Siganos (UCR)

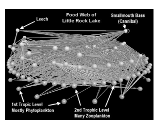




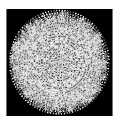
## Introduction



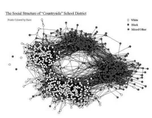
Internet Map
[lumeta.com]



Food Web
[Martinez '91]



Protein Interactions
[genomebiology.com]



Friendship Network
[Moody '01]

Graphs are everywhere!





## Physical graphs

- Physical networks
- Physical Internet
- Telephone lines
- Commodity distribution networks





## Networks derived from "behavior"

- Telephone call patterns
- Email, Blogs, Web, Databases, XML
- Language processing
- Web of trust, epinions.com





## Outline

Topology, 'laws' and generators
- 'Laws' and patterns
- Generators
- Tools

## Motivating questions

- What do real graphs look like?
  - What properties of nodes, edges are important to model?
  - What local and global properties are important to measure?
- How to generate realistic graphs?





## Why should we care?

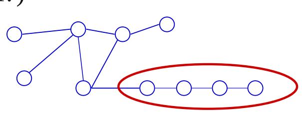
- **A1: extrapolations**: how will the Internet/Web look like next year?
- **A2: algorithm design**: what is a realistic network topology,
  - to try a new routing protocol?
  - to study virus/rumor propagation, and immunization?





## Why should we care? (cont'd)

- **A3: Sampling**: How to get a 'good' sample of a network?
- **A4: Abnormalities**: is this sub-graph / sub-community / sub-network 'normal'? (what **is** normal?)







## Virus propagation

- **Who is the best person/computer to immunize against a virus?**





## Outline

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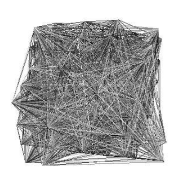
## Topology

How does the Internet look like? Any rules?



(Looks random – right?)

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## Laws and patterns

Real graphs are NOT random!!

- Diameter
- in- and out- degree distributions
- other (surprising) patterns



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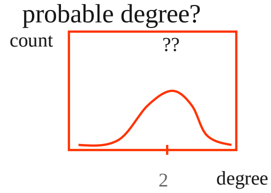
## Laws – degree distributions

- Q: avg degree is ~2 - what is the most probable degree?



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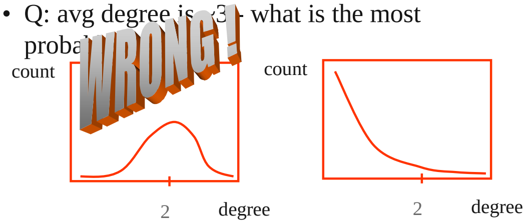
## Laws – degree distributions

- Q: avg degree is ~3 - what is the most probable degree?



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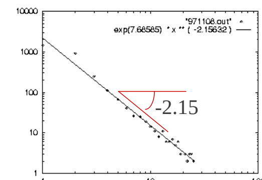
## I.Power-law: outdegree *O*

Frequency




Outdegree

Exponent = slope

$O = -2.15$

Nov'97

The plot is linear in log-log scale [FFF'99]

$$freq = degree^{(-2.15)}$$



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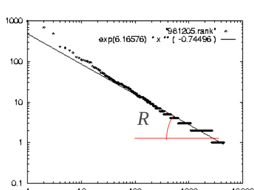
## II.Power-law: rank *R*

outdegree




Rank: nodes in decreasing outdegree order

Exponent = slope

$R = -0.74$

Dec'98

- The plot is a line in log-log scale



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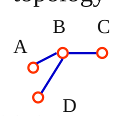
## III. Eigenvalues

- Let $A$ be the adjacency matrix of graph
- $\lambda$ and $v$ is an eigenvalue/eigenvector pair if:

$$A\, v = \lambda\, v$$

- Eigenvalues are strongly related to graph topology




| | A | B | C | D |
|---|---|---|---|---|
| A | | 1 | | |
| B | 1 | | 1 | 1 |
| C | | 1 | | |
| D | | 1 | | |

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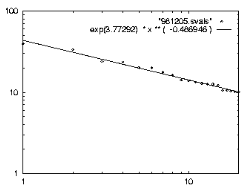
## III.Power-law: eigen *E*

Eigenvalue



Exponent = slope

*E = -0.48*

Dec'98

Rank of decreasing eigenvalue

- Eigenvalues in decreasing order (first 20)
- [Mihail+, 02]: $R = 2 * E$

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## IV. The Node Neighborhood

- $N(h)$ = # of pairs of nodes within $h$ hops

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## IV. The Node Neighborhood

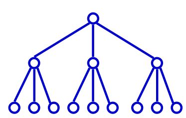
- Q: average degree = 3 - how many neighbors should I expect within 1,2,… *h* hops?
- Potential answer:

  1 hop -> 3 neighbors

  2 hops -> 3 * 3

  …

  *h* hops -> $3^h$



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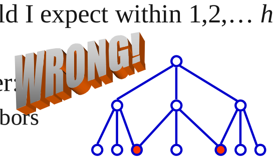


**WE HAVE DUPLICATES!**

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## IV. The Node Neighborhood

- Q: average degree = 3 - how many neighbors should I expect within 1,2,… *h* hops?
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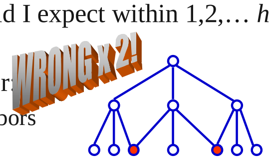


**'avg' degree: meaningless!**

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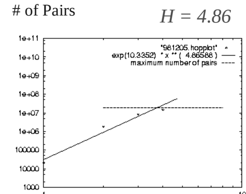
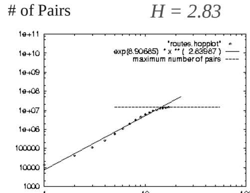
## IV. Power-law: hopplot *H*

# of Pairs *H = 4.86*



Hops Dec 98

# of Pairs *H = 2.83*



Hops Router level '95

Pairs of nodes as a function of hops $N(h)= h^H$

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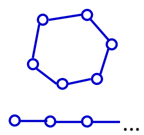
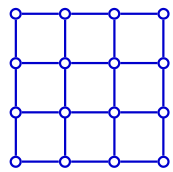
## Observation

- Q: Intuition behind ‘hop exponent’?
- A: ‘intrinsic=fractal dimensionality’ of the network



N($h$) ~ $h^1$



N($h$) ~ $h^2$

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## Hop plots

- More on fractal/intrinsic dimensionalities: very soon

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## But:

- Q1: How about graphs from other domains?
- Q2: How about temporal evolution?

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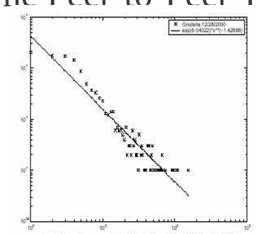
## The Peer-to-Peer Topology




[Jovanovic+]

- Frequency versus degree
- Number of adjacent peers follows a power-law

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## More Power laws

- Also hold for other web graphs [Barabasi+, ‘99], [Kumar+, ‘99]
- citation graphs (see later)
- and many more

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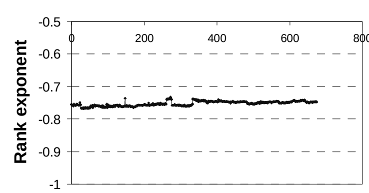
## Time Evolution: rank *R*




***Domain level***

**#days since Nov. ‘97**

The rank exponent has not changed! [Siganos+, ‘03]

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## Outline

Part 1: Topology, ‘laws’ and generators
- ‘Laws’ and patterns
  - Power laws for degree, eigenvalues, hop-plot
  - ???
- Generators
- Tools

Part 2: PageRank, HITS and eigenvalues





## Any other ‘laws’?

Yes!





## Any other ‘laws’?

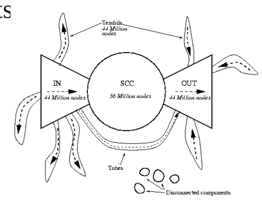
Yes!
- Small diameter
  – six degrees of separation / ‘Kevin Bacon’
  – small worlds [Watts and Strogatz]





## Any other ‘laws’?

- Bow-tie, for the web [Kumar+ ‘99]
- IN, SCC, OUT, ‘tendrils’
- disconnected components

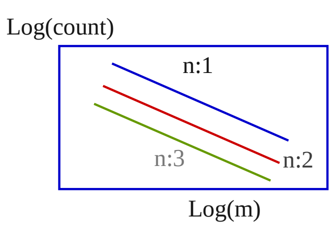


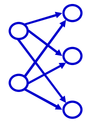





## Any other ‘laws’?

- power-laws in communities (bi-partite cores) [Kumar+, ‘99]



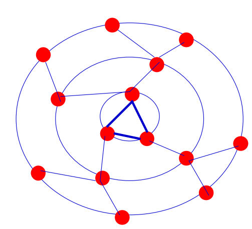
2:3 core
(m:n core)





## Any other ‘laws’?

- “Jellyfish” for Internet [Tauro+ ’01]
- core: ~clique
- ~5 concentric layers
- many 1-degree nodes



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## How do graphs evolve?

- degree-exponent seems constant - anything else?



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## Evolution of diameter?

- Prior analysis, on power-law-like graphs, hints that

  diameter ~ O(log(N)) or

  diameter ~ O( log(log(N)))
- i.e.., slowly increasing with network size
- Q: What is happening, in reality?



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## Evolution of diameter?

- Prior analysis, on power-law-like graphs, hints that

  diameter ~ O(log(N)) or

  diameter ~ O( log(log(N)))
- i.e.., slowly increasing with network size
- Q: What is happening, in reality?
- A: It **shrinks**(!!), towards a constant value



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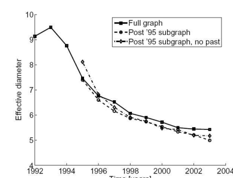
## Shrinking diameter

ArXiv physics papers and their citations

[Leskovec+05a]




(a) arXiv citation graph



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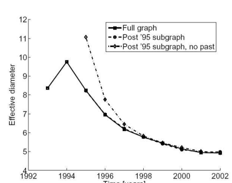
## Shrinking diameter

ArXiv: who co-authored with whom




(b) Affiliation network



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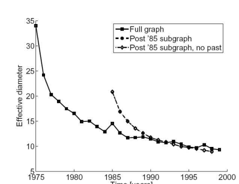
## Shrinking diameter

U.S. patents citing each other




(c) Patents

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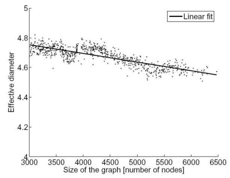
## Shrinking diameter

Autonomous systems




(d) AS

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## Temporal evolution of graphs

- N(t) nodes; E(t) edges at time t
- suppose that

  N(t+1) = 2 * N(t)
- Q: what is your guess for

  E(t+1) =? ...* E(t)

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## Temporal evolution of graphs

- N(t) nodes; E(t) edges at time t
- suppose that

  N(t+1) = 2 * N(t)
- Q: what is your guess for

  E(t+1) =? ...* E(t)
- A: over-doubled!

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## Temporal evolution of graphs

- A: over-doubled - but obeying:

  $\boxed{E(t) \sim N(t)^a \quad \text{for all } t}$

  where $1<a<2$

  a=1: constant avg degree

  a=2: ~full clique
- Real graphs **densify** over time [Leskovec+05a]

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## Temporal evolution of graphs

- A: over-doubled - but obeying:

  $\boxed{E(t) \sim N(t)^a \quad \text{for all } t}$
- Identically:

  $\boxed{\log(E(t)) / \log(N(t)) = \text{constant}}$ for all $t$

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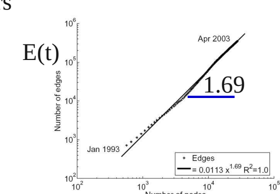
## Densification Power Law

ArXiv: Physics papers and their citations




(a) arXiv

N(t)

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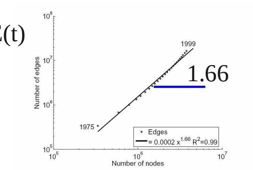
## Densification Power Law

U.S. Patents, citing each other

E(t)




(b) Patents

N(t)

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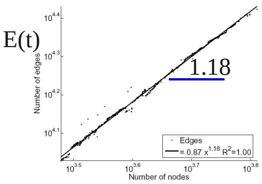
## Densification Power Law

Autonomous Systems

E(t)




(c) Autonomous Systems

N(t)

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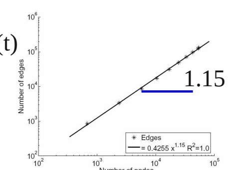
## Densification Power Law

ArXiv: who co-authored with whom

E(t)




(d) Affiliation network

N(t)

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## Summary of ‘laws’

- Power laws for degree distributions
- …………... for eigenvalues, bi-partite cores
- Small & **shrinking** diameter (‘6 degrees’)
- ‘Bow-tie’ for web; ‘jelly-fish’ for internet
- ``Densification Power Law’’, over time

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## Outline

Part 1: Topology, ‘laws’ and generators
- ‘Laws’ and patterns
- ➡ Generators
- Tools

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## Generators

- How to generate random, realistic graphs?
  - Erdos-Renyi model: beautiful, but unrealistic
  - process-based generators
  - recursive generators

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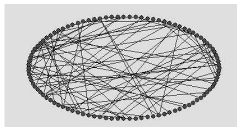
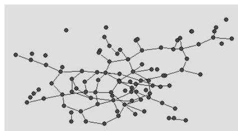
## Erdos-Renyi

- random graph – 100 nodes, avg degree = 2
- Fascinating properties (phase transition)
- But: unrealistic (Poisson degree distribution != power law)







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## Process-based

- Barabasi; Barabasi-Albert: Preferential attachment -> power-law tails!
  - ‘rich get richer’
- [Kumar+]: preferential attachment + mimic
  - Create ‘communities’



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## Process-based (cont’d)

- [Fabrikant+, ‘02]: H.O.T.: connect to closest, high connectivity neighbor
- [Pennock+, ‘02]: Winner does NOT take all
- ... and many more



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## Recursive generators - intuition

- recursion <-> self-similarity <-> power laws
  (see details later)
- Recursion -> communities within communities within communities



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## Wish list for a generator:

- Power-law-tail in- and out-degrees
- Power-law-tail scree plots
- **shrinking/constant** diameter
- Densification Power Law
- communities-within-communities

Q: how to achieve all of them?



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## Wish list for a generator:

- Power-law-tail in- and out-degrees
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Q: how to achieve all of them?

A: Kronecker matrix product [Leskovec+05b]

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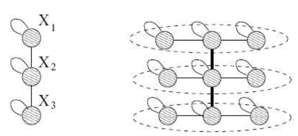
# Kronecker product




(a) Graph $G_1$ (b) Intermediate stage



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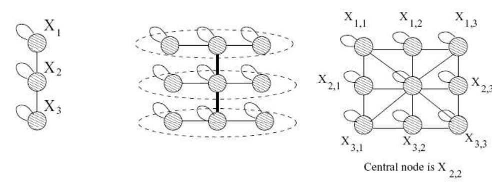
# Kronecker product




(a) Graph $G_1$ (b) Intermediate stage (c) Graph $G_2 = G_1 \otimes G_1$



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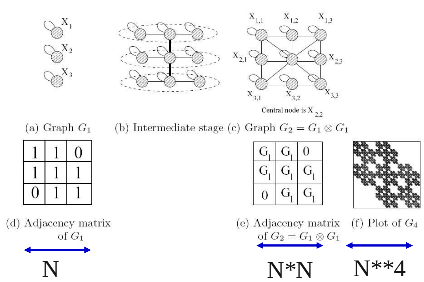
# Kronecker product



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# Properties of Kronecker graphs:

- Power-law-tail in- and out-degrees
- Power-law-tail scree plots
- **constant** diameter
- perfect Densification Power Law
- communities-within-communities



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# Properties of Kronecker graphs:

- Power-law-tail in- and out-degrees
- Power-law-tail scree plots
- **constant** diameter
- perfect Densification Power Law
- communities-within-communities

**and we can <u>prove all</u> of the above**

(first and only generator that does that)



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# Properties of Kronecker graphs:

- ‘stochastic’ version gives even better results and
  - Includes Erdos-Renyi as special case
  - Includes ‘RMAT’ as special case [Chakrabarti+,’04]
- (stochastic version: generate Kronecker matrix; decimate edges with some probability)

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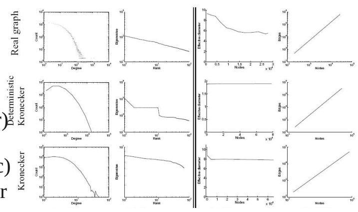
# Kronecker - ArXiv

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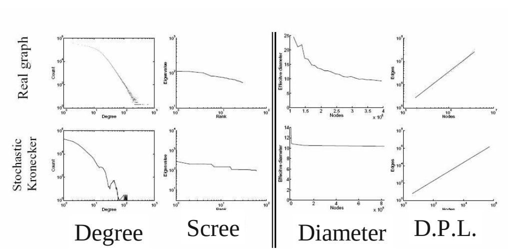
# Kronecker - patents

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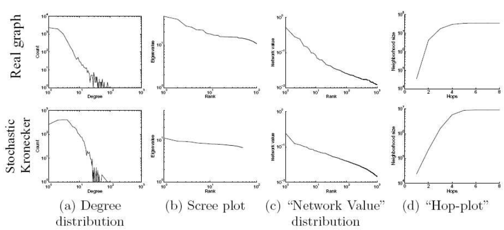
# Kronecker - A.S.

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# Conclusions

'Laws' and patterns:

- Power laws for degrees, eigenvalues, 'communities'/cores
- Small / Shrinking diameter
- Bow-tie; jelly-fish

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# Conclusions, cont'd

Generators

- Preferential attachment (Barabasi)
- Variations
- Recursion – Kronecker product & RMAT

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# Outline

Topology, 'laws' and generators

- 'Laws' and patterns
- Generators
- ➡ Tools

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# Outline

Part 1: Topology, ʻlawsʼ and generators

- ʻLawsʼ and patterns
- Generators
- Tools: power laws and fractals
  - Why so many power laws?
  - Self-similarity, power laws, fractal dimension



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# Power laws

- Q1: Are they only in graph-related settings?
- A1:
- Q2: Why so many?
- A2:



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# Power laws

- Q1: Are they only in graph-related settings?
- A1: NO!
- Q2: Why so many?
- A2: self-similarity; ‘rich-get-richer’



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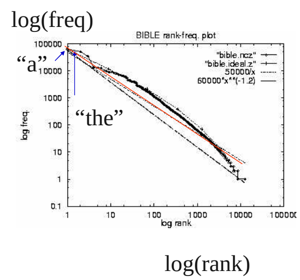
# A famous power law: Zipf’s law




- Bible - **rank** vs **frequency** (log-log)



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# Power laws, cont’ed

- length of file transfers [Bestavros+]
- web hit counts [Huberman]
- magnitude of earthquakes (Guttenberg-Richter law)
- sizes of lakes/islands (Korcak’s law)
- Income distribution (Pareto’s law)



# Click-stream data

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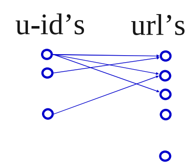
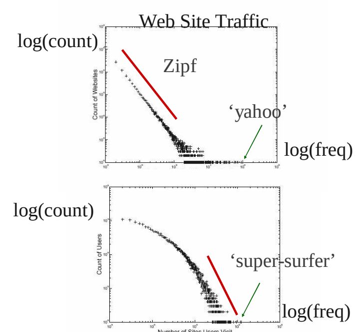
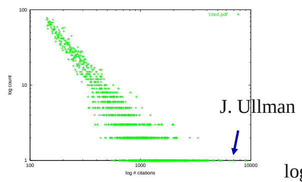


## Lotka's law

(Lotka's law of publication count); and citation counts: (*citeseer.nj.nec.com* 6/2001)

log(count)




log(#citations)





## Power laws

- Q1: Are they only in graph-related settings?
- A1: NO!
- Q2: Why so many?
- A2: self-similarity; 'rich-get-richer'





## Fractals and power laws

- Power laws and fractals are closely related
- And fractals appear in **MANY** cases
  - coast-lines: 1.1-1.5
  - brain-surface: 2.6
  - rain-patches: 1.3
  - tree-bark: ~2.1
  - stock prices / random walks: 1.5
  - ... [see Mandelbrot; or Schroeder]





## Digression: intro to fractals

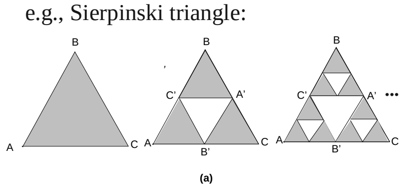
- Fractals: sets of points that are self similar





## A famous fractal

e.g., Sierpinski triangle:




**zero area;**
**infinite length!**

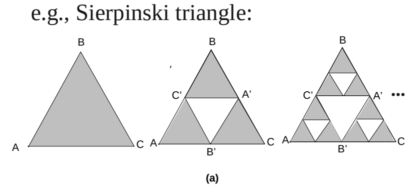
dimensionality = ??





## A famous fractal

e.g., Sierpinski triangle:

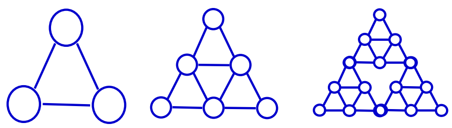



**zero area;**
**infinite length!**

dimensionality = log(3)/log(2) = 1.58

## A famous fractal

equivalent graph:







## Intrinsic ('fractal') dimension

How to estimate it?





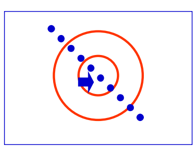
## Intrinsic ('fractal') dimension

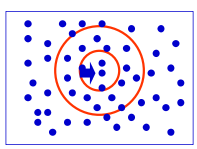
- Q: fractal dimension of a line?
- A: nn ( <= r ) ~ r^1

('power law': y=x^a)

- Q: fd of a plane?
- A: nn ( <= r ) ~ r^2

fd== slope of (log(nn) vs log(r) )





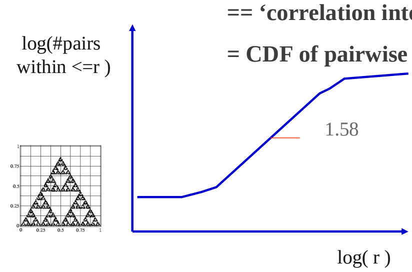




## Sierpinsky triangle

**== 'correlation integral'**

**= CDF of pairwise distances**





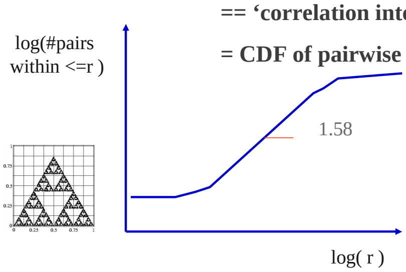



## Sierpinsky triangle

**== hopplot**

**== 'correlation integral'**

**= CDF of pairwise distances**



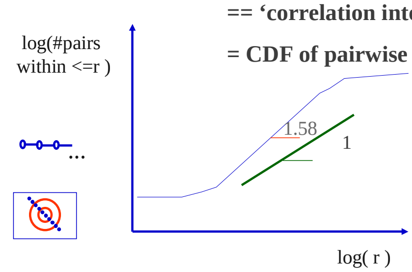





## Line

**== 'correlation integral'**

**= CDF of pairwise distances**



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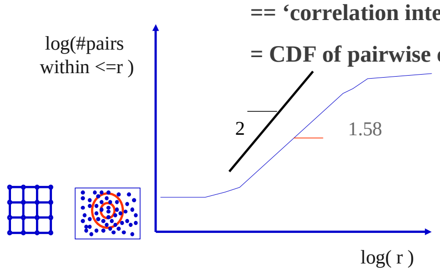
## 2-d (Plane)

== ‘correlation integral’

= CDF of pairwise distances

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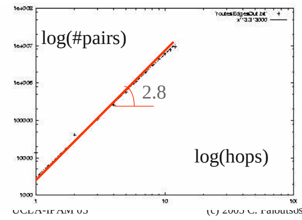
## Recall: Hop Plot

- Internet routers: how many neighbors within *h* hops? (= **correlation integral**!)




Reachability function: number of neighbors within r hops, vs r (log-log).

Mbone routers, 1995

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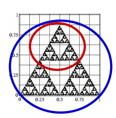
## Fractals and power laws

They are related concepts:

- fractals <=>
- self-similarity <=>
- scale-free <=>
- power laws ( $y= x^a$ )
  - $F = C r^{(-2)}$



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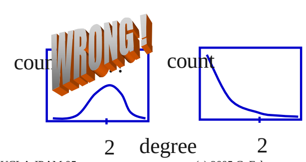
## Conclusions

- Real settings/graphs: skewed distributions
  - ‘mean’ is meaningless

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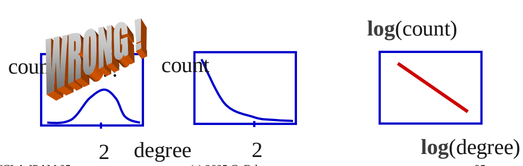
## Conclusions

- Real settings/graphs: skewed distributions
  - ‘mean’ is meaningless
  - slope of power law, instead

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## Conclusions: Tools:

- rank-frequency plot (a’la Zipf)
- Correlation integral (= neighborhood function)

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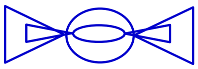
## Conclusions (cont’d)

- Recursion/self-similarity
  - May reveal non-obvious patterns (e.g., bow-ties within bow-ties within bow-ties) [Dill+, ‘01]



**“To iterate is human, to recurse is divine”**

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## Resources

Generators:

- RMAT (deepay AT cs.cmu.edu)
- Kronecker ({deepay,jure} AT cs.cmu.edu)
- BRITE http://www.cs.bu.edu/brite/
- INET: http://topology.eecs.umich.edu/inet

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## Other resources

Visualization - graph algo’s:

- Graphviz: http://www.graphviz.org/
- pajek: http://vlado.fmf.uni-lj.si/pub/networks/pajek/

Kevin Bacon web site: http://www.cs.virginia.edu/oracle/

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## Thank you!

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## EXTRA
## Virus propagation



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## Outline

Topology, 'laws' and generators

EXTRA: Virus Propagation



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## Problem definition

- Q1: How does a virus spread across an arbitrary network?
- Q2: will it create an epidemic?

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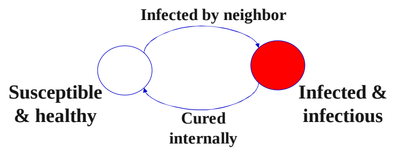
## Framework

- Susceptible-Infected-Susceptible (SIS) model
  - Cured nodes immediately become susceptible

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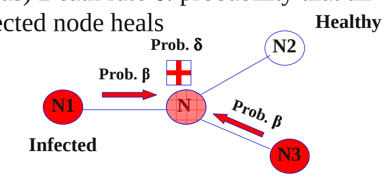
## The model

- (virus) Birth rate $\beta$: probability than an infected neighbor attacks
- (virus) Death rate $\delta$: probability that an infected node heals

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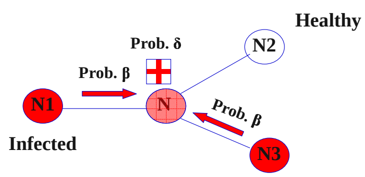
## The model

- Virus 'strength' s= $\beta/\delta$

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## Epidemic threshold $\tau$

of a graph, defined as the value of $\tau$, such that

if strength $s = \beta / \delta < \tau$

an epidemic can not happen

Thus,

- given a graph
- compute its epidemic threshold

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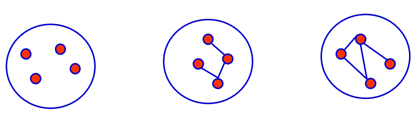
## Epidemic threshold $\tau$

What should $\tau$ depend on?

- avg. degree? and/or highest degree?
- and/or variance of degree?
- and/or third moment of degree?



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## Epidemic threshold

- [Theorem] We have no epidemic, if

$$\boxed{\beta/\delta < \tau = 1/\lambda_{1,A}}$$

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## Epidemic threshold

- [Theorem] We have no epidemic, if

recovery prob. epidemic threshold

$$\beta/\delta < \tau = 1/\lambda_{1,A}$$

attack prob. largest eigenvalue of adj. matrix ***A***

Proof: [Wang+03]

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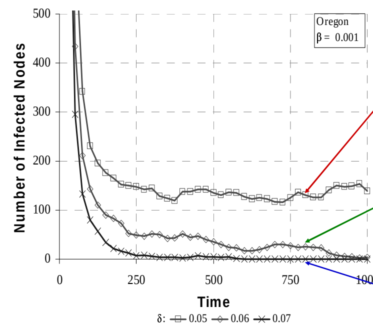
## Experiments (Oregon)




**β/δ > τ (above threshold)**

**β/δ = τ (at the threshold)**

**β/δ < τ (below threshold)**

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## Our result:

- Holds for **any** graph
- includes older results as special cases

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(really done this time ☺ )

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