Mining Graphs and Tensors

Christos Faloutsos
CMU

Thank you!

- Charlie Van Loan
- Lenore Mullin
- Frank Olken

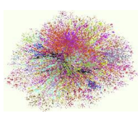
Outline

- Introduction – Motivation
- Problem#1: Patterns in static graphs
- Problem#2: Patterns in tensors / time evolving graphs
- Problem#3: Which tensor tools?
- Problem#4: Scalability
- Conclusions

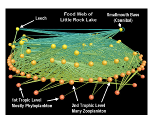
Motivation

➡ Data mining: ~ find patterns (rules, outliers)
- Problem#1: How do real graphs look like?
- Problem#2: How do they evolve?
- Problem#3: What tools to use?
- Problem#4: Scalability to GB, TB, PB?

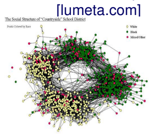
Graphs - why should we care?



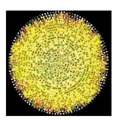
Internet Map
[lumeta.com]



Food Web
[Martinez '91]



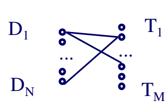
Friendship Network
[Moody '01]



Protein Interactions
[genomebiology.com]

Graphs - why should we care?

- IR: bi-partite graphs (doc-terms)



- web: hyper-text graph
- ... and more:

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Graphs - why should we care?

- network of companies & board-of-directors members
- 'viral' marketing
- web-log ('blog') news propagation
- computer network security: email/IP traffic and anomaly detection
- ....

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Outline

Data mining: ~ find patterns (rules, outliers)

- Problem#1: How do real graphs look like?
  - Degree distributions
  - Eigenvalues
  - Triangles
  - weights
- Problem#2: How do they evolve?
- …

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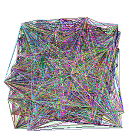
Problem #1 - network and graph mining



- How does the Internet look like?
- How does the web look like?
- What is 'normal'/'abnormal'?
- which patterns/laws hold?

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Graph mining

- Are real graphs random?

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Laws and patterns

- Are real graphs random?
- A: NO!!
  - Diameter
  - in- and out- degree distributions
  - other (surprising) patterns

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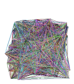
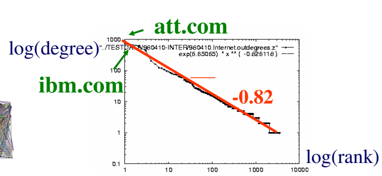
Solution#1.1

- Power law in the degree distribution [SIGCOMM99]

internet domains





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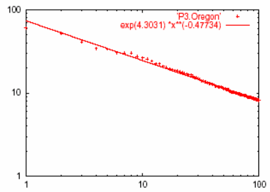
Solution#1.2: Eigen Exponent *E*

Eigenvalue



Rank of decreasing eigenvalue

Exponent = slope

E = -0.48

May 2001

- A2: power law in the eigenvalues of the adjacency matrix

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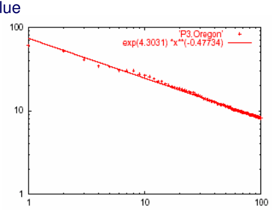
Solution#1.2: Eigen Exponent *E*

Eigenvalue



Rank of decreasing eigenvalue

Exponent = slope

E = -0.48

May 2001

- [Mihail, Papadimitriou ’02]: slope is ½ of rank exponent

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But:

How about graphs from other domains?

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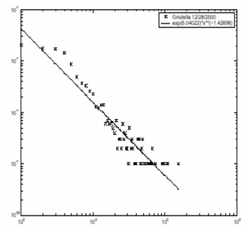
The Peer-to-Peer Topology



[Jovanovic+]

- Count versus degree
- Number of adjacent peers follows a power-law

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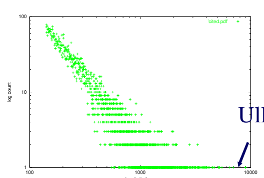
More settings w/ power laws:

citation counts: (*citeseer.nj.nec.com* 6/2001)

log(count)



log(#citations)

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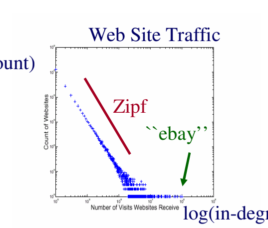
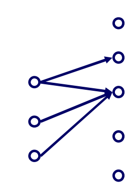
More power laws:

- web hit counts [w/ A. Montgomery]

log(count)



log(in-degree)



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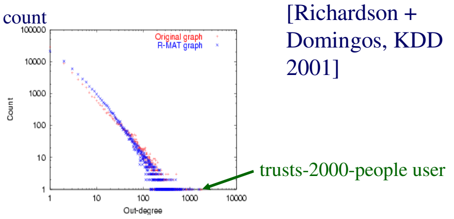
epinions.com

count



(out) degree

- who-trusts-whom [Richardson + Domingos, KDD 2001]

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Data mining: ~ find patterns (rules, outliers)

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- …

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How about triangles?

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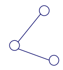
Solution# 1.3: Triangle 'Laws'



- Real social networks have a lot of triangles

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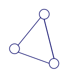
Triangle 'Laws'



- Real social networks have a lot of triangles
  - Friends of friends are friends
- Any patterns?

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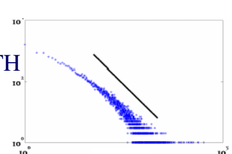
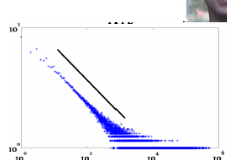
Triangle Law: #1
[Tsourakakis ICDM 2008]

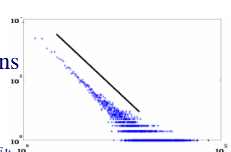
HEP-TH





ASN

Epinions



X-axis: # of Triangles
a node participates in
Y-axis: count of such nodes

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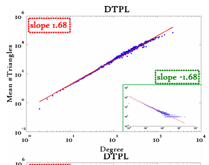
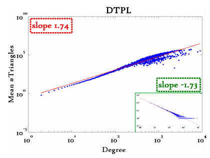
Triangle Law: #2
[Tsourakakis ICDM 2008]

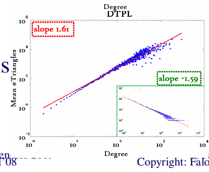
Reuters



SN



Epinions



X-axis: degree
Y-axis: mean # triangles
Notice: slope ~ degree exponent (insets)

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Triangle Law: Computations
[Tsourakakis ICDM 2008]

But: triangles are expensive to compute
(3-way join; several approx. algos)
Q: Can we do that quickly?

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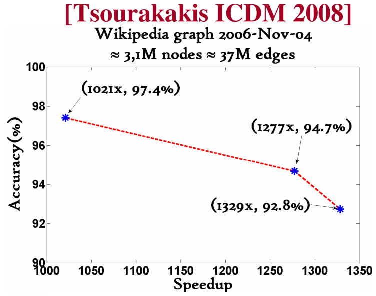
Triangle Law: Computations
[Tsourakakis ICDM 2008]

But: triangles are expensive to compute
(3-way join; several approx. algos)
Q: Can we do that quickly?
A: Yes!
#triangles = 1/6 Sum (λ_i^3)
(and, because of skewness, we only need the top few eigenvalues!

Triangle Law: Computations

[Tsourakakis ICDM 2008]



1000x+ speed-up, high accuracy

Outline

Data mining: ~ find patterns (rules, outliers)

- Problem#1: How do real graphs look like?
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  - Eigenvalues
  - Triangles
  - weights
- Problem#2: How do they evolve?
- …

How about weighted graphs?

- A: even more 'laws'!

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Solution#1.4: fortification

Q: How do the weights of nodes relate to degree?

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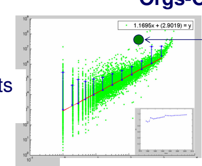
Solution#1.4: fortification: Snapshot Power Law

- At any time, total incoming weight of a node is proportional to in-degree with PL exponent 'iw':
  - *i.e. 1.01* < iw < 1.26, *super-linear*
- ***More donors, even more $***



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Motivation

Data mining: ~ find patterns (rules, outliers)

- Problem#1: How do real graphs look like?
- Problem#2: How do they evolve?
  - Diameter
  - GCC, and NLCC
  - Blogs, linking times, cascades
- Problem#3: Tensor tools?
- …

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Problem#2: Time evolution

- with Jure Leskovec (CMU/MLD)

- and Jon Kleinberg (Cornell – sabb. @ CMU)

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Evolution of the Diameter

- Prior work on Power Law graphs hints at **slowly growing diameter**:
  - diameter ~ O(log N)
  - diameter ~ O(log log N)
- What is happening in real data?

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Evolution of the Diameter

- Prior work on Power Law graphs hints at **slowly growing diameter**:
  - diameter ~ O(log N)
  - diameter ~ O(log log N)
- What is happening in real data?
- Diameter **shrinks** over time

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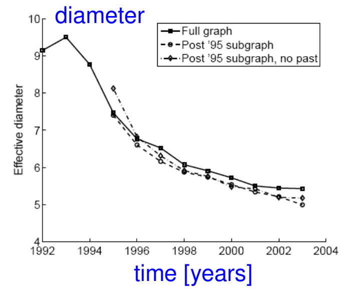
Diameter – ArXiv citation graph

- Citations among physics papers
- 1992 –2003
- One graph per year



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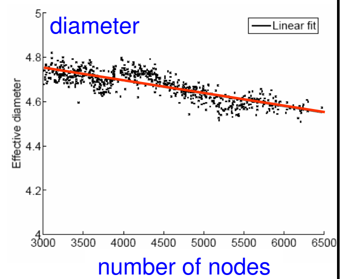
Diameter – “Autonomous Systems”

- Graph of Internet
- One graph per day
- 1997 – 2000



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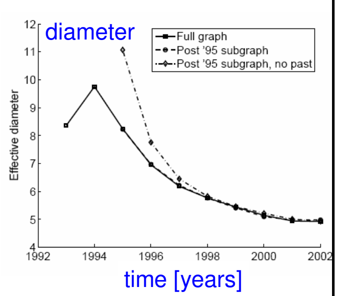
Diameter – “Affiliation Network”

- Graph of collaborations in physics – authors linked to papers
- 10 years of data



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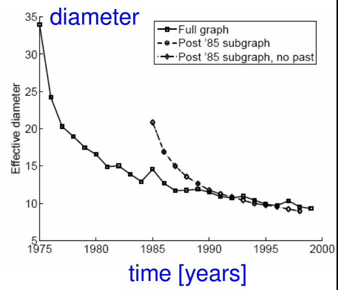
Diameter – "Patents"

- Patent citation network
- 25 years of data



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Temporal Evolution of the Graphs

- N(t) … nodes at time t
- E(t) … edges at time t
- Suppose that

  N(t+1) = 2 * N(t)
- Q: what is your guess for

  E(t+1) =? 2 * E(t)

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Temporal Evolution of the Graphs

- N(t) … nodes at time t
- E(t) … edges at time t
- Suppose that

  N(t+1) = 2 * N(t)
- Q: what is your guess for

  E(t+1) =? 2 * E(t)
- A: over-doubled!
  - But obeying the ``Densification Power Law''

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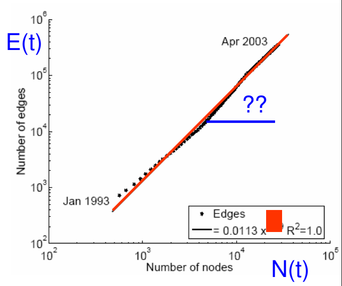
Densification – Physics Citations

- Citations among physics papers
- 2003:
  - 29,555 papers, 352,807 citations



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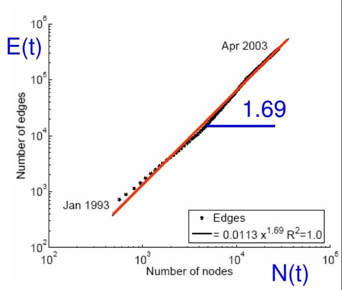

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Densification – Physics Citations

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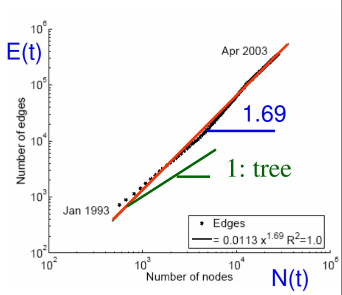

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Densification – Physics Citations

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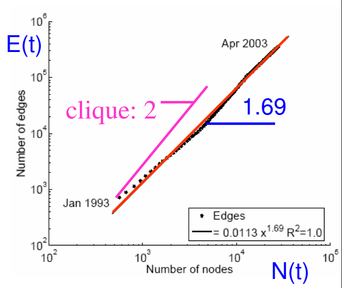

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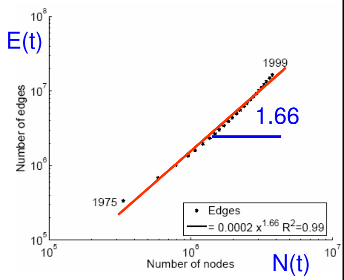
Densification – Patent Citations

- Citations among patents granted
- 1999
  - 2.9 million nodes
  - 16.5 million edges
- Each year is a datapoint



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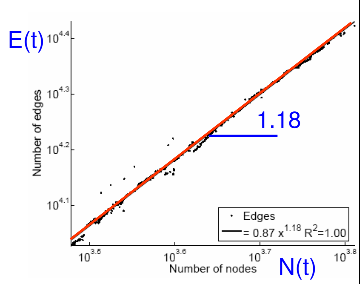
Densification – Autonomous Systems

- Graph of Internet
- 2000
  - 6,000 nodes
  - 26,000 edges
- One graph per day



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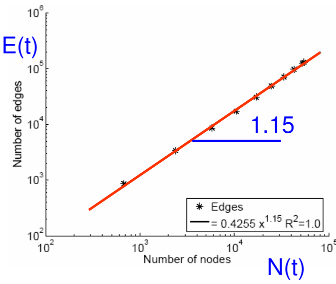
Densification – Affiliation Network

- Authors linked to their publications
- 2002
  - 60,000 nodes
    - 20,000 authors
    - 38,000 papers
  - 133,000 edges



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Motivation

Data mining: ~ find patterns (rules, outliers)

- Problem#1: How do real graphs look like?
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  - Diameter
  - ➡ GCC, and NLCC
  - Blogs, linking times, cascades
- Problem#3: Tensor tools?
- …

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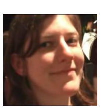
More on Time-evolving graphs




M. McGlohon, L. Akoglu, and C. Faloutsos *Weighted Graphs and Disconnected Components: Patterns and a Generator. SIG-KDD* 2008

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Observation 1: Gelling Point

Q1: How does the GCC emerge?

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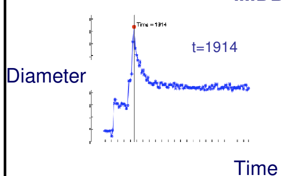
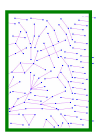
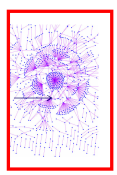
Observation 1: Gelling Point

- Most real graphs display a gelling point
- After gelling point, they exhibit typical behavior. This is marked by a spike in diameter.

IMDB







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Observation 2: NLCC behavior

Q2: How do NLCC's emerge and join with the GCC?

(``NLCC'' = non-largest conn. components)

– Do they continue to grow in size?
– or do they shrink?
– or stabilize?

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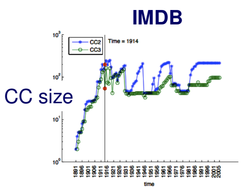
Observation 2: NLCC behavior

- After the gelling point, the GCC takes off, but NLCC's remain ~constant (actually, oscillate).



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How do new edges appear?

[LBKT'08] [Microscopic Evolution of Social Networks](#)
Jure Leskovec, Lars Backstrom, Ravi Kumar, Andrew Tomkins.
(ACM KDD), 2008.

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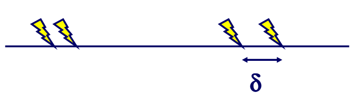
How do edges appear in time? [LBKT'08]

Edge gap δ(d): inter-arrival time between d^{th} and $d+1^{st}$ edge



What is the PDF of **δ?**
Poisson**?**

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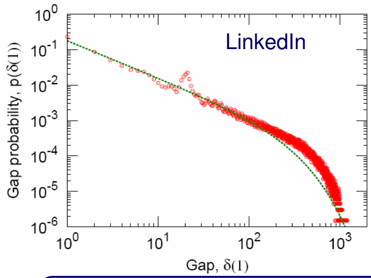
How do edges appear in time? [LBKT'08]



Edge gap δ(d): inter-arrival time between d^{th} and $d+1^{st}$ edge

$$p_g(\delta(d);\alpha,\beta) \propto \delta(d)^{-\alpha} e^{-\beta\delta(d)}$$

1-58

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  - GCC, and NLCC
  - ➡ linking times, blogs, cascades
- Problem#3: Tensor tools?
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Blog analysis

- with Mary McGlohon (CMU)
- Jure Leskovec (CMU)
- Natalie Glance (now at Google)
- Mat Hurst (now at MSR)

[SDM'07]

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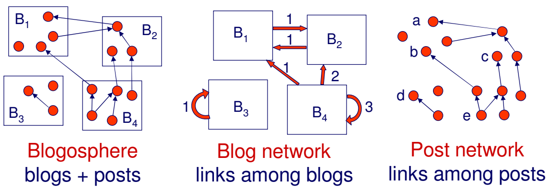
Cascades on the Blogosphere



Q1: popularity-decay of a post?
Q2: degree distributions?

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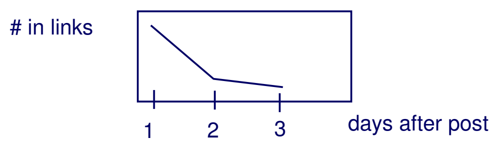
Q1: popularity over time



Post popularity drops-off – exponentially?

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Days after post

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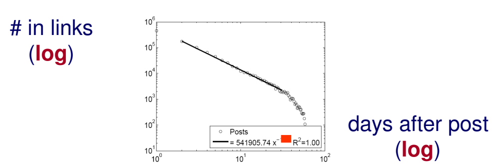
Q1: popularity over time



Post popularity drops-off – exponentially?
POWER LAW!
Exponent?

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Days after post

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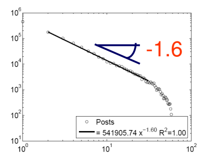
Q1: popularity over time



Post popularity drops-off – exponentially?
POWER LAW!
Exponent? -1.6 (close to -1.5: Barabasi's stack model)

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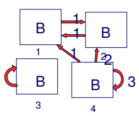
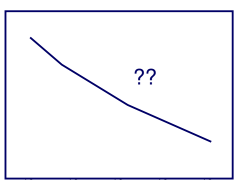
Q2: degree distribution

44,356 nodes, 122,153 edges. Half of blogs belong to largest connected component.





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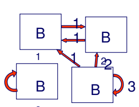

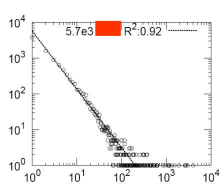

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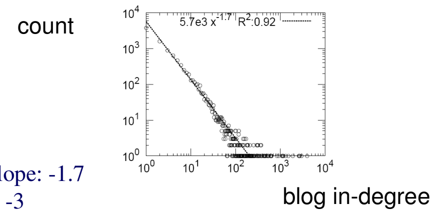

in-degree slope: -1.7
out-degree: -3
'rich get richer'

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Motivation

Data mining: ~ find patterns (rules, outliers)

- Problem#1: How do real graphs look like?
- Problem#2: How do they evolve?
- ➡ Problem#3: What tools to use?
  - PARAFAC/Tucker, CUR, MDL
  - generation: Kronecker graphs
- Problem#4: Scalability to GB, TB, PB?

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Tensors for time evolving graphs

- [Jimeng Sun+ KDD'06]
- [" , SDM'07]
- [CF, Kolda, Sun, SDM'07 tutorial]

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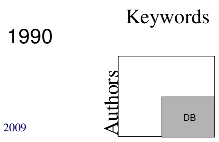
Social network analysis

- Static: find community structures



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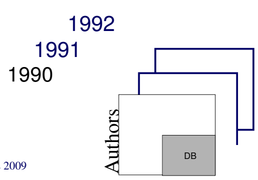
Social network analysis

- Static: find community structures



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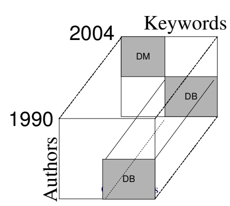
Social network analysis

- Static: find community structures
- Dynamic: monitor community structure evolution; spot abnormal individuals; abnormal time-stamps



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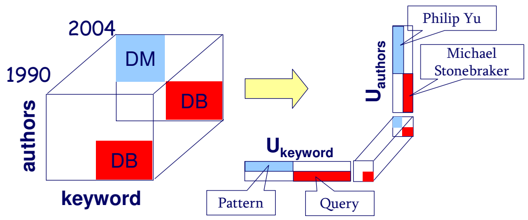
Application 1: Multiway latent semantic indexing (LSI)



- Projection matrices specify the clusters
- Core tensors give cluster activation level

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Bibliographic data (DBLP)

- Papers from VLDB and KDD conferences
- Construct 2nd order tensors with yearly windows with <author, keywords>
- Each tensor: 4584×3741
- 11 timestamps (years)

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Multiway LSI

Authors	Keywords	Year
michael carey, michael stonebraker, h. jagadish, hector garcia-molina	queri,parallel,optimization,concurr, objectorient	1995
surajit chaudhuri,mitch cherniack,michael stonebraker,ugur etintemel	distribut,systems,view,storage,servic,process,cache	2004
jiawei han,jian pei,philip s. yu, jianyong wang,charu c. aggarw[illegible]	streams,pattern,support, cluster, [illegible] gener,queri	2004

DB

DM

- Two groups are correctly identified: Databases and Data mining
- People and concepts are drifting over time

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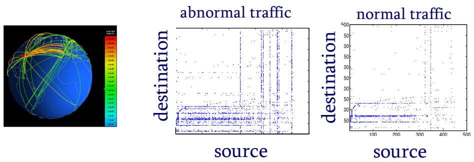
Network forensics

- Directional network flows
- A large ISP with 100 POPs, each POP 10Gbps link capacity [Hotnets2004]
  - 450 GB/hour with compression
- Task: Identify abnormal traffic pattern and find out the cause



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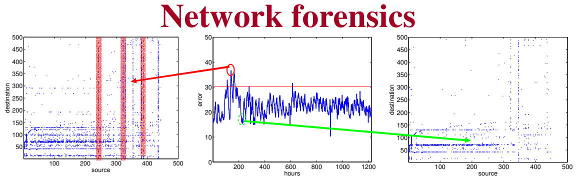
Network forensics



Abnormal traffic | Reconstruction error over time | Normal traffic

- Reconstruction error gives indication of anomalies.
- Prominent difference between normal and abnormal ones is mainly due to the unusual scanning activity (confirmed by the campus admin).

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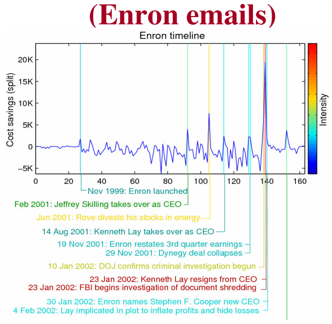
MDL mining on time-evolving graph (Enron emails)



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GraphScope [w. Jimeng Sun, Spiros Papadimitriou and Philip Yu, KDD'07]

Motivation

Data mining: ~ find patterns (rules, outliers)

- Problem#1: How do real graphs look like?
- Problem#2: How do they evolve?
- Problem#3: What tools to use?
  - PARAFAC/Tucker, CUR, MDL
  - generation: Kronecker graphs
- Problem#4: Scalability to GB, TB, PB?

Problem#3: Tools - Generation

- Given a growing graph with count of nodes N_1, N_2, …
- Generate a realistic sequence of graphs that will obey all the patterns

Problem Definition

- Given a growing graph with count of nodes N_1, N_2, …
- Generate a realistic sequence of graphs that will obey all the patterns
  - Static Patterns
    - Power Law Degree Distribution
    - Power Law eigenvalue and eigenvector distribution
    - Small Diameter
  - Dynamic Patterns
    - Growth Power Law
    - Shrinking/Stabilizing Diameters

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Problem Definition

- Given a growing graph with count of nodes N_1, N_2, …
- Generate a realistic sequence of graphs that will obey all the patterns
- **Idea: Self-similarity**
  - Leads to power laws
  - Communities within communities
  - …

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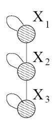
Kronecker Product – a Graph



1	1	0
1	1	1
0	1	1

G_1

Adjacency matrix

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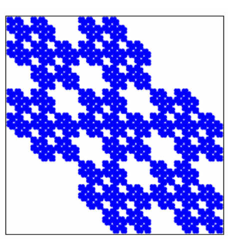
Kronecker Product – a Graph

- Continuing multiplying with G_1 we obtain G_4 and so on …



G_4 adjacency matrix

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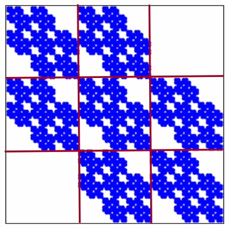
Kronecker Product – a Graph

- Continuing multiplying with G_1 we obtain G_4 and so on …



G_4 adjacency matrix

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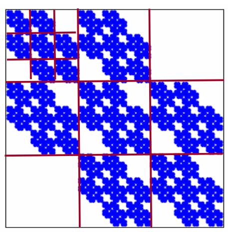
Kronecker Product – a Graph

- Continuing multiplying with G_1 we obtain G_4 and so on …



G_4 adjacency matrix

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Properties:

- We can PROVE that
  - Degree distribution is multinomial ~ power law
  - Diameter: constant
  - Eigenvalue distribution: multinomial
  - First eigenvector: multinomial
- See [Leskovec+, PKDD'05] for proofs

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Problem Definition

- Given a growing graph with nodes N_1, N_2, …
- Generate a realistic sequence of graphs that will obey all the patterns
  - Static Patterns
    - ✔ Power Law Degree Distribution
    - ✔ Power Law eigenvalue and eigenvector distribution
    - ✔ Small Diameter
  - Dynamic Patterns
    - ✔ Growth Power Law
    - ✔ Shrinking/Stabilizing Diameters
- First and only generator for which we can **prove** all these properties

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skip

Stochastic Kronecker Graphs

- Create $N_1 \times N_1$ probability matrix P_1
- Compute the k^{th} Kronecker power P_k
- For each entry p_{uv} of P_k include an edge (u,v) with probability p_{uv}

0.4	0.2
0.1	0.3

P_1

Kronecker multiplication →

0.16	0.08	0.08	0.04
0.04	0.12	0.02	0.06
0.04	0.02	0.12	0.06
0.01	0.03	0.03	0.09

P_k

→ flip biased coins

Instance Matrix G_2

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Experiments

- How well can we match real graphs?
  - Arxiv: physics citations:
    - 30,000 papers, 350,000 citations
    - 10 years of data
  - U.S. Patent citation network
    - 4 million patents, 16 million citations
    - 37 years of data
  - Autonomous systems – graph of internet
    - Single snapshot from January 2002
    - 6,400 nodes, 26,000 edges
- We show both static and temporal patterns

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(Q: how to fit the parm's?)

A:

- Stochastic version of Kronecker graphs +
- Max likelihood +
- Metropolis sampling
- [Leskovec+, ICML'07]

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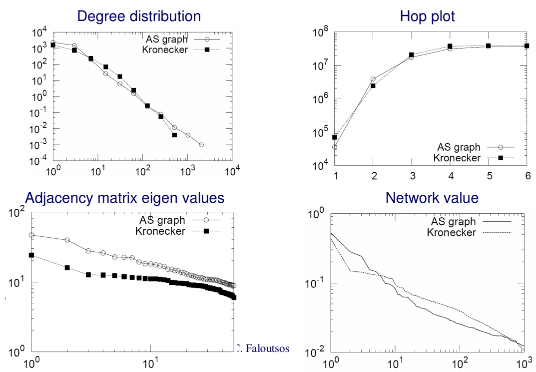
Experiments on real AS graph



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Conclusions

- Kronecker graphs have:
  - All the static properties
    - ✓Heavy tailed degree distributions
    - ✓Small diameter
    - ✓Multinomial eigenvalues and eigenvectors
  - All the temporal properties
    - ✓Densification Power Law
    - ✓Shrinking/Stabilizing Diameters
  - We can formally prove these results

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How to generate realistic tensors?

- A: 'RTM' [Akoglu+, ICDM'08]
  - do a tensor-tensor Kronecker product
  - resulting tensors (= time evolving graphs) have bursty addition of edges, over time.

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Motivation

Data mining: ~ find patterns (rules, outliers)

- Problem#1: How do real graphs look like?
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Scalability

- How about if graph/tensor does not fit in core?
- How about handling huge graphs?

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Scalability

- How about if graph/tensor does not fit in core?
- ['MET': Kolda, Sun, ICMD'08, best paper award]
- How about handling huge graphs?

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Scalability

- Google: > 450,000 processors in clusters of ~2000 processors each [Barroso, Dean, Hölzle, *"Web Search for a Planet: The Google Cluster Architecture"* IEEE Micro 2003]
- Yahoo: 5Pb of data [Fayyad, KDD'07]
- Problem: machine failures, on a daily basis
- How to parallelize data mining tasks, then?
- A: map/reduce – hadoop (open-source clone) http://hadoop.apache.org/

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2' intro to hadoop

- master-slave architecture; n-way replication (default n=3)
- 'group by' of SQL (in parallel, fault-tolerant way)
- e.g, find histogram of word frequency
  - compute local histograms
  - then merge into global histogram

```
select course-id, count(*)
from ENROLLMENT
group by course-id
```

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2’ intro to hadoop

- master-slave architecture; n-way replication (default n=3)
- ‘group by’ of SQL (in parallel, fault-tolerant way)
- e.g, find histogram of word frequency
  - compute local histograms
  - then merge into global histogram

select course-id, count(*)   reduce
from ENROLLMENT
group by course-id   map

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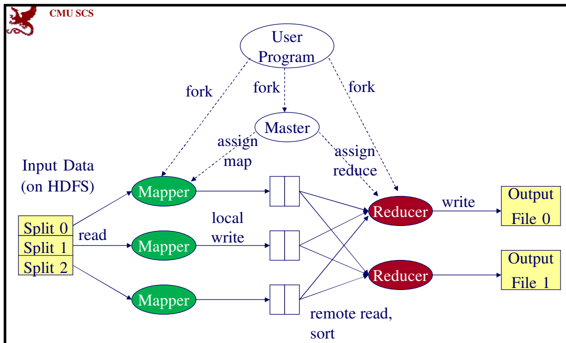

By default: 3-way replication;
Late/dead machines: ignored, **transparently** (!)

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D.I.S.C.

- ‘Data Intensive Scientific Computing’ [R. Bryant, CMU]
  - ‘big data’
  - http://www.cs.cmu.edu/~bryant/pubdir/cmu-cs-07-128.pdf

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E.g.: self-* and DCO systems @ CMU

- >200 nodes
- target: 1 PetaByte
- Greg Ganger +:
  - www.pdl.cmu.edu/SelfStar
  - www.pdl.cmu.edu/DCO

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OVERALL CONCLUSIONS

- Graphs/tensors pose a wealth of fascinating problems
- self-similarity and power laws work, when textbook methods fail!
- New patterns (densification, fortification, -1.5 slope in blog popularity over time
- New generator: Kronecker
- Scalability / cloud computing -> PetaBytes

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THANK YOU!

Contact info:

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(w/ papers, datasets, code, etc)

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