# CDM

# Model Checking ECA

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SPRING 2021

1 **ECA 90 Fixed**

2 **ECA 150 Fixed**

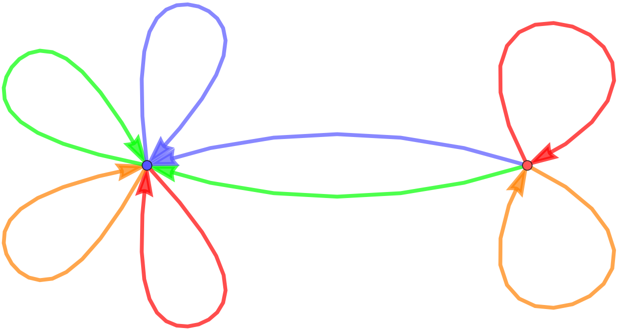
3 **ECA 90 Cyclic**

Existential quantifiers are easier than universal ones, so we check non-reversibility:

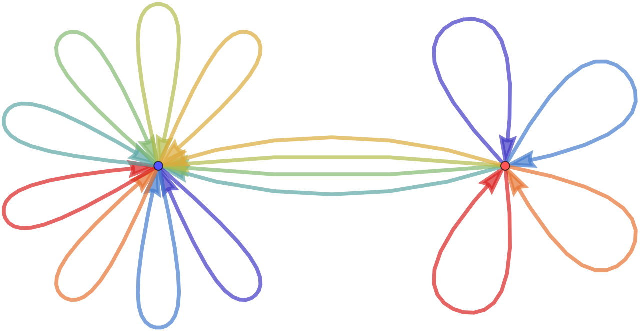
$$\varphi = \exists\, x, y, z\, (x \rightarrow z \wedge y \rightarrow z \wedge x \neq y)$$

We are working with finite words, not infinite ones, so we get the injectivity spectrum rather than specific answers (for $\mathbb{N}$ or $\mathbb{Z}$ grids).

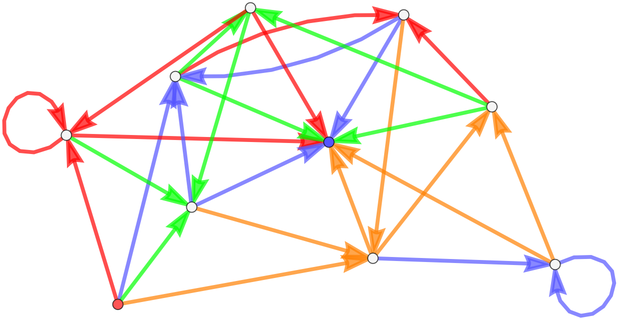
$$\mathsf{spec}(\varphi) = \{\, n \in \mathbb{N} \mid \langle \mathbf{2}^n, \rightarrow \rangle \models \varphi \,\}$$



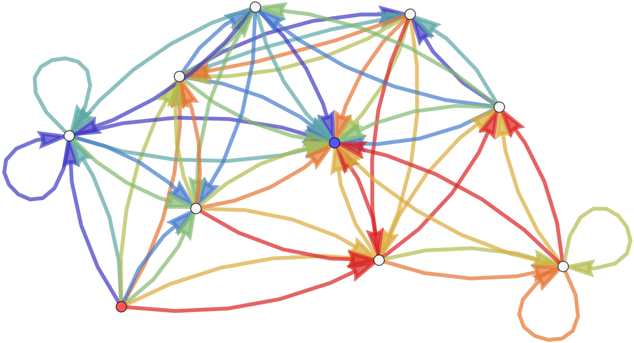
Basic synchronous automaton over $\mathbf{2} \times \mathbf{2}$ checking inequality.



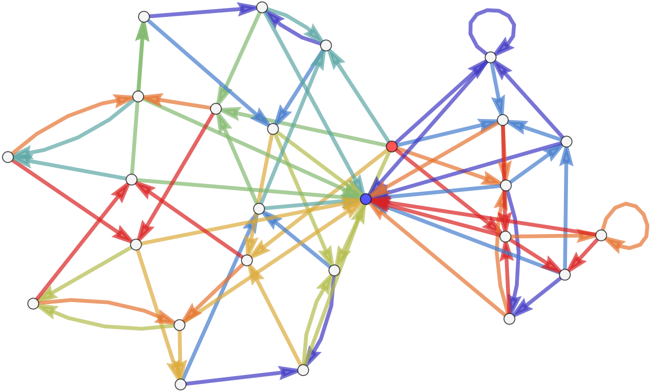
Inflated to alphabet $\mathbf{2} \times \mathbf{2} \times \mathbf{2}$, uses tracks 1 and 2 to check $x \neq y$.



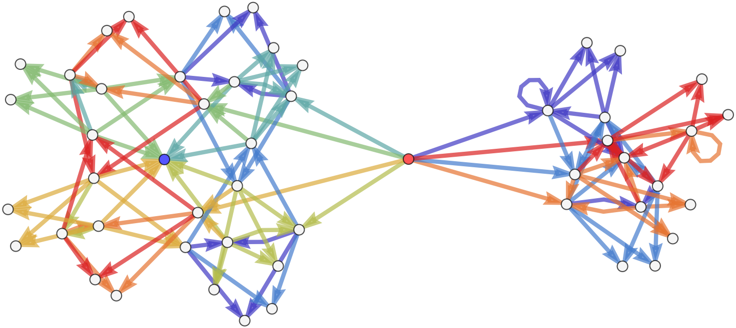
Basic synchronous automaton $\mathcal{A}_{\rightarrow}$ over $\mathbf{2} \times \mathbf{2}$ checking one step in ECA 90 with fixed boundary conditions.



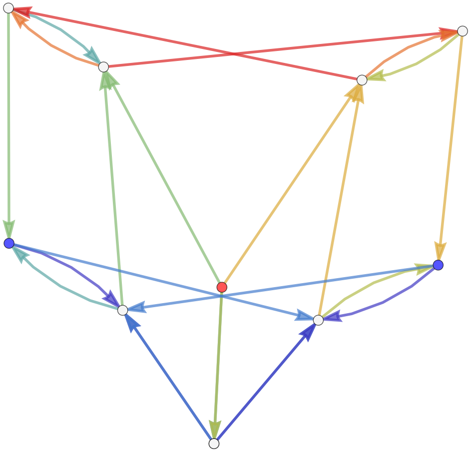
$\mathcal{A}_{\rightharpoondown,1,3}$ using tracks 1 and 3 to check $x \rightharpoondown z$.



$\mathcal{A}_{\rightarrowtriangle,1,3} \times \mathcal{A}_{\rightarrowtriangle,2,3}$ checking $x \rightarrowtriangle z \wedge y \rightarrowtriangle z$.



$\mathcal{A}_{\rightharpoonup,1,3} \times \mathcal{A}_{\rightharpoonup,2,3} \times \mathsf{UE}_{1,2}$ checking $x \rightharpoonup z \wedge y \rightharpoonup z \wedge x \neq y$.

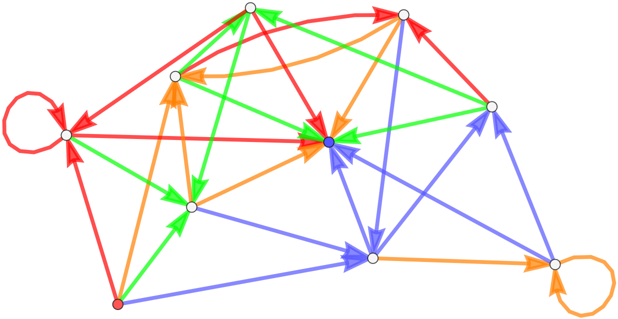


The symmetry is a consequence of the nice algebraic properties of ECA 90,

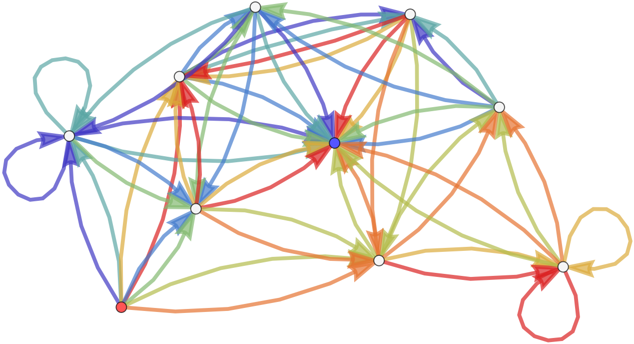
Hence ECA 90 fails to be injective on a lattice of length $n$ iff $n \in 3 + 2\mathbb{N}$.

The spectrum is regular as a language over $a^\star$; equivalently, a semilinear set over $\mathbb{N}$.

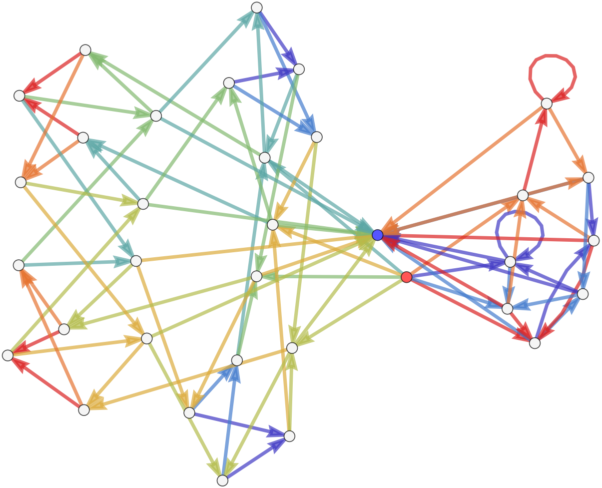
One could argue that $n = 1$ actually makes sense here (it's non-injective), but our construction assumes $n \geq 2$. .



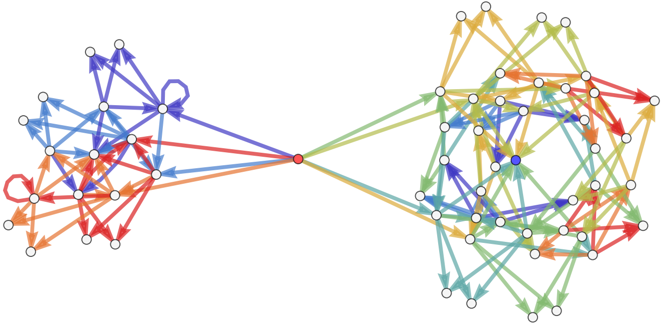
Basic synchronous automaton $\mathcal{A}_{\rightarrow}$ over $\mathbf{2} \times \mathbf{2}$ checking one step in ECA 150 with fixed boundary conditions.



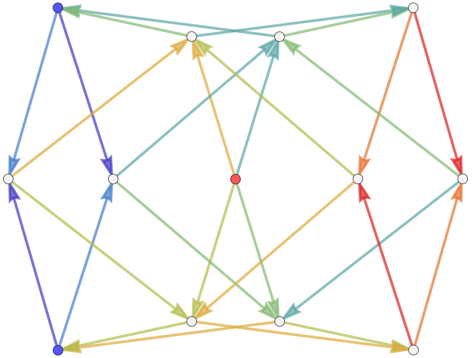
$\mathcal{A}_{\twoheadrightarrow,1,3}$ using tracks 1 and 3 to check $x \twoheadrightarrow z$.



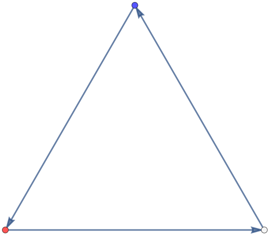
$\mathcal{A}_{\rightarrow,1,3} \times \mathcal{A}_{\rightarrow,2,3}$ checking $x \rightarrow z \wedge y \rightarrow z$.



$\mathcal{A}_{\rightarrow,1,3} \times \mathcal{A}_{\rightarrow,2,3} \times \mathsf{UE}_{1,2}$ checking $x \rightarrow z \wedge x \rightarrow z \wedge y \neq y$.

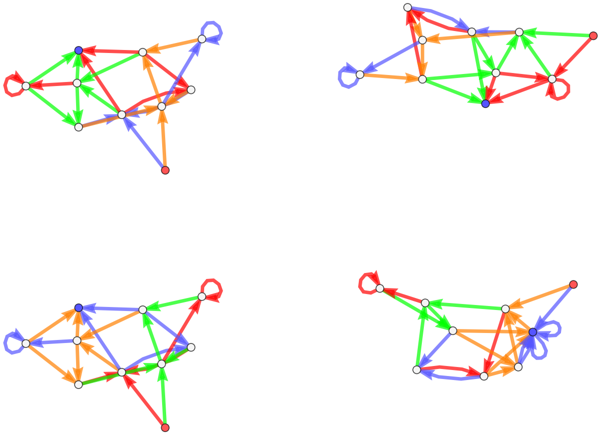


Again, the symmetry is a consequence of algebra.

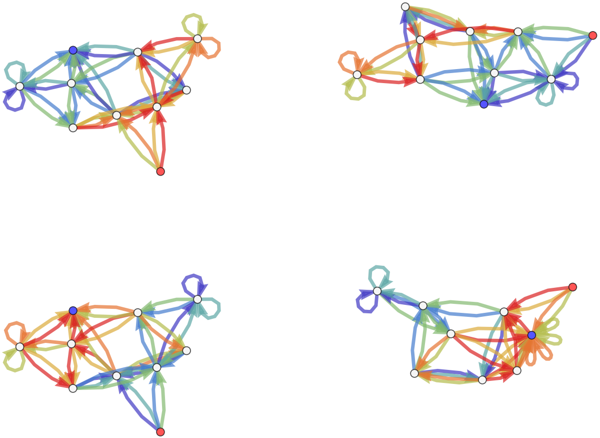


ECA 150 fails to be injective on a lattice of length $n$ iff $n = 2 \pmod 3$.

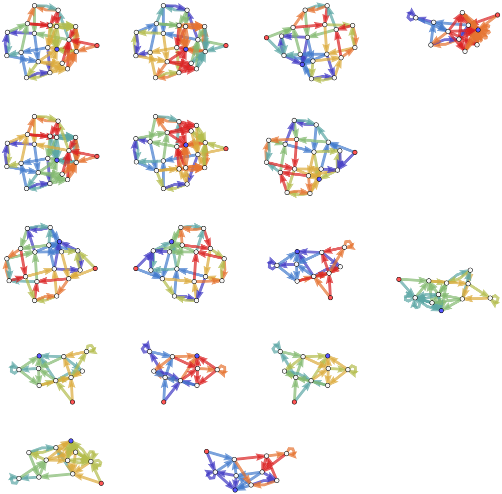
This time we are getting $n = 1$ right, by accident.



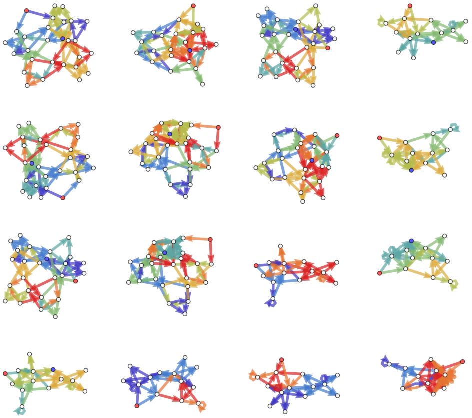
Basic synchronous automaton $\mathcal{A}_{\rightarrow}$ over $\mathbf{2} \times \mathbf{2}$ checking one step in ECA 30 with fixed boundary conditions.



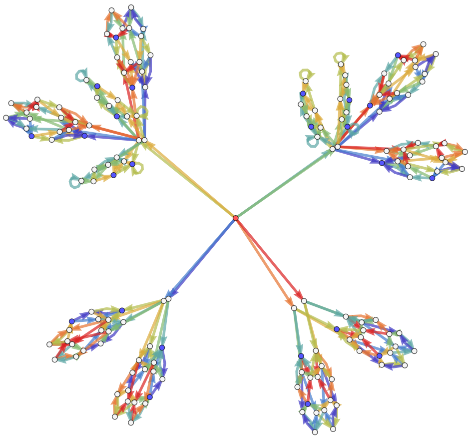
$\mathcal{A}_{\rightarrow,1,3}$ using tracks 1 and 3 to check $x \rightarrow z$.



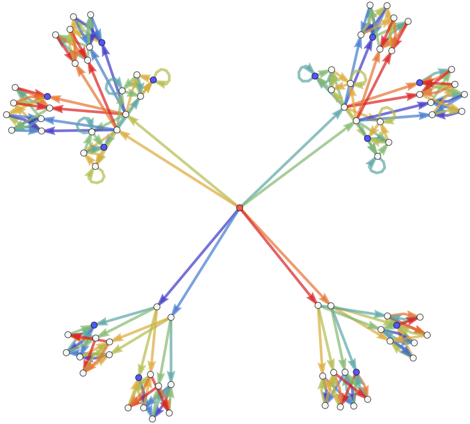
$\mathcal{A}_{\rightarrow,1,3} \times \mathcal{A}_{\rightarrow,2,3}$ checking $x \rightarrow z \wedge y \rightarrow z$.



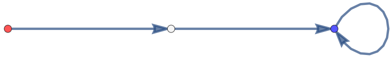
$\mathcal{A}_{\rightarrow,1,3} \times \mathcal{A}_{\rightarrow,2,3} \times \mathsf{UE}_{1,2}$ checking $x \rightarrow z \wedge y \rightarrow z \wedge x \neq y$.



Determinization forces a single initial state.



Minimization rearranges things a little.



ECA 90 with cyclic boundary conditions is never injective.

Alas, for $n = 1$ our construction fails.