

CDM

Safra's Algorithm

KLAUS SUTNER

CARNEGIE MELLON UNIVERSITY

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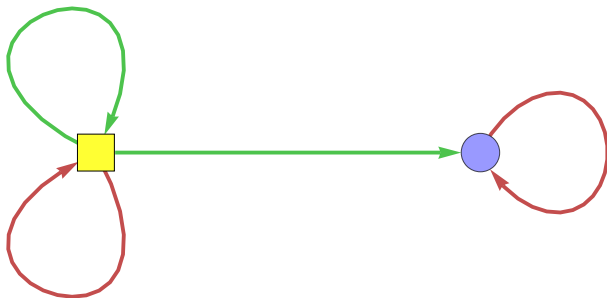


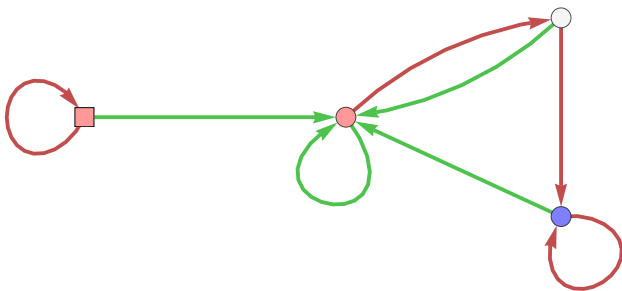
1 **Reasonable Examples**

2 **Blow-Up**

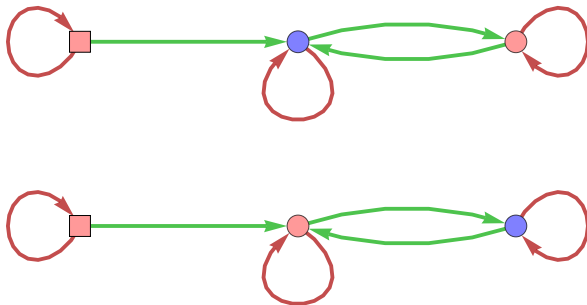
$$0 < \#_b < \infty$$

2

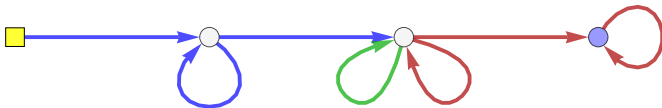




Note how an input a resets the machine. Branch first, 1 Rabin pair.

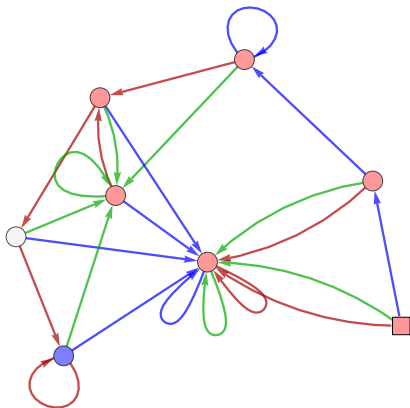


Transitions first, 2 Rabin pairs.



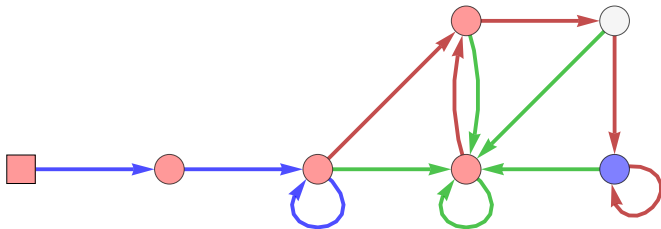
$$L = ccc^*(a + b)^*aa^*$$

This could be managed by hand ...



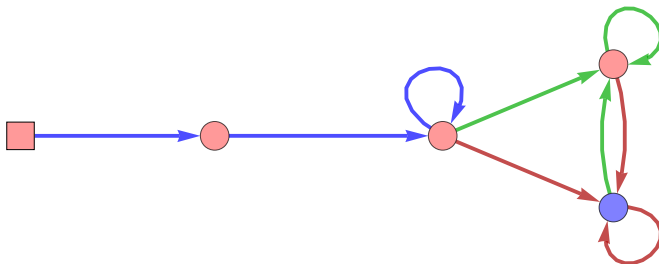
Branching first, 1 Rabin pair.

Note the accepting trap.



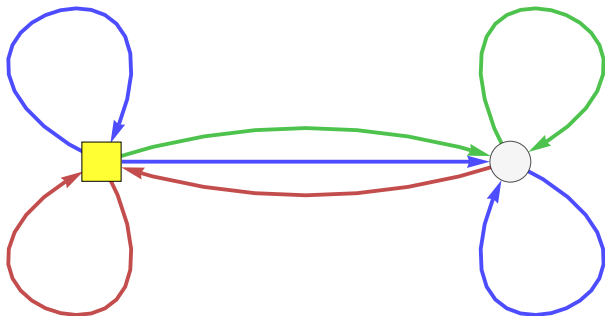
Without the sink.

Input b resets the right part of the automaton.

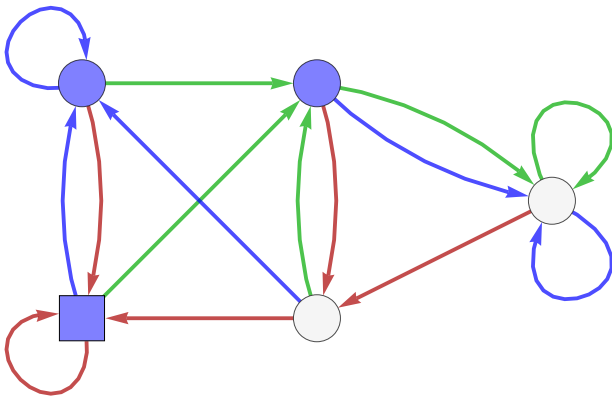


Transitions first, 1 Rabin pair. Clearly better.

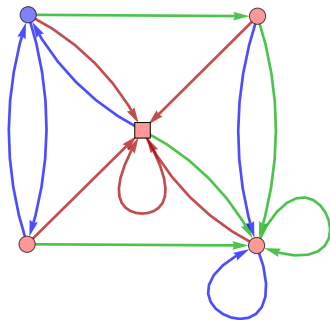
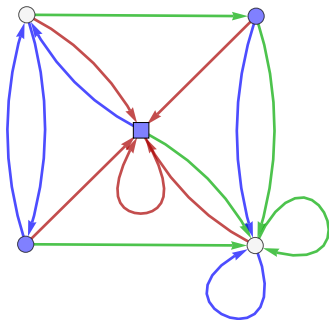
Figure out how to construct this machine from the previous one by state-merging.



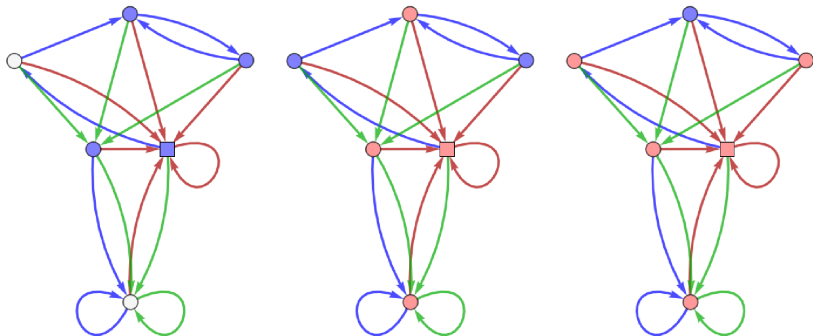
A surprisingly messy example from the Pin/Perrin book.



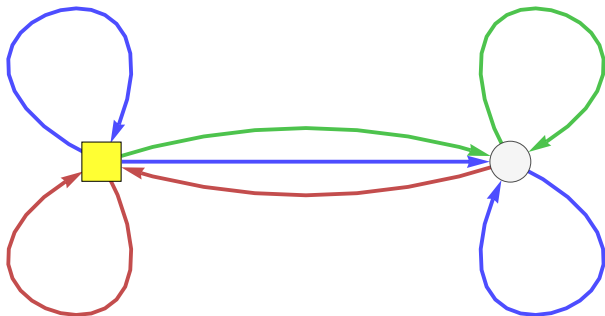
Branching first.



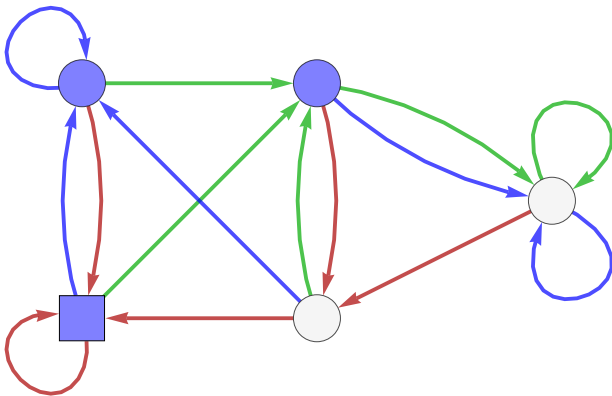
Transitions first. 2 Rabin pairs.



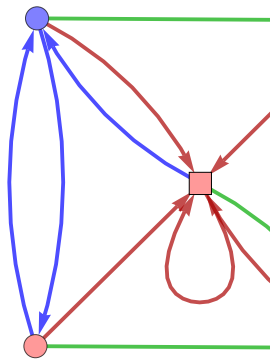
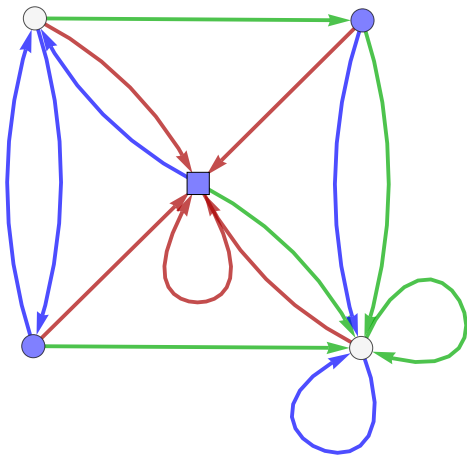
A different algorithm due to Muller and Schupp.



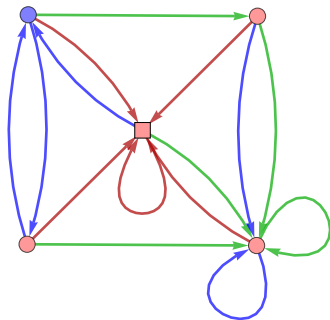
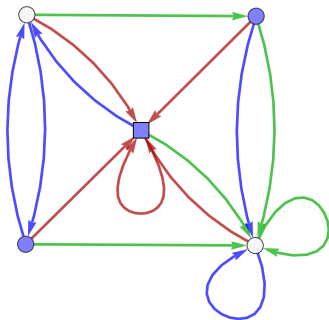
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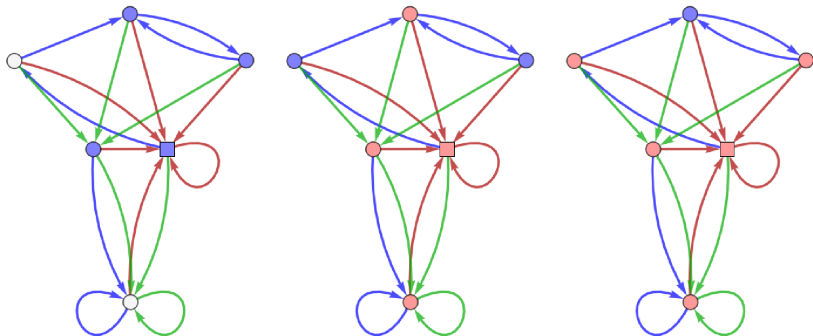
Branching first.



Transitions first.



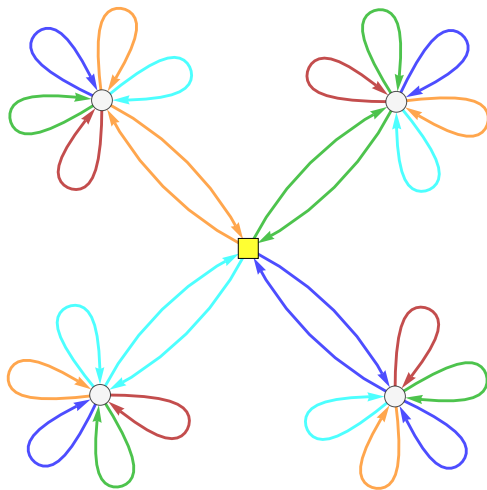
Transitions first. 2 Rabin pairs.



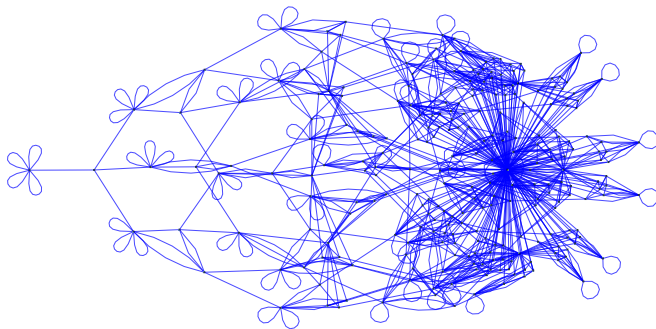
A different algorithm due to Muller and Schupp.

1 Reasonable Examples

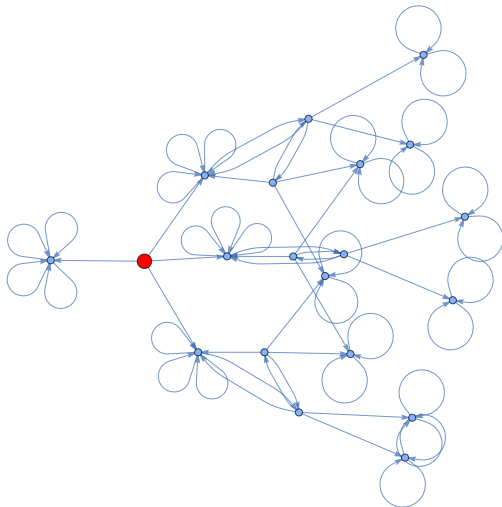
2 **Blow-Up**

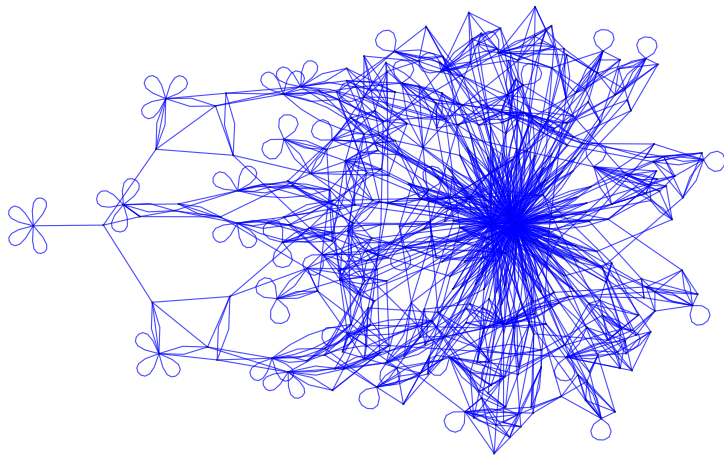


The Michel "infinite path" automaton for $k = 4$.

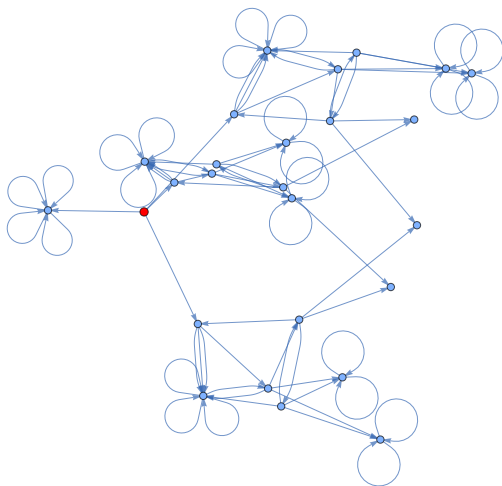


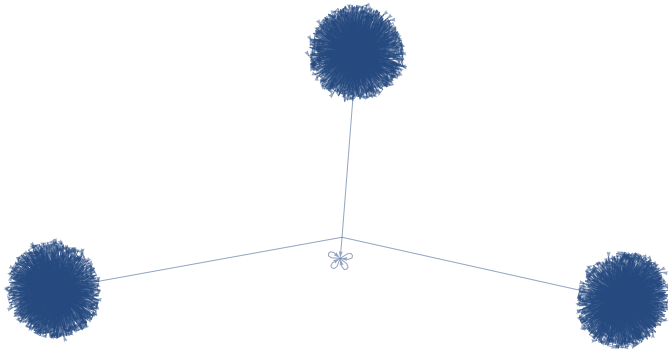
257 states—from a 4-state machine!





Even worse, this time there are 385 states.





Muller-Schupp produces a machine with a whopping 13,907 states.
The picture is the 2-neighborhood of state 1, containing 1919 states.

$n = 3$	385	<1 sec	branch first
	257	<1 sec	trans first
	13,907	20 sec	Muller-Schupp
$n = 4$	13,349	19 sec	branch first
	10,369	17 sec	trans first

This is a straightforward implementation in Mathematica and should be taken with a grain of salt.