Putting CART on the CM

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Advanced Decision Systems

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Roadmap

- What is CART?
- CART performance
- General lessons about programming the CM
- Details of CART on the CM

Concept Induction With CART - 4, 5

Levels of parallelism in CART

- Multiple datasets (not implemented)
- Growing multiple trees at the same time
- Breadth first search
- Evaluation of nodes
- Exploits parallelism based on the size and complexity of the dataset and the number of trees.

CART capabilities

- Implemented
  - Catagorical and numeric features
  - Missing values
  - Assignment of priors to classes
  - Cross validation
  - Feature rankings
- Unimplemented
  - Connection to Sun user interface
  - Bootstap error estimation
  - Incremental learning
  - Linear and boolean combinations of features

CART algorithm

- Read and format data from disk
- Allocate pvars and copy the dataset into the CM
- Initialize the root of each tree
- Basic loop
  - Compute summaries
  - Compute impurity
  - Update nodes
  - Find primary splits
  - Split summaries
  - Mark examples
  - Find surrogate splits
  - Record the splits
  - Split the summaries
  - Split the examples
- Find optimal tree

CART Results

CART-318 CM monitoring

Function	Percent	Calls	Seconds
COMPUTE-SUMMARIES:	0.24	237	.297073
SPLIT-EXAMPLES:	0.14	113	.354944
FIND-SURROGATES:	0.12	113	0.31167
MARK-EXAMPLES:	0.12	113	.310254
ALLOCATE-SUMMARIES!!:	0.11	1	32.1899
RECORD-SPLITS:	0.07	113	.189723
COMPUTE-IMPURITIES:	0.06	237	0.07087
PRIMARY-SPLITS:	0.05	113	.130962
READ-FILE:	0.03	1	7.6799
SETUP-EXAMPLES:	0.02	1	6.0699
UPDATE-NODES:	0.01	237	.010744
ALLOCATE-DATASET!!:	0.01	1	2.4199
SPLIT-SUMMARIES:	0.01	113	.016095
OPTIMAL-TREE:	0.00	1	0.8999
INIT-ROOTS:	0.00	1	0.1499

CART-318 SUN monitoring

Function	Percent	Calls	Seconds
COMPUTE-SUMMARIES:	0.31		
PRIMARY-SPLITS			
SPLIT-SUMMARIES			
MARK-EXAMPLES			
SPLIT-EXAMPLES	0.31		
COMPUTE-IMPURITIES:	0.15		
GET-MAX-DELTA:	0.31	180	32.3702
NUMERICAL-UPDATE:	0.21	168872	.023394
FIND-SURROGATES:	0.14	78	33.2818
GINI-COMPUTED-PRIORS:	0.09	166592	.009973
IMPURITY-FUNC:	0.06	152413	.007817
MEMBER:	0.06	418483	.002661
SORT:	0.04	7904	.083289

CM Instructions

- Static instructions: 28 scan!!, 11 pref!!, 9 *pset, 3 rank!!
- Dynamic instruction usage:

Function	Percent	Calls	Seconds
CM:DEALLOCATE-UPTO-STACK-FIELD:	0.13	216739	0.00133
CM:SEND-WITH-S-ADD-1L:	0.12	6220	.040315
CM:ALLOCATE-STACK-FIELD:	0.09	391818	.000485
CM:DEALLOCATE-STACK-THROUGH:	0.08	317298	.000571
CM:SEND-WITH-OVERWRITE-1L:	0.04	2486	0.03498
CM:MOVE-ALWAYS:	0.04	132508	.000572
CM:SET-VP-SET:	0.03	149818	.000497
CM:SCAN-WITH-F-ADD-1L:	0.03	1422	.046764
CM:UNSIGNED-RANK:	0.03	340	.167635
CM:MOVE:	0.03	92120	0.00059
CM:SCAN-WITH-COPY-1L:	0.02	5230	.007996
CM:UNSIGNED-NEW-SIZE:	0.02	71134	.000581
CM:ENUMERATE-AND-COUNT:	0.02	10371	.003907
CM:MOVE-CONSTANT:	0.02	68089	.000583
CM:UNSIGNED-RESIZE-FIELD-ALWAYS:	0.01	5458	.005111
CM:MOVE-CONSTANT-ALWAYS:	0.01	43849	.000552
CM:GET-1L:	0.01	5085	.004093
CM:UNSIGNED-ADD:	0.01	29748	.000674
CM:MY-CUBE-ADDRESS:	0.01	5341	.003195
CM:DEPOSIT-NEWS-COORDINATE-1L:	0.01	6690	.002465
CM:GEOMETRY-SEND-ADDRESS-LENGTH:	0.01	33461	.000457
CM:GLOBAL-LOGIOR:	0.01	27687	0.00055
CM:F-EQ-CONSTANT-1L:	0.01	4046	.003756
CM:ENUMERATE:	0.01	3946	.003344
CM:S-RESIZE-FIELD-ALWAYS:	0.01	2679	.004921

Memory usage

```
*Lisp system memory utilization

Vp Set *DEFAULT-VP-SET*, (16384)

  Stack memory usage   : 0
  Heap memory usage    : 792
  *Defvar memory usage : 0
  Overhead             : 16
  Total for Vp Set     : 808

Unnamed Vp Set, (8192), #1=#<VP-SET Name: NIL, Dimensions (8192), Geometry-id: 4>
  Stack memory usage   : 0
  Heap memory usage    : 378
  *Defvar memory usage : 0
  Overhead             : 8
  Total for Vp Set     : 386

Overall totals

Stack memory usage   : 0
Heap memory usage    : 1170
*Defvar memory usage : 0
Overhead             : 32
Total                : 1202
```

Not enough processors!

- A crucial consideration is how to balance the computation and data structures between the front-end and the CM
- The CM is SIMD, but you can do reasonable MIMD simulation if you are careful
- Forced to make a decision between speed now and speed later
- Duplicate dataset for each tree (unimplemented)
- Duplicate catagorical features for offsets (unimplemented)
- Grow all trees breadth first in parallel (implemented)
- Current implementation would be much faster with more processors because more nodes could be processed in parallel

*LISP simulator

- Generic CL implementation that is reasonable for testing
- Does not know about types, so types need to be conditionalized
- Can help find assumed value bugs
- Does not help to write compiled code

*LISP Compiler

- Still has lots of bugs (10 over the summer)
- Painful to use because of exceptions
- Can speed up the code (6 times)
- Very, very slow at compiling

Grid versus cube addresses

- The grid address is the address that the programmer works with
- The cube address is the address that the machine works with
- Must be very careful to know which you are dealing with when you have a 1d processor arrangement

Typical development cycle

- Develop new code on simulator using interpreter
- Try out code on CM using interpreter and check cube/grid bugs
- Use simulator to check value reference bugs (an error indicating that something is too big -¿ that you may have such an assumption)
- Get code to compile:
  - Compile code using compiler (don't need to be attached)
  - Add declarations to code that didn't compile
  - Repeat
  - Compile code in simulator
- Performance monitoring
- Test performance tradeoffs
- Repeat

Scan!! is your friend

- A very useful and common instruction
- Generate a sequence of <= values
- Generate totals in designated processors (usually first)
- Propagate values from designated processor through segments
- Find the value in the next selected processor (scan!! pvar!! 'copy!! :include-self nil :direction :backward :dimension 0)

Making it fast

- Getting data out is > 12 times faster than getting it in
- Use the compiler if you can...
- Precompute all you can
  - Flags at the start and end of segments
  - Pointers to the start of segments
  - Address calculations
  - Anything that is not constant in processors
- If you are looping, you could probably do it in parallel by using more processors

*LISP programming style

- Encourages FORTRAN like programming style
- Lots of global variables = global resources
- Makes type propagation a little easier
- If I did it again, I would not use this style
- Have to understand that since *LISP is not integrated with GC, temporary allocations must be resolved
- *DEFUN is a macro that allows the compiler to propagate type information and do deallocation
- A normal defun is fine as long as all references are resolved

Processor allocation

- Dataset = (next-cm-size (* number-of-features number-of-examples))
- Summary-block-size = number-of-unique-feature-values number-of-classes
- Number of summary blocks:
  - Depth-first = (next-cm-size summary-block-size)
  - Processors = (next-cm-size processors)
  - Otherwise start at (next-cm-size summary-block-size) and grow
- Depth-first is the fastest because virtual processors and allocating more processors take too long

Dataset

Example	Class	N1	N2	C1	C2
0	A	***	2.5	D	Y
1	B	1.5	2.0	E	N
2	C	1.0	2.5	***	Y
3	B	1.0	2.0	E	N
4	A	1.0	2.0	F	Y

Example	Class	N1	N2	C1	C2
0	A	***	2.5	D	Y
2	C	1.0	2.5	***	Y
3	B	1.0	2.0	E	N

Examples initial state

```
Examples 0-3 Layout
           Pvar:    0   1   2   3   4   5   6   7   8   9  10  11
    EXAMPLE-P!!:    T   T   T   T   T   T   T   T   T   T   T   T
        START!!:    0   0   0   0   4   4   4   4   8   8   8   8
      START-P!!:    T NIL NIL NIL   T NIL NIL NIL   T NIL NIL NIL
      FEATURE!!:    0   1   2   3   0   1   2   3   0   1   2   3
        CLASS!!:    0   0   0   0   1   1   1   1   2   2   2   2
     NUMBER-P!!:  NIL   T NIL NIL   T   T NIL NIL   T   T NIL NIL
       NUMBER!!:  *** 2.5 *** *** 1.5 2.0 *** *** 1.0 2.5 *** ***
        SET-P!!:  NIL NIL   T   T NIL NIL   T   T NIL NIL NIL   T
          SET!!:    0   0   1   1   0   0   2   2   0   0   0   1
    MISSING-P!!:    T NIL NIL NIL NIL NIL NIL NIL NIL NIL   T NIL
     TREE-P-0!!:    T   T   T   T   T   T   T   T   T   T   T   T
  POINTER-P-0!!:    T   T   T   T   T   T   T   T   T   T   T   T
      POINTER-0:    0   0   0   0   0   0   0   0   0   0   0   0
     LEFT-P-0!!:  NIL NIL NIL NIL NIL NIL NIL NIL NIL NIL NIL NIL
     TREE-P-1!!:    T   T   T   T NIL NIL NIL NIL   T   T   T   T
  POINTER-P-1!!:    T   T   T   T   T   T   T   T   T   T   T   T
      POINTER-1:   36  36  36  36  36  36  36  36  36  36  36  36
     LEFT-P-1!!:  NIL NIL NIL NIL NIL NIL NIL NIL NIL NIL NIL NIL
     TREE-P-2!!:  NIL NIL NIL NIL   T   T   T   T NIL NIL NIL NIL
  POINTER-P-2!!:    T   T   T   T   T   T   T   T   T   T   T   T
      POINTER-2:   72  72  72  72  72  72  72  72  72  72  72  72
     LEFT-P-2!!:  NIL NIL NIL NIL NIL NIL NIL NIL NIL NIL NIL NIL
   OFFSET-P-0!!:  NIL   T   T   T   T   T   T   T   T   T NIL   T
     OFFSET-0!!:    0   9  12  30   4   7  22  34   2  11   0  32
BASE-OFFSET-0!!:    0   9  12  30   3   6  21  33   0   9   0  30
   OFFSET-P-1!!:  NIL NIL   T NIL NIL NIL   T NIL NIL NIL NIL NIL
     OFFSET-1!!:    0   0  15   0   0   0  16   0   0   0   0   0
BASE-OFFSET-1!!:    0   0  15   0   0   0  15   0   0   0   0   0
   OFFSET-P-1!!:  NIL NIL   T NIL NIL NIL   T NIL NIL NIL NIL NIL
     OFFSET-2!!:    0   0  18   0   0   0  25   0   0   0   0   0
BASE-OFFSET-2!!:    0   0  18   0   0   0  24   0   0   0   0   0
```

Initial summary

```
Summary 72 Layout
                    Pvar:   72  73  74  75  76  77  78  79  80  81  82  83  84
                         :   85  86  87  88  89  90  91  92  93  94  95  96  97
                         :   98  99 100 101 102 103 104 105 106 107
      SUMMARY-ACTIVE-P!!:    T   T   T   T   T   T   T   T   T   T   T   T   T
                         :    T   T   T   T   T   T   T   T   T   T   T   T   T
                         :    T   T   T   T   T   T   T   T   T   T
        SUMMARY-START!!:  104 104 104 104 104 104 104 104 104 104 104 104 104
                         :  104 104 104 104 104 104 104 104 104 104 104 104 104
                         :  104 104 104 104 104 104 104 104 104 104
       SUMMARY-START-P!!:    T   T   T NIL NIL NIL NIL NIL NIL NIL NIL NIL NIL
                         :  NIL NIL NIL NIL NIL NIL NIL NIL NIL NIL NIL NIL NIL
                         :  NIL NIL NIL NIL NIL NIL NIL NIL NIL NIL
        SUMMARY-END-P!!:  NIL NIL NIL NIL NIL NIL NIL NIL NIL NIL NIL NIL NIL
                         :  NIL NIL NIL NIL NIL NIL NIL NIL NIL NIL NIL NIL NIL
                         :  NIL NIL NIL NIL NIL NIL NIL   T   T   T
      SUMMARY-EXAMPLES!!:    0   0   0   0   0   0   0   0   0   0   0   0   0
                         :    0   0   0   0   0   0   0   0   0   0   0   0   0
                         :    0   0   0   0   0   0   0   0   0   0
  CHILDREN-LEFT!!-RESULT:    0   0   0   0   0   0   0   0   0   0   0   0   0
                         :    0   0   0   0   0   0   0   0   0   0   0   0   0
                         :    0   0   0   0   0   0   0   0   0   0
CHILDREN-RIGHT!!-RESULT:    0   0   0   0   0   0   0   0   0   0   0   0   0
                         :    0   0   0   0   0   0   0   0   0   0   0   0   0
                         :    0   0   0   0   0   0   0   0   0   0
       SUMMARY-SPLIT-P!!:  NIL NIL NIL NIL NIL NIL NIL NIL NIL NIL NIL NIL NIL
                         :  NIL NIL NIL NIL NIL NIL NIL NIL NIL NIL NIL NIL NIL
                         :  NIL NIL NIL NIL NIL NIL NIL NIL NIL NIL
      SUMMARY-IMPURITY!!:  0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0
                         :  0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0
                         :  0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0
      FEATURE-NUMBER-P!!:    T   T   T   T   T   T   T   T   T   T   T   T NIL
                         :  NIL NIL NIL NIL NIL NIL NIL NIL NIL NIL NIL NIL NIL
                         :  NIL NIL NIL NIL NIL NIL NIL NIL NIL NIL
```

```
      FEATURE-START!!:  104 104 104 104 104 104 110 110 110 110 110 110 116
                     :  116 116 116 116 116 116 116 116 116 116 116 116 116
                     :  116 116 116 116  86  86  86  86  86  86
       FEATURE-START-P!!:   T   T   T NIL NIL NIL   T   T   T NIL NIL NIL   T
                     :    T   T NIL NIL NIL NIL NIL NIL NIL NIL NIL NIL NIL
                     :  NIL NIL NIL NIL   T   T   T NIL NIL NIL
         FEATURE-END-P!!:  NIL NIL NIL   T   T   T NIL NIL NIL   T   T   T NIL
                     :  NIL NIL NIL NIL NIL NIL NIL NIL NIL NIL NIL NIL NIL
                     :  NIL   T   T   T NIL NIL NIL   T   T   T
      FEATURE-SET-BITS!!:    0   0   0   0   0   0   0   0   0   0   0   0   3
                     :    3   3   3   3   3   3   3   3   3   3   3   3   3
                     :    3   3   3   3   1   1   1   1   1   1
      FEATURE-EXAMPLES!!:    0   0   0   0   0   0   0   0   0   0   0   0   0
                     :    0   0   0   0   0   0   0   0   0   0   0   0   0
                     :    0   0   0   0   0   0   0   0   0   0
         FEATURE-ERROR!!:    0   0   0   0   0   0   0   0   0   0   0   0   0
                     :    0   0   0   0   0   0   0   0   0   0   0   0   0
                     :    0   0   0   0   0   0   0   0   0   0
       FEATURE-SPLIT-P!!:  NIL NIL NIL NIL NIL NIL NIL NIL NIL NIL NIL NIL NIL
                     :  NIL NIL NIL NIL NIL NIL NIL NIL NIL NIL NIL NIL NIL
                     :  NIL NIL NIL NIL NIL NIL NIL NIL NIL NIL
  SPLIT-ADDRESS!!-RESULT:  104 105 106 107 108 109 110 111 112 113 114 115 116
                     :  117 118 119 120 121 122 123 124 125 126 127  80  81
                     :   82  83  84  85  86  87  88  89  90  91
   SPLIT-MARKER!!-RESULT:  NIL NIL NIL NIL NIL NIL NIL NIL NIL NIL NIL NIL NIL
                     :  NIL NIL NIL NIL NIL NIL NIL NIL NIL NIL NIL NIL NIL
                     :  NIL NIL NIL NIL NIL NIL NIL NIL NIL NIL
         FEATURE-RANK!!:    0   0   0   0   0   0   0   0   0   0   0   0   0
                     :    0   0   0   0   0   0   0   0   0   0   0   0   0
                     :    0   0   0   0   0   0   0   0   0   0
       VALUE-START-P!!:    T NIL NIL   T NIL NIL   T NIL NIL   T NIL NIL   T
                     :  NIL NIL   T NIL NIL   T NIL NIL   T NIL NIL   T NIL
                     :  NIL   T NIL NIL   T NIL NIL   T NIL NIL
         VALUE-END-P!!:  NIL NIL   T NIL NIL   T NIL NIL   T NIL NIL   T NIL
                     :  NIL   T NIL NIL   T NIL NIL   T NIL NIL   T NIL NIL
                     :    T NIL NIL   T NIL NIL   T NIL NIL   T
       VALUE-EMPTY-P!!:  NIL NIL NIL NIL NIL NIL NIL NIL NIL NIL NIL NIL NIL
                     :  NIL NIL NIL NIL NIL NIL NIL NIL NIL NIL NIL NIL NIL
                     :  NIL NIL NIL NIL NIL NIL NIL NIL NIL NIL
    VALUE-NO-SPLIT-P!!:  NIL NIL NIL NIL NIL NIL NIL NIL NIL NIL NIL NIL NIL
```

:	NIL	NIL	NIL	NIL	NIL	NIL	NIL	NIL	NIL	NIL	NIL	NIL	NIL	NIL
:	NIL	NIL	NIL	NIL	NIL	NIL	NIL	NIL	NIL	NIL				
VALUE-SPLIT!!:	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
:	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
:	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0				
VALUE-NUMBER!!:	1.0	1.0	1.0	1.5	1.5	1.5	2.0	2.0	2.0	2.5	2.5	2.5	0.0	
:	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	
:	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0				
VALUE-SET!!:	0	0	0	0	0	0	0	0	0	0	0	0	1	
:	1	1	3	3	3	5	5	5	2	2	2	6	6	
:	6	4	4	4	1	1	1	2	2	2				
VALUE-LEFT!!:	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	
:	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	
:	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0				
VALUE-RIGHT!!:	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	
:	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	
:	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0				
VALUE-DELTA!!:	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	
:	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	
:	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0				
VALUE-DIRECTION!!:	0	0	0	0	0	0	0	0	0	0	0	0	0	
:	0	0	0	0	0	0	0	0	0	0	0	0	0	
:	0	0	0	0	0	0	0	0	0	0				
VALUE-MIRROR-P!!:	NIL	NIL	NIL	NIL	NIL	NIL	NIL	NIL	NIL	NIL	NIL	NIL	NIL	NIL
:	NIL	NIL	NIL	NIL	NIL	NIL	NIL	NIL	NIL	NIL	NIL	NIL	NIL	NIL
:	NIL	NIL	NIL	NIL	NIL	NIL	NIL	NIL	NIL	NIL				
CLASS-CLASS!!:	0	1	2	0	1	2	0	1	2	0	1	2	0	
:	1	2	0	1	2	0	1	2	0	1	2	0	1	
:	2	0	1	2	0	1	2	0	1	2				
CLASS-PRIOR!!:	0.4	.05	0.1	0.4	.05	0.1	0.4	.05	0.1	0.4	.05	0.1	0.4	
:	.05	0.1	0.4	.05	0.1	0.4	.05	0.1	0.4	.05	0.1	0.4	.05	
:	0.1	0.4	.05	0.1	0.4	.05	0.1	0.4	.05	0.1				
CLASS-COUNT!!:	0	0	0	0	0	0	0	0	0	0	0	0	0	
:	0	0	0	0	0	0	0	0	0	0	0	0	0	
:	0	0	0	0	0	0	0	0	0	0				
CLASS-<=!!:	0	0	0	0	0	0	0	0	0	0	0	0	0	
:	0	0	0	0	0	0	0	0	0	0	0	0	0	
:	0	0	0	0	0	0	0	0	0	0				
CLASS->!!:	0	0	0	0	0	0	0	0	0	0	0	0	0	
:	0	0	0	0	0	0	0	0	0	0	0	0	0	

: 0 0 0 0 0 0 0 0 0 0

After primary split

```
Summary 72 Layout
                    Pvar:   72  73  74  75  76  77  78  79  80  81  82  83  84
                         :   85  86  87  88  89  90  91  92  93  94  95  96  97
                         :   98  99 100 101 102 103 104 105 106 107
      SUMMARY-ACTIVE-P!!:    T   T   T   T   T   T   T   T   T   T   T   T   T
                         :    T   T   T   T   T   T   T   T   T   T   T   T   T
                         :    T   T   T   T   T   T   T   T   T   T
         SUMMARY-START!!:  104 104 104 104 104 104 104 104 104 104 104 104 104
                         :  104 104 104 104 104 104 104 104 104 104 104 104 104
                         :  104 104 104 104 104 104 104 104 104 104
       SUMMARY-START-P!!:    T   T   T NIL NIL NIL NIL NIL NIL NIL NIL NIL NIL
                         :  NIL NIL NIL NIL NIL NIL NIL NIL NIL NIL NIL NIL NIL
                         :  NIL NIL NIL NIL NIL NIL NIL NIL NIL NIL
         SUMMARY-END-P!!:  NIL NIL NIL NIL NIL NIL NIL NIL NIL NIL NIL NIL NIL
                         :  NIL NIL NIL NIL NIL NIL NIL NIL NIL NIL NIL NIL NIL
                         :  NIL NIL NIL NIL NIL NIL NIL   T   T   T
      SUMMARY-EXAMPLES!!:    1   2   0   1   2   0   1   2   0   1   2   0   1
                         :    2   0   1   2   0   1   2   0   1   2   0   1   2
                         :    0   1   2   0   1   2   0   1   2   0
  CHILDREN-LEFT!!-RESULT:   72  72  72  72  72  72  72  72  72  72  72  72  72
                         :   72  72  72  72  72  72  72  72  72  72  72  72  72
                         :   72  72  72  72  72  72  72  72  72  72
CHILDREN-RIGHT!!-RESULT:  108 108 108 108 108 108 108 108 108 108 108 108 108
                         :  108 108 108 108 108 108 108 108 108 108 108 108 108
                         :  108 108 108 108 108 108 108 108 108 108
       SUMMARY-SPLIT-P!!:  NIL NIL NIL NIL NIL NIL NIL NIL NIL NIL NIL NIL NIL
                         :  NIL NIL NIL NIL NIL NIL NIL NIL NIL NIL NIL NIL NIL
                         :  NIL NIL NIL NIL NIL NIL NIL NIL NIL NIL
      SUMMARY-IMPURITY!!:  .16 .48 0.5 .16 .48 0.5 .16 .48 0.5 .16 .48 0.5 .16
                         :  .48 0.5 .16 .48 0.5 .16 .48 0.5 .16 .48 0.5 .16 .48
                         :  0.5 .16 .48 0.5 .16 .48 0.5 .16 .48 0.5
      FEATURE-NUMBER-P!!:    T   T   T   T   T   T   T   T   T   T   T   T NIL
                         :  NIL NIL NIL NIL NIL NIL NIL NIL NIL NIL NIL NIL NIL
                         :  NIL NIL NIL NIL NIL NIL NIL NIL NIL NIL
```

```
      FEATURE-START!!:  104 104 104 104 104 104 110 110 110 110 110 110 116
                     :  116 116 116 116 116 116 116 116 116 116 116 116 116
                     :  116 116 116 116  86  86  86  86  86  86
       FEATURE-START-P!!:   T   T   T NIL NIL NIL   T   T   T NIL NIL NIL   T
                     :    T   T NIL NIL NIL NIL NIL NIL NIL NIL NIL NIL NIL
                     :  NIL NIL NIL NIL   T   T   T NIL NIL NIL
         FEATURE-END-P!!:  NIL NIL NIL   T   T   T NIL NIL NIL   T   T   T NIL
                     :  NIL NIL NIL NIL NIL NIL NIL NIL NIL NIL NIL NIL NIL
                     :  NIL   T   T   T NIL NIL NIL   T   T   T
      FEATURE-SET-BITS!!:    0   0   0   0   0   0   0   0   0   0   0   0   3
                     :    3   3   3   3   3   3   3   3   3   3   3   3   3
                     :    3   3   3   3   1   1   1   1   1   1
      FEATURE-EXAMPLES!!:    1   2   0   1   2   0   1   2   0   1   2   0   1
                     :    2   0   1   2   0   1   2   0   1   2   0   1   2
                     :    0   1   2   0   1   2   0   1   2   0
         FEATURE-ERROR!!:    0   0   1   0   0   1   1   0   1   1   0   1   1
                     :    0   0   1   0   0   1   0   0   1   0   0   1   0
                     :    0   1   0   0   1   0   1   1   0   1
       FEATURE-SPLIT-P!!:  NIL NIL NIL NIL NIL NIL NIL NIL NIL NIL NIL NIL NIL
                     :  NIL NIL NIL NIL NIL NIL NIL NIL   T NIL NIL NIL NIL
                     :  NIL NIL NIL NIL NIL NIL NIL NIL NIL NIL
  SPLIT-ADDRESS!!-RESULT:  104 104 104 104 104 104 110 110 110 110 110 110 125
                     :  125 125 125 125 125 125 125 125 125 125 125 125 125
                     :  125 125 125 125  86  86  86  86  86  86
   SPLIT-MARKER!!-RESULT:  NIL NIL NIL NIL NIL NIL NIL NIL NIL NIL NIL NIL   T
                     :    T   T   T   T   T   T   T   T   T   T   T   T   T
                     :    T   T   T   T NIL NIL NIL NIL NIL NIL
          FEATURE-RANK!!:    0   0   0   0   0   0   0   0   0   0   0   0   0
                     :    0   0   0   0   0   0   0   0   0   0   0   0   0
                     :    0   0   0   0   0   0   0   0   0   0
         VALUE-START-P!!:    T NIL NIL   T NIL NIL   T NIL NIL   T NIL NIL   T
                     :  NIL NIL   T NIL NIL   T NIL NIL   T NIL NIL   T NIL
                     :  NIL   T NIL NIL   T NIL NIL   T NIL NIL
           VALUE-END-P!!:  NIL NIL   T NIL NIL   T NIL NIL   T NIL NIL   T NIL
                     :  NIL   T NIL NIL   T NIL NIL   T NIL NIL   T NIL NIL
                     :    T NIL NIL   T NIL NIL   T NIL NIL   T
         VALUE-EMPTY-P!!:  NIL NIL   T NIL NIL   T NIL NIL   T   T   T   T   T
                     :    T   T NIL NIL   T NIL   T   T NIL NIL   T NIL NIL
                     :    T NIL   T   T NIL   T   T NIL NIL   T
      VALUE-NO-SPLIT-P!!:  NIL   T   T   T   T   T   T   T   T   T   T   T   T
```

:	T	T	NIL	T	T	NIL	T	T	NIL	T	T	T	T	
:	T	NIL	T	T	NIL	T	T	NIL	T	T				
VALUE-SPLIT!!:	1.2	0.0	0.0	1.7	0.0	0.0	1.5	0.0	0.0	0.0	0.0	0.0	0.0	
:	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	
:	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0				
VALUE-NUMBER!!:	1.0	1.0	1.0	1.5	1.5	1.5	2.0	2.0	2.0	2.5	2.5	2.5	0.0	
:	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	
:	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0				
VALUE-SET!!:	0	0	0	0	0	0	0	0	0	0	0	0	1	
:	1	1	3	3	3	5	5	5	2	2	2	6	6	
:	6	4	4	4	1	1	1	2	2	2				
VALUE-LEFT!!:	.09	.44	.45	.16	.48	0.5	.16	.48	0.5	.16	.48	0.5	0.0	
:	0.0	0.0	0.0	0.0	0.1	0.0	0.4	0.4	0.0	0.0	0.1	.16	.48	
:	0.5	0.0	0.4	0.4	0.0	0.4	0.4	0.0	0.0	0.1				
VALUE-RIGHT!!:	0.0	0.0	.05	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	.16	
:	.48	0.5	0.0	0.4	0.4	0.0	0.0	0.1	0.0	0.4	0.4	0.0	0.0	
:	0.0	0.0	0.0	0.1	0.0	0.0	0.1	0.0	0.4	0.4				
VALUE-DELTA!!:	.07	.04	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	
:	0.0	0.0	.16	.08	0.0	.16	0.0	0.0	.16	.08	0.0	0.0	0.0	
:	0.0	.16	0.0	0.0	.16	0.0	0.0	.16	.08	0.0				
VALUE-DIRECTION!!:	0	0	0	0	0	0	0	0	0	0	0	0	108	
:	108	108	72	72	72	108	108	108	72	72	72	72	72	
:	72	108	108	108	0	0	0	0	0	0				
VALUE-MIRROR-P!!:	NIL	NIL	NIL	NIL	NIL	NIL	NIL	NIL	NIL	NIL	NIL	NIL	NIL	
:	NIL	NIL	NIL	NIL	NIL	NIL	NIL	NIL	NIL	NIL	NIL	NIL	NIL	
:	NIL	NIL	NIL	NIL	NIL	NIL	NIL	NIL	NIL	NIL				
CLASS-CLASS!!:	0	1	2	0	1	2	0	1	2	0	1	2	0	
:	1	2	0	1	2	0	1	2	0	1	2	0	1	
:	2	0	1	2	0	1	2	0	1	2				
CLASS-PRIOR!!:	0.4	.05	0.1	0.4	.05	0.1	0.4	.05	0.1	0.4	.05	0.1	0.4	
:	.05	0.1	0.4	.05	0.1	0.4	.05	0.1	0.4	.05	0.1	0.4	.05	
:	0.1	0.4	.05	0.1	0.4	.05	0.1	0.4	.05	0.1				
CLASS-COUNT!!:	1	1	0	0	1	0	1	2	0	0	0	0	0	
:	0	0	0	2	0	1	0	0	0	2	0	1	2	
:	0	1	0	0	1	0	0	0	2	0				
CLASS-<=!!:	1	1	0	1	2	0	1	2	0	1	2	0	0	
:	0	0	0	2	0	1	0	0	0	2	0	1	2	
:	0	1	0	0	1	0	0	0	2	0				
CLASS->!!:	0	1	0	0	0	0	0	0	0	0	0	0	1	
:	2	0	1	0	0	0	2	0	1	0	0	0	0	

: 0 0 2 0 0 2 0 1 0 0

After surrogates

```
Summary 72 Layout
                    Pvar:  72  73  74  75  76  77  78  79  80  81  82  83  84
                         :  85  86  87  88  89  90  91  92  93  94  95  96  97
                         :  98  99 100 101 102 103 104 105 106 107
      SUMMARY-ACTIVE-P!!:   T   T   T   T   T   T   T   T   T   T   T   T   T
                         :   T   T   T   T   T   T   T   T   T   T   T   T   T
                         :   T   T   T   T   T   T   T   T   T   T
         SUMMARY-START!!: 104 104 104 104 104 104 104 104 104 104 104 104 104
                         : 104 104 104 104 104 104 104 104 104 104 104 104 104
                         : 104 104 104 104 104 104 104 104 104 104
       SUMMARY-START-P!!:   T   T   T NIL NIL NIL NIL NIL NIL NIL NIL NIL NIL
                         : NIL NIL NIL NIL NIL NIL NIL NIL NIL NIL NIL NIL NIL
                         : NIL NIL NIL NIL NIL NIL NIL NIL NIL NIL
         SUMMARY-END-P!!: NIL NIL NIL NIL NIL NIL NIL NIL NIL NIL NIL NIL NIL
                         : NIL NIL NIL NIL NIL NIL NIL NIL NIL NIL NIL NIL NIL
                         : NIL NIL NIL NIL NIL NIL NIL   T   T   T
      SUMMARY-EXAMPLES!!:   1   2   0   1   2   0   1   2   0   1   2   0   1
                         :   2   0   1   2   0   1   2   0   1   2   0   1   2
                         :   0   1   2   0   1   2   0   1   2   0
  CHILDREN-LEFT!!-RESULT: 108 108 108 108 108 108  72  72  72  72  72  72 108
                         : 108 108 108 108 108 108 108 108 108 108 108 108 108
                         : 108 108 108 108 108 108 108 108 108 108
CHILDREN-RIGHT!!-RESULT:  72  72  72  72  72  72 108 108 108 108 108 108  72
                         :  72  72  72  72  72  72  72  72  72  72  72  72  72
                         :  72  72  72  72  72  72  72  72  72  72
       SUMMARY-SPLIT-P!!: NIL NIL NIL NIL NIL NIL NIL NIL NIL NIL NIL NIL NIL
                         : NIL NIL NIL NIL NIL NIL NIL NIL NIL NIL NIL NIL NIL
                         : NIL NIL NIL NIL NIL NIL NIL NIL NIL NIL
      SUMMARY-IMPURITY!!: .16 .48 0.5 .16 .48 0.5 .16 .48 0.5 .16 .48 0.5 .16
                         : .48 0.5 .16 .48 0.5 .16 .48 0.5 .16 .48 0.5 .16 .48
                         : 0.5 .16 .48 0.5 .16 .48 0.5 .16 .48 0.5
      FEATURE-NUMBER-P!!:   T   T   T   T   T   T   T   T   T   T   T   T NIL
                         : NIL NIL NIL NIL NIL NIL NIL NIL NIL NIL NIL NIL NIL
                         : NIL NIL NIL NIL NIL NIL NIL NIL NIL NIL
```

```
      FEATURE-START!!:  104 104 104 104 104 104 110 110 110 110 110 110 116
                      :  116 116 116 116 116 116 116 116 116 116 116 116 116
                      :  116 116 116 116  86  86  86  86  86  86
       FEATURE-START-P!!:   T   T   T NIL NIL NIL   T   T   T NIL NIL NIL   T
                      :    T   T NIL NIL NIL NIL NIL NIL NIL NIL NIL NIL NIL
                      :  NIL NIL NIL NIL   T   T   T NIL NIL NIL
         FEATURE-END-P!!:  NIL NIL NIL   T   T   T NIL NIL NIL   T   T   T NIL
                      :  NIL NIL NIL NIL NIL NIL NIL NIL NIL NIL NIL NIL NIL
                      :  NIL   T   T   T NIL NIL NIL   T   T   T
      FEATURE-SET-BITS!!:    0   0   0   0   0   0   0   0   0   0   0   0   3
                      :    3   3   3   3   3   3   3   3   3   3   3   3   3
                      :    3   3   3   3   1   1   1   1   1   1
      FEATURE-EXAMPLES!!:    1   2   0   1   2   0   1   2   0   1   2   0   1
                      :    2   0   1   2   0   1   2   0   1   2   0   1   2
                      :    0   1   2   0   1   2   0   1   2   0
         FEATURE-ERROR!!:    0   0   1   0   0   1   1   0   1   1   0   1   1
                      :    0   0   1   0   0   1   0   0   1   0   0   1   0
                      :    0   1   0   0   1   0   1   1   0   1
       FEATURE-SPLIT-P!!:    T NIL NIL NIL NIL NIL   T NIL NIL NIL NIL NIL NIL
                      :  NIL NIL NIL NIL NIL NIL NIL NIL NIL NIL NIL NIL NIL
                      :  NIL   T NIL NIL   T NIL NIL NIL NIL NIL
  SPLIT-ADDRESS!!-RESULT:  104 104 104 104 104 104 110 110 110 110 110 110  83
                      :   83  83  83  83  83  83  83  83  83  83  83  83  83
                      :   83  83  83  83  86  86  86  86  86  86
   SPLIT-MARKER!!-RESULT:    T   T   T   T   T   T   T   T   T   T   T   T   T
                      :    T   T   T   T   T   T   T   T   T   T   T   T   T
                      :    T   T   T   T   T   T   T   T   T   T
         FEATURE-RANK!!:    2   2   2   2   2   2   5   5   5   5   5   5   0
                      :    0   0   0   0   0   0   0   0   0   0   0   0   0
                      :    0   0   0   0   1   1   1   1   1   1
       VALUE-START-P!!:    T NIL NIL   T NIL NIL   T NIL NIL   T NIL NIL   T
                      :  NIL NIL   T NIL NIL   T NIL NIL   T NIL NIL   T NIL
                      :  NIL   T NIL NIL   T NIL NIL   T NIL NIL
         VALUE-END-P!!:  NIL NIL   T NIL NIL   T NIL NIL   T NIL NIL   T NIL
                      :  NIL   T NIL NIL   T NIL NIL   T NIL NIL   T NIL NIL
                      :    T NIL NIL   T NIL NIL   T NIL NIL   T
       VALUE-EMPTY-P!!:  NIL NIL   T NIL NIL   T NIL NIL   T   T   T   T   T
                      :    T   T NIL NIL   T NIL   T   T NIL NIL   T NIL NIL
                      :    T NIL   T   T NIL   T   T NIL NIL   T
    VALUE-NO-SPLIT-P!!:  NIL   T   T   T   T   T   T   T   T   T   T   T   T
```

:	T	T	NIL	T	T	NIL	T	T	NIL	T	T	T	T	
:	T	NIL	T	T	NIL	T	T	NIL	T	T				
VALUE-SPLIT!!:	1.2	0.0	0.0	1.7	0.0	0.0	1.5	0.0	0.0	0.0	0.0	0.0	0.0	
:	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	
:	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0				
VALUE-NUMBER!!:	1.0	1.0	1.0	1.5	1.5	1.5	2.0	2.0	2.0	2.5	2.5	2.5	0.0	
:	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	
:	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0				
VALUE-SET!!:	0	0	0	0	0	0	0	0	0	0	0	0	1	
:	1	1	3	3	3	5	5	5	2	2	2	6	6	
:	6	4	4	4	1	1	1	2	2	2				
VALUE-LEFT!!:	.09	.44	.45	.16	.48	0.5	.16	.48	0.5	.16	.48	0.5	0.0	
:	0.0	0.0	0.0	0.0	0.1	0.0	0.4	0.4	0.0	0.0	0.1	.16	.48	
:	0.5	0.0	0.4	0.4	0.0	0.4	0.4	0.0	0.0	0.1				
VALUE-RIGHT!!:	0.0	0.0	.05	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	.16	
:	.48	0.5	0.0	0.4	0.4	0.0	0.0	0.1	0.0	0.4	0.4	0.0	0.0	
:	0.0	0.0	0.0	0.1	0.0	0.0	0.1	0.0	0.4	0.4				
VALUE-DELTA!!:	.07	.04	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	
:	0.0	0.0	.16	.08	0.0	.16	0.0	0.0	.16	.08	0.0	0.0	0.0	
:	0.0	.16	0.0	0.0	.16	0.0	0.0	.16	.08	0.0				
VALUE-DIRECTION!!:	108	108	108	72	72	72	72	72	72	108	108	108	72	
:	72	72	72	72	72	108	108	108	72	72	72	108	108	
:	108	108	108	108	108	108	108	72	72	72				
VALUE-MIRROR-P!!:	T	T	T	T	T	T	NIL	NIL	NIL	NIL	NIL	NIL	T	
:	T	T	T	T	T	T	T	T	T	T	T	T	T	
:	T	T	T	T	T	T	T	T	T	T				
CLASS-CLASS!!:	0	1	2	0	1	2	0	1	2	0	1	2	0	
:	1	2	0	1	2	0	1	2	0	1	2	0	1	
:	2	0	1	2	0	1	2	0	1	2				
CLASS-PRIOR!!:	0.4	.05	0.1	0.4	.05	0.1	0.4	.05	0.1	0.4	.05	0.1	0.4	
:	.05	0.1	0.4	.05	0.1	0.4	.05	0.1	0.4	.05	0.1	0.4	.05	
:	0.1	0.4	.05	0.1	0.4	.05	0.1	0.4	.05	0.1				
CLASS-COUNT!!:	1	1	0	0	1	0	1	2	0	0	0	0	0	
:	0	0	0	2	0	1	0	0	0	2	0	1	2	
:	0	1	0	0	1	0	0	0	2	0				
CLASS-<=!!:	1	1	0	1	2	0	1	2	0	1	2	0	0	
:	0	0	0	2	0	1	0	0	0	2	0	1	2	
:	0	1	0	0	1	0	0	0	2	0				
CLASS->!!:	0	1	0	0	0	0	0	0	0	0	0	0	1	
:	2	0	1	0	0	0	2	0	1	0	0	0	0	

: 0 0 2 0 0 2 0 1 0 0

Examples after splitting

```
Examples 0-3 Layout
           Pvar:    0   1   2   3   4   5   6   7   8   9  10  11
    EXAMPLE-P!!:    T   T   T   T   T   T   T   T   T   T   T   T
        START!!:    0   0   0   0   4   4   4   4   8   8   8   8
      START-P!!:    T NIL NIL NIL   T NIL NIL NIL   T NIL NIL NIL
      FEATURE!!:    0   1   2   3   0   1   2   3   0   1   2   3
        CLASS!!:    0   0   0   0   1   1   1   1   2   2   2   2
     NUMBER-P!!:  NIL   T NIL NIL   T   T NIL NIL   T   T NIL NIL
       NUMBER!!:  *** 2.5 *** *** 1.5 2.0 *** *** 1.0 2.5 *** ***
        SET-P!!:  NIL NIL   T   T NIL NIL   T   T NIL NIL NIL   T
          SET!!:    0   0   1   1   0   0   2   2   0   0   0   1
    MISSING-P!!:    T NIL NIL NIL NIL NIL NIL NIL NIL NIL   T NIL
     TREE-P-0!!:    T   T   T   T   T   T   T   T   T   T   T   T
  POINTER-P-0!!:    T   T   T   T   T   T   T   T   T   T   T   T
      POINTER-0:  180 180 180 180   0   0   0   0 180 180 180 180
     LEFT-P-0!!:  NIL NIL NIL NIL   T   T   T   T NIL NIL NIL NIL
     TREE-P-1!!:    T   T   T   T NIL NIL NIL NIL   T   T   T   T
  POINTER-P-1!!:    T   T   T   T   T   T   T   T   T   T   T   T
      POINTER-1:  144 144 144 144  36  36  36  36 144 144 144 144
     LEFT-P-1!!:  NIL NIL NIL NIL   T   T   T   T NIL NIL NIL NIL
     TREE-P-2!!:  NIL NIL NIL NIL   T   T   T   T NIL NIL NIL NIL
  POINTER-P-2!!:    T   T   T   T   T   T   T   T   T   T   T   T
      POINTER-2:   72  72  72  72  72  72  72  72 108 108 108 108
     LEFT-P-2!!:  NIL NIL NIL NIL   T   T   T   T NIL NIL NIL NIL
   OFFSET-P-0!!:  NIL   T   T   T   T   T   T   T   T   T NIL   T
     OFFSET-0!!:    0   9  12  30   4   7  22  34   2  11   0  32
BASE-OFFSET-0!!:    0   9  12  30   3   6  21  33   0   9   0  30
   OFFSET-P-1!!:  NIL NIL   T NIL NIL NIL   T NIL NIL NIL NIL NIL
     OFFSET-1!!:    0   0  15   0   0   0  16   0   0   0   0   0
BASE-OFFSET-1!!:    0   0  15   0   0   0  15   0   0   0   0   0
   OFFSET-P-1!!:  NIL NIL   T NIL NIL NIL   T NIL NIL NIL NIL NIL
     OFFSET-2!!:    0   0  18   0   0   0  25   0   0   0   0   0
BASE-OFFSET-2!!:    0   0  18   0   0   0  24   0   0   0   0   0
```

Final tree

```
If C1 <- (E)
      C2 <> (Y)
      N2 <= 2.25
      N1 > 1.25 then (B)
else
   If N2 <= 2.25 then (A)
   else ((A 0.8) (C 0.2))
```