# Energy-Based Control of Coronal Biped Balance and Stepping

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## Abstract

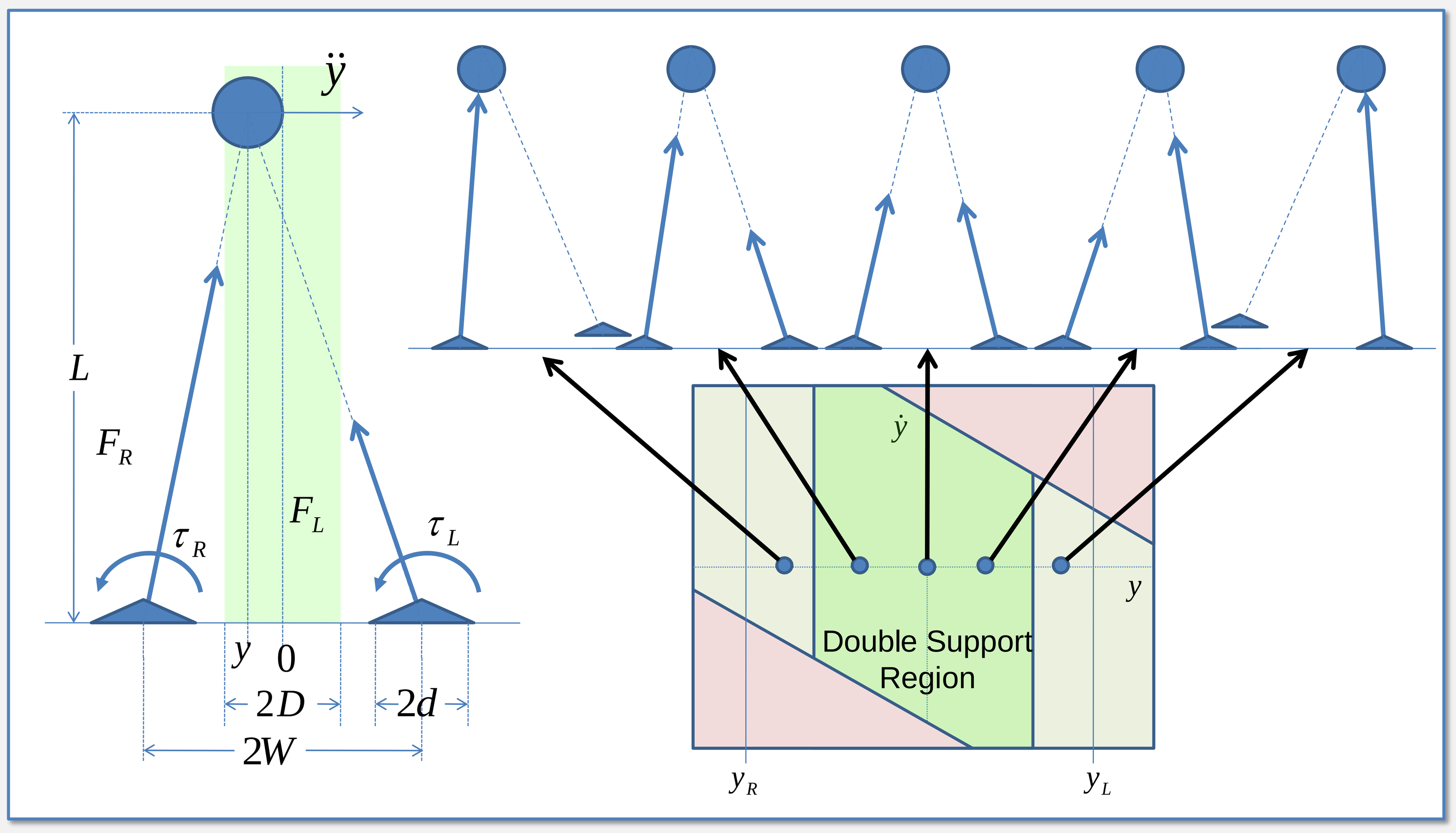
We have developed the Linear Biped Model (LiBM) to focus on the motion of the center of mass with respect to the base of support. We present the dynamics of the model, an energy controller that regulates periodic motion of the system, and a foot placement controller for push recovery and locomotion.

## Linear Biped Model

The linear inverted pendulum model (LIPM) has long been used in the study and control of biped systems. The Linear Biped Model is an extension of this model that models the effects of two feet. The dynamics are expressed as the weighted combination of two LIPMs.

$$\ddot{y} = w_L \frac{g}{L}(y - y_L) + w_R \frac{g}{L}(y - y_R) + \frac{u}{mL}$$

$$\tau_L = w_L u, \tau_R = w_R u$$




We define the weights using the position of the center of mass. While in a "double support region," the weights vary linearly. In single support, one weight is zero and the other is one.

$$w_L = \begin{cases} 1 & , \quad y > D \\ \frac{(D - y)}{2D} & , \quad |y| < D \\ 0 & , \quad y < -D \end{cases}$$

$$w_R = 1 - w_L$$

One advantage of this model is we can predict the forces on the two feet

$$F_{LZ} = w_L mg, F_{RZ} = w_R mg$$

$$F_{LY} = \frac{F_{LZ}}{L}(y - y_L) + \frac{\tau_L}{L}, F_{RY} = \frac{F_{RZ}}{L}(y - y_R) + \frac{\tau_R}{L}$$

Additionally, because the center of pressure must remain under the foot, we can write the constraints on the input torques.

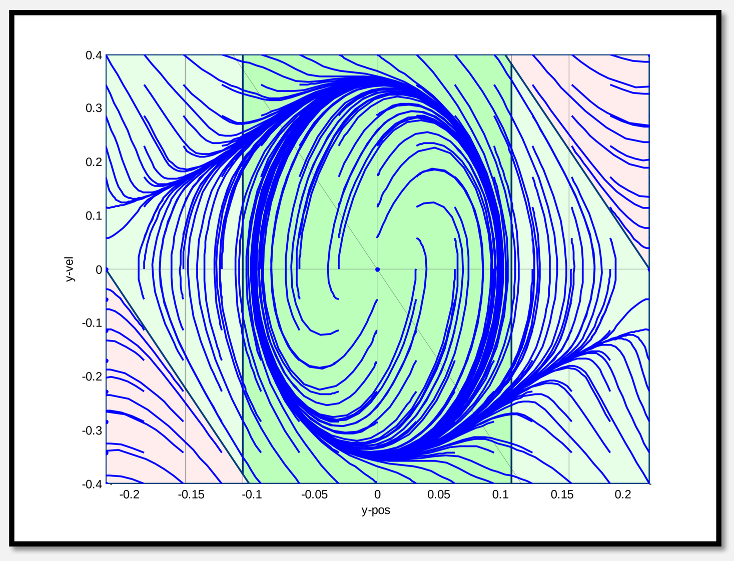
$$\left.\begin{matrix} |\tau_L| \le F_{LZ} d \\ |\tau_L| \le F_{RZ} d \end{matrix}\right\} \Rightarrow |\tau_L + \tau_R| \le mgd \Rightarrow |u| \le mgd$$

## Coronal Energy Control

The "orbital energy" of the system defines an ellipse in the phase plane of the center of mass. In the coronal plane, this corresponds to the side-to-side motion during periodic activities like marching in place or walking.

$$E = \frac{1}{2}\dot{y}^2 + \frac{g}{2L}y^2$$

$$\frac{1}{2}\dot{y}^2 + \frac{g}{2L}y^2 - E_d = 0 \Rightarrow y = \sqrt{\frac{2LE_d}{g}}\sin\left(\sqrt{\frac{g}{L}}t\right)$$



A controller can be derived to drive the energy error asymptotically to zero. We can then use the inverse dynamics of the linear biped model to generate nonlinear ankle torque policies for any desired energy.

$$e = E_d - E \qquad \dot{e} + Ke = 0 \Rightarrow -\left(\ddot{y} + \frac{g}{L}y\right)\dot{y} + Ke = 0$$

$$\therefore \ddot{y} = -\frac{g}{L}y + K\dot{y}(E_d - E)$$

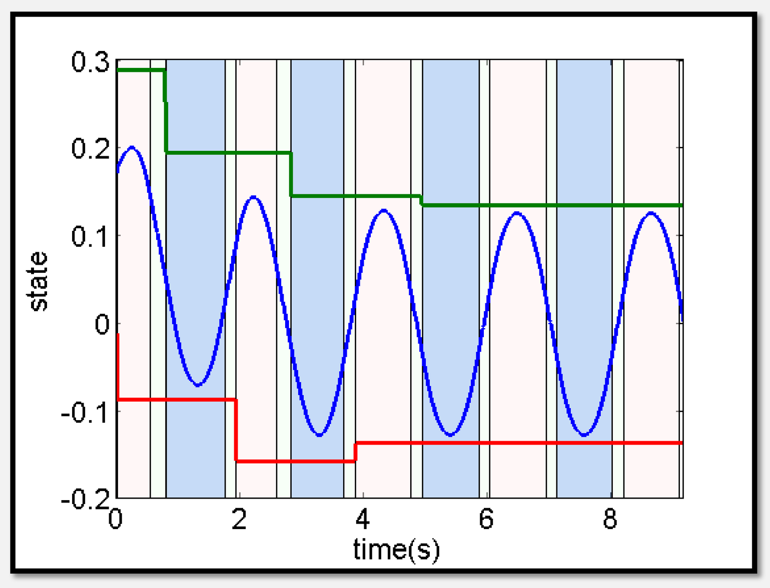
$$u = mL\left(w_L \frac{g}{L}(y - y_L) + w_R \frac{g}{L}(y - y_R) + \frac{g}{L}y - K\dot{y}(E_d - E)\right)$$

## Coronal Stepping Control

The LiBM assumes that the transition between single and double support occurs at the edge of the double support region. If we assume the energy controller above is driving the system, the trajectory is fully determined by the initial state and the step distance. We use an N-step lookahead controller which determines the step distances that will minimize the stance width error and the position error.

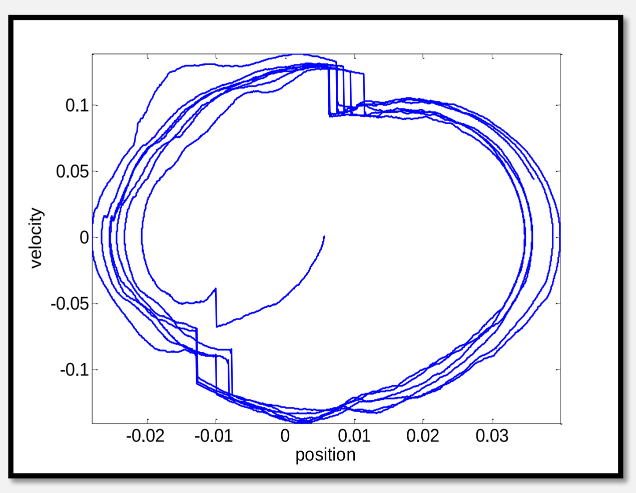
$$cost = K_1 \times (\Delta stance\_width)^2 + K_2 \times (\Delta y)^2$$

This controller results in a closed loop controller which can recover from large initial state disturbances. We found that while a 2-step lookahead found a trajectory of a lower cost, the improvement was only marginal and a 1-step lookahead is adequate.



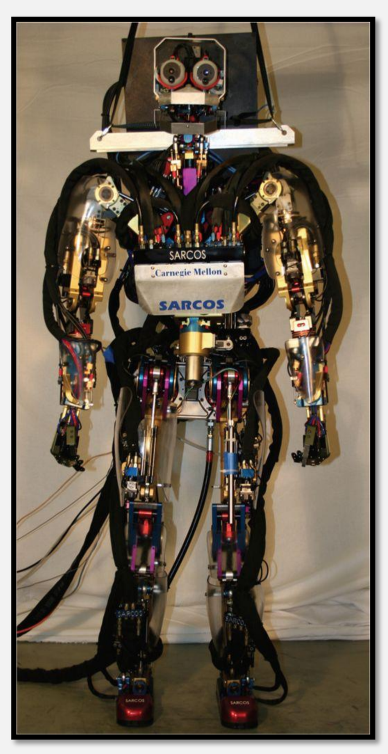

## Robot Experiments

We have performed preliminary experiments on our humanoid robot. The LiBM is applied for center of mass estimation and feed-forward control.




**Robot Description**

Mass = 94kg
CoM height = 0.9m
DOF = 41 (3D)
Power provided by off-board hydraulic pump



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