

Formal Hardware Verification by Symbolic Ternary Trajectory Evaluation*

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Abstract

Symbolic trajectory evaluation is a new approach to formal hardware verification combining the circuit modeling capabilities of symbolic logic simulation with some of the analytic methods found in temporal logic model checkers. We have created such an evaluator by extending the symbolic switch-level simulator COSMOS. This program gains added efficiency by exploiting the ability of COSMOS to evaluate circuit operation over a ternary logic model, where the third value X represents an unknown logic value. This program can formally verify systems containing complex features such as switch-level models, detailed timing, and pipelining.

1. Introduction

Formal verification seeks to overcome the weakness of informal design testing by simulation. Using our verifier, one can prove that a switch-level model of the transistor circuit correctly implements a formal description of the desired behavior for all possible system operations. Formal verification becomes increasingly desirable as system designs become more complex. With the introduction of pipelining and concurrently-operating subsystems, it becomes increasingly difficult using informal methods to evaluate the many subtle interactions between logically unrelated system activities.

In this paper we describe a new approach to formal verification that augments the circuit modeling capabilities of symbolic, switch-level simulation with some of the analytic capabilities found in temporal logic model checkers. With this increased analytic capability, we can express such temporal aspects of circuit behavior as clocking methodology and the skewing in time caused by pipelining. This paper describes the operation and application of our evaluator using a number of circuit design examples. A detailed presentation of the formal logic is presented in [2].

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Symbolic trajectory evaluation extends symbolic simulation with some of the analytic capability of finite state system analyzers. The user specifies the desired behavior of the system by assertions expressed as temporal logic formulas. Our temporal logic is quite restricted; it allows us to express properties of the circuit over *trajectories*: bounded-length sequences of circuit states. Our program verifies these formulas by a modified form of symbolic simulation, avoiding the need to extract a finite state machine representation. Furthermore, we exploit the 3-valued modeling

3. Symbolic Trajectory Evaluation

to represent even symbolically. Recent versions of these programs encode the states symbolically and hence can analyze systems having very large numbers of states. For circuits involving large amounts of storage, such as memories, data paths, and processors, the automata become too large to represent even symbolically. Recent versions of these programs encode the states symbolically and hence can analyze systems having very large numbers of states. For circuits involving large amounts of storage, such as memories, data paths, and processors, the automata become too large to represent even symbolically. Recent versions of these programs encode the states symbolically and hence can analyze systems having very large numbers of states. For circuits involving large amounts of storage, such as memories, data paths, and processors, the automata become too large to represent even symbolically.

Most automated approaches to formal verification (as opposed to more manual methods based on theorem proving) are based either on *symbolic simulation* or on *state machine analysis*. A symbolic simulator evaluates circuit behavior using symbolic variables (typically Boolean-valued) to encode a range of circuit operating conditions. In one simulation run, a symbolic simulator can compute what would require many runs of a conventional simulator. Symbolic simulation can support detailed circuit models and can handle large circuits. In addition to a method for evaluating circuit operation, however, formal verification requires a methodology for specifying the desired behavior and for checking the correspondence between the desired and realized behaviors. Straightforward symbolic simulation is adequate for verifying combinational circuits or the combinational portion of sequential circuits [5]. For verifying sequential systems where the state storage elements are not identified, or for which the behavioral specification is not based on the explicit state encoding, however, a more powerful methodology is required.

2. Heritage

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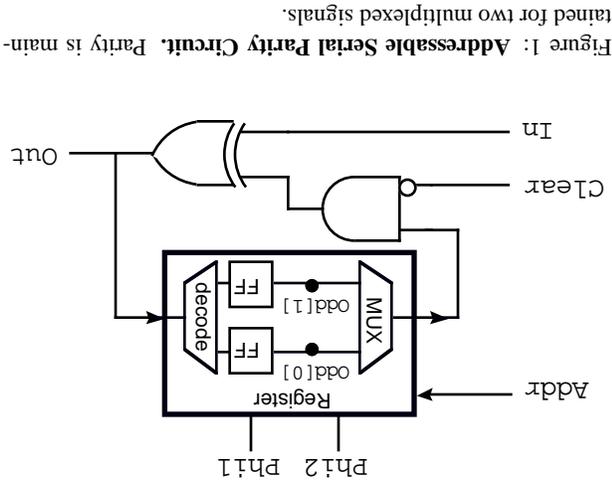


Figure 1: Addressable Serial Parity Circuit. Parity is maintained for two multiplexed signals.

represent an entire clock cycle as a single state transition. Potentially, our method could be extended to even more detailed timing models, including ones modeling real time.

Our algorithm checks only one basic form, the *assertion*, in the form of an implication $A \implies C$; the antecedent A gives the stimulus and current state, and the consequent C gives the desired response and state transition. System states and stimuli are given as trajectories over fixed length sequences of phases.

We describe each of these trajectories with a temporal *formula*. Primitive formulas specify Boolean values for circuit nodes, i.e., $n = 1$ or $n = 0$, and express the property that node n has the specified value for the entire phase. The only combining form is conjunction $F_1 \wedge F_2$, and the only temporal operator is the “next-phase” operator $\mathbf{P}F$, stating that F must hold in the following phase. This operator is similar to the next-time operator \mathbf{X} found in linear temporal logic [6]. In addition, a restriction operator creates a formula $B \rightarrow F$ stating that the property represented by formula F need only hold for those assignments satisfying Boolean expression B .

The temporal logic supported by our evaluator is far weaker than that of other model checkers. It lacks such basic forms as disjunction and negation, along with temporal operators expressing properties of unbounded state sequences. The logic was designed as a compromise between expressive power and ease of evaluation. It is powerful enough to express the timing and state transition behavior of circuits, while allowing assertions to be verified by an extended form of symbolic simulation.

4.2. Specification Example

We illustrate our methodology with the addressable serial parity circuit shown in Figure 1, designed to maintain the parity of two channels multiplexed onto a single line. On each cycle, the channel to be updated is specified by the *Addr* input. Setting *Clear* to 1 has the effect of setting the previous parity value for the channel to 0. Internally, the circuit consists of some combinational logic, plus a two-bit register file. Circuit nodes *Odd[0]* and *Odd[1]* are identified as representing the parity values for the two channels.

capability of the simulator, where the third logic value X indicates an unknown or indeterminate value. By judicious use of X as “don’t care” values, we can reduce the complexity of the symbolic manipulations considerably. For example, as will be shown, we can verify the correctness of a data path containing m registers of n bits each by performing a symbolic evaluation involving just $O(n + \log m)$ variables, whereas approaches based on symbolic finite state machine analysis require manipulations of functions involving $O(m)$ Boolean variables. Finally, since our verifier is based on simulation, we can more easily model timing details that are normally abstracted away by state machine models.

Our verifier supports a methodology in which the user expresses the desired system behavior as a set of assertions about the state transitions of an abstract state machine. In addition, the user provides temporal formulas defining such circuit details as the clocking methodology, the timing of input and output signals, and how the circuit realizes the abstract state both spatially and temporally. This form of specification works well for circuits that are normally viewed as *state transformation systems*, i.e., where each operation is viewed as updating the circuit state. Examples of such systems include memories, data paths and processors. For such systems, the complex analysis permitted by state machine analyzers is not required.

4. Specifying Circuit Behavior

4.1. Specification Logic

We model a circuit as operating over logic levels 0, 1, and a third level X representing an indeterminate or unknown level. These values can be partially ordered by their “information content” as $X \sqsubseteq 0$ and $X \sqsubseteq 1$, i.e., X conveys no information about the node value, while 0 and 1 are fully defined values. The only constraint we place on the circuit model—apart from the obvious requirement that it accurately model the physical system—is monotonicity over the information ordering. Intuitively, changing an input from X to a binary value (i.e., 0 or 1) must not cause an observed node to change from a binary value to X or to the opposite binary value. In extending to symbolic evaluation, the circuit nodes can take on arbitrary ternary functions over a set of Boolean variables \mathcal{V} .

Symbolic circuit evaluation can be thought of as computing circuit behavior for many different operating conditions simultaneously, with each possible assignment of 0 or 1 to the variables in \mathcal{V} indicating a different condition. Formally, this is expressed by defining an *assignment* ϕ to be a particular mapping from the elements of \mathcal{V} to binary values. A formula F in our logic expresses some property of the circuit in terms of the symbolic variables. It may hold for only a subset \mathcal{D} of the possible assignments. Such a subset can be represented as the Boolean domain function d over \mathcal{V} yielding 1 for precisely the assignments in \mathcal{D} . The constant functions 0 and 1, for example, represent the empty and universal sets, respectively.

Our verifier analyzes the temporal behavior of the circuit at a *phase* level, where one phase represents a period of time in which all external inputs are held fixed and the circuit operates until it reaches a stable state. This timing level was chosen to allow more detailed analysis than traditional state machine models, which

Define *Clocks* as describing the clocking behavior over the entire trajectory:

$$Clocks \stackrel{\text{def}}{=} CLK10 \vee P^1 CLK00 \vee P^2 CLK01 \vee P^3 CLK00 \vee P^4 CLK10$$

where P^i denotes i repetitions of the next-phase operator.

We can then write the specification as:

$$Clocks \vee (In = a) \vee (Clear = c) \vee (Addr = n) \vee (Odd [v] = b)$$

$$\begin{aligned} n = v &\rightarrow (P^2(Out = a \oplus bc) \vee P^4(Odd [v] = a \oplus bc)) \vee \\ &\rightarrow (P^4(Odd [v] = b)) \end{aligned}$$

We use several abbreviations to keep the specification concise. The notation $n = a$ stands for the formula: $(a \rightarrow n = 1) \vee (\bar{a} \rightarrow n = 0)$. The notation $Odd [v] = b$ stands for: $(\bar{v} \rightarrow Odd [0] = b) \vee (v \rightarrow Odd [1] = b)$.

5. Verifying Circuits

5.1. Evaluation Algorithm

The constraints we place on assertions make it possible to verify an assertion by a single evaluation of the circuit over a number of phases determined by the deepest nesting of the next-phase operators. In essence, we simulate the circuit over the unique weakest (in information content) trajectory allowed by the antecedent, while checking that the resulting behavior satisfies the consequent. In this process we compute a Boolean function OK expressing those assignments for which the assertion holds. For a correct circuit, this function should equal 1; otherwise, we can determine which cases failed by examining it.

More precisely, we first rewrite the antecedent into a form $A_0 \vee P^1 A_1 \vee P^2 A_2 \vee \dots \vee P^k A_k$, where each component A_i is *instantaneous*, i.e., it does not contain any next-phase operators. We rewrite the consequent similarly. Due to our restricted formula syntax, each such instantaneous formula obeys the property that for any assignment ϕ , one of the following cases must hold:

1. The formula has an internal inconsistency (e.g., $n = a \wedge n = b$ for assignments where $a \neq b$).

2. There exists a unique circuit state, minimal in information content, satisfying the formula.

We can combine this information for all possible assignments symbolically by associating with each instantaneous formula F a Boolean function OK_F denoting those assignments where the formula has no internal inconsistencies, and a symbolic state vector \bar{a}_F which for a given assignment describes the minimal circuit state if the formula is consistent, and has all elements equal to X otherwise.

As an example, consider the instantaneous formula F defined as $Odd [n] = a \vee Odd [v] = b$. The associated domain function is $OK_F = (a \oplus b) + (n \oplus v)$, i.e., the formula is consistent so long as we do not try to assign opposite binary values to the same node. The minimal state values assigned to nodes $Odd [0]$ and $Odd [1]$ would be ternary functions over the variables a, b, n , and v given by the tables:

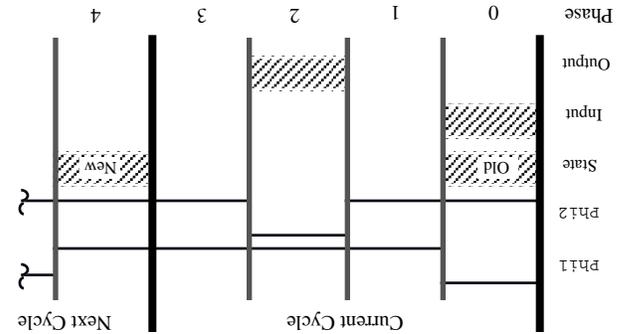


Figure 2: **Circuit Timing.** Each clock cycle is modeled as four phases, with the state, input, and output data valid in the phases indicated. Verification requires evaluating a trajectory of 5 phases.

The desired behavior of this circuit can be expressed by a single assertion stating that each register should be updated appropriately when it is addressed and should hold its value when it is not. In developing this assertion, we incorporate the timing information illustrated in Figure 2. Each cycle consists of 4 phases including the nonoverlapping periods of the clock nodes. Inputs are applied during the initial phase, and the output is guaranteed to be valid two phases later. Furthermore, the internal state is held stable during the initial phase. Thus, we must evaluate circuit operation over a trajectory of 5 phases—one full clock cycle plus the first phase of the next. These phases are numbered from 0 to 4 to match the nesting of next-phase operators in the specification.

Informally, the assertion states:

- Given:
 - The clocks are cycled correctly.
 - Inputs values a, c , and n are applied during phase 0 to In, Clear, and Addr, respectively.
 - Node $Odd [v]$ has value b during phase 0
- Then, for the case where $n = v$:
 - Value $a \oplus bc$ will appear on Out during phase 2
 - Node $Odd [v]$ will have this same value in the initial phase of the following cycle (phase 4).
- Otherwise ($n \neq v$):
 - Node $Odd [v]$ will have value b in the initial phase of the following cycle.

We construct the antecedent by first defining the operation of the clocks. As shorthand, define formulas representing the possible signals applied to the clocks:

$$\begin{aligned} CLK10 &\stackrel{\text{def}}{=} Ph11 = 0 \vee Ph12 = 1 \\ CLK00 &\stackrel{\text{def}}{=} Ph11 = 0 \vee Ph12 = 0 \\ CLK10 &\stackrel{\text{def}}{=} Ph11 = 1 \vee Ph12 = 0 \end{aligned}$$

We now illustrate verification of more complex sequential circuits, including pipelining. Consider the addressable accumulator shown in Figure 3. This circuit can maintain the sum of signals for m different channels, storing the sums in its register array. Timing is similar to that of the parity circuit. A specification of this circuit has the same general form as the specification for the serial parity circuit. Instead of using single Boolean variables (e.g., u, a) to indicate possible address and data values, however, we use vectors of Boolean variables (e.g., \vec{u}, \vec{a}). The desired output and new state values are defined in terms of the binary representation of the addition function. Finally, rather than fixing the nodes representing the stored values in the abstract specification, we write the assertions in terms of an abstract predicate Reg , where $Reg[\vec{u}, \vec{a}]$ states that value \vec{a} is held in some abstract register \vec{u} . The mapping between the abstract state and the actual circuit state is specified by defining this predicate in terms of values on the circuit nodes.

6. Verifying Complex Circuits

Unlike a simulator, the evaluator will set an input node to X whenever it is not constrained by the antecedent. In this way, the evaluator makes no assumptions about input values except those explicit in the antecedent. The consequent component C_i defines some checks that should be made of the circuit state during phase i . We require that each component of the observed circuit state remain greater or equal to the corresponding element of \vec{a}_i throughout the phase. Thus, the program keeps track of the assignments that satisfy these checks, represented by a Boolean function $Check$. The program also computes function OK_A (respectively, OK_C), representing the assignments where the antecedent (resp., consequent) contains no inconsistencies. Once the circuit has been evaluated for its entire trajectory, the program computes the function $OK = OK_A + \overline{Check} + (OK_C \cdot Check)$, representing the set of assignments for which the entire assertion is valid, i.e., where the consequent holds whenever the antecedent can be established.

Entries labeled ‘-’ indicate cases where the operation is overconstrained, i.e., we are attempting to assign opposite binary values to a single node. As the evaluation progresses, the program keeps track of the assignments that do not have overconstrained behavior, represented by a Boolean function Try . Unlike a simulator, the evaluator will set an input node to X whenever it is not constrained by the antecedent. In this way, the evaluator makes no assumptions about input values except those explicit in the antecedent.

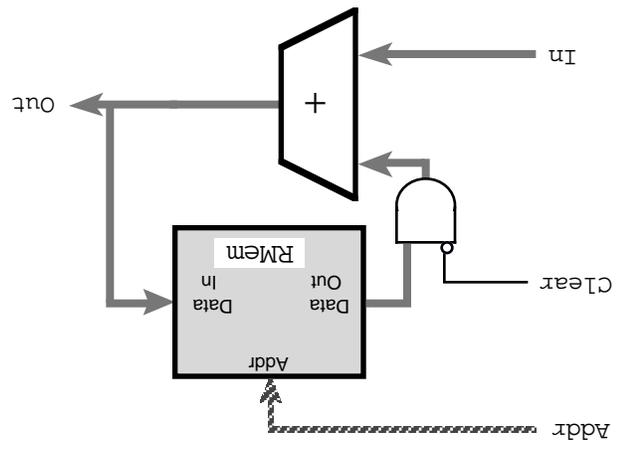
\square	0	1	X
0	0	-	0
1	-	1	1
X	0	1	X

Symbolic trajectory evaluation is implemented by modifying this algorithm. For each phase i of the trajectory, the antecedent component A_i defines some constraints on the input and internal nodes. These constraints are imposed by setting each element of the circuit state to the least upper bound (denoted by \square) of the current value and the corresponding element of \vec{a}_i , where this operation has the function table:

The assertion syntax we have introduced is rather primitive. To facilitate generating more abstract notations, we have developed a front end to the evaluator by embedding a specification language in Scheme, a dialect of Lisp. This language allows the user to write formulas in terms of vectors of Boolean values, apply operations such as binary arithmetic and bitwise logical operations, and extend the language. These formulas are then expanded into Boolean operations automatically, generating a command file for the evaluator.

5.2. Implementation

Figure 3: Addressable Accumulator. Sums can be accumulated in m different registers.



Starting from a setting of the inputs and an initial state of the internal nodes, a simulator such as COSMOS simulates one phase of circuit operation by repeatedly updating the internal nodes according to their state excitation value until a stable state is reached. This process can be viewed as the cycling of a finite state automaton with the new values of the internal nodes computed according to their excitation functions, and the new values of the input nodes equal to their old values.

Observe that some of the X entries in these tables arise due to overconstrained cases, where OK_F yields 0. Others arise due to unspecified cases. For example, when $n = v = 1$, no value is specified for node $Odd[0]$.

\square	Overconstrained
\circ	Unspecified

$Odd[0]$	00	01	10	11
a	0	0	X	X
b	0	1	0	X
	00	01	10	11

$Odd[1]$	00	01	10	11
a	X	0	0	0
b	X	1	0	X
	00	01	10	11

I. K. McWilliam designed the data path, and R. Luthi implemented it at the switch level.

To gain an understanding of how this form of verification scales to larger designs, we verified switch level implementations of a family of pipelined data paths containing a register file, ALU, pipe registers, and a simple controller. The data path processes each instruction in four pipe stages: instruction read, operand read, execute, and write back. Such a pipeline has a *read-after-write*

more CPU time to verify than do simpler circuits. More typically, complex pipelines require some mapping. This is due to particular features of the assertion and state-lator can be verified more quickly than the unpipelined accumulator. It is interesting to note that the pipelined accumulator scales less than quadratically, while the memory required scales at either the word size or the number of registers, the time required grows larger by increasing the number of registers. As the circuits grow larger by increasing the number of registers, the time required grows larger by increasing the number of registers. As the circuits grow larger by increasing the number of registers, the time required grows larger by increasing the number of registers.

Table 1 shows the performance of our verifier running on a SUN-4/110 for some of the circuits discussed above. As the table indicates, we separately verified the address and register file (implemented with a static RAM) before assembling the accumulator. In our experience, verification is much more manageable when conducted as the designs are constructed. Attempting to verify existing designs created by other people without precise information about interface timing and state encoding tends to be time-consuming and frustrating. Often, a fair amount of reverse engineering is required.

7. Experimental Results and Observations

In this mapping, we use a vector of existentially quantified variables w to indicate that register $OldAddr$ may hold an arbitrary address independent of the current operation. Given such an address, abstract register \bar{w} maps to either the holding register ($\bar{w} = w$), or to register \bar{w} in the register file ($\bar{w} \neq w$). Adding quantified variables to our assertion logic is straightforward. The quantifiers map directly into operations on the OBDDs.

$$[\exists w \left[\begin{array}{l} OldAddr = w \quad \vee \quad (\bar{w} = w \rightarrow Hold = a) \\ \vee \quad (\bar{w} \neq w \rightarrow RMem[\bar{w}] = a) \end{array} \right]]$$

register file or in Hold, depending on the previous address. This is expressed by defining the register predicate $Reg[\bar{w}, a]$ as:

Table 1: Verifier performance for example circuits. Measured on SUN-4/110.

Circuit	Num. of Regs.	Word Size	Num. of CPU Secs.	Word Size	Num. of CPU Secs.
Addr	-	32	1186	7	0.2
SRAM	32	32	6666	92	1.0
Accum.	2	32	2352	22	0.3
	32	32	4603	290	1.0
	16	32	5665	267	1.8
	32	32	8332	883	1.9
Pipelined	2	32	2823	21	0.4
Accum.	32	32	8875	591	1.9

The abstract specification for this circuit is identical to that of the unpipelined circuit. The state mapping, however, is quite different. The contents of abstract register \bar{w} may be in either the control logic that includes a register $OldAddr$ storing the address from the previous cycle.

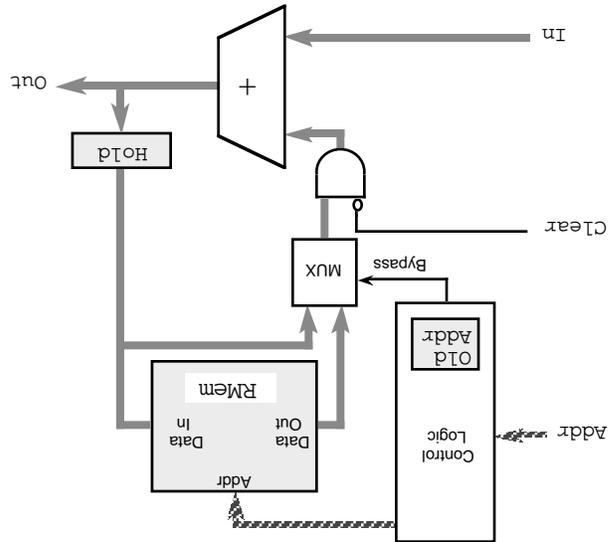
Consider, for example, a pipelined version of the addressable accumulator illustrated in Figure 4. This circuit exhibits the same external behavior as the one illustrated in Figure 3, but achieves greater performance by overlapping the address and register write operations. That is, on each cycle a value is first read from the register file. Then, while the address is computing the sum for the current cycle, the value from the previous cycle is written into the register array. When the same address is used in succession, the previous address output, stored in register Hold, is transferred by control logic that includes a register $OldAddr$ storing the address from the previous cycle.

include this information in the state mapping. Instead, we system state should not reflect the pipeline structure. Instead, we find that the pipelined system realizes an unpipelined specification, and hence the behavior specification in terms of abstract to prove that the pipelined system realizes an unpipelined specification for formal verification. In verifying such a system, we want a difficulty of designing such systems makes them an important bypass circuitry makes the system appear to be unpipelined. The amount of concurrent activity. Most pipelined systems are designed to be transparent to the outside. That is, interlock and Pipelining enhances circuit performance by increasing the amount of concurrent activity.

6.1. Pipelined Circuits

Ray RMem is in exact correspondence with the abstract system registers. Thus, we can define the state mapping as $Reg[\bar{w}, a] =_{def} (RMem[\bar{w}] = a)$, using a vector extension of the indexing notation introduced earlier.

Figure 4: Pipelined Addressable Accumulator Circuit. Register access and address operation occur simultaneously. Bypass logic handles the case where the same address is used on consecutive operations.



extremely large, and it is difficult to relate errors from the verifier back to the part of the original specification that failed. Furthermore, some form of graphical interface to specify the input, output, and clock timing would be helpful.

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Word Size	2	4	8	16
Registers	2	4	8	16
	60s	196s	323s	1119s
	120s	274s	398s	1289s
	178s	449s	704s	2290s
	320s	669s	1116s	2290s
	0.5M	1.0M	1.7M	5.9M
	0.7M	1.1M	1.8M	6.0M
	0.8M	1.8M	3.2M	6.1M
	1.1M	1.9M	3.3M	6.1M

Table 2: Verifier Performance for Pipelined Data Paths. Measured on DECstation 3100.

data hazard. When an instruction i reads a register which is the destination register of instruction $i-1$ or $i-2$, those instructions $i-1$ and $i-2$ have not yet written back their results into the register file. The controller detects this hazard and compensates using two features. First, the register file write operations occur in the first half of a clock cycle, while read operations occur in the second half, so the controller can write a value into a register and read the newly written value from the same register in one clock cycle. Second, the data path contains a register bypass to pass the result of one instruction directly to the following instruction via the appropriate pipe register. The controller also includes logic to detect a no-operation op code and inhibit the appropriate writeback pipe stage. The ALU implements 10 operations.

Our specification of the data path consists of two kinds of assertions. First, we separately specified each ALU operation, and second, we specified that the NO-OP instruction should preserve register state. Space precludes a more detailed description of this specification.

Performance of our prototype verifier on the data path example is shown in Table 2. Note that execution time scales less than quadratically with word size and less than linearly with the number of registers. The memory requirement scales even more gradually. Attempting to verify larger data paths overloaded the Scheme interpreter we were using to implement the front end interface.

8. Conclusions

Our experience to date indicates that this is a promising approach for verifying circuits viewed as state transformation systems. It can operate on detailed switch-level models and verify timing at the phase level. It can be used to verify pipelined systems using high-level specifications that express the desired behavior without explicitly referencing the pipeline structure. It requires the definition of both assertions and implementation mappings.

For future work, we plan to extend our program, improve its performance, and test it on more ambitious circuit designs such as microprocessors. Although we have found an embedded language to provide powerful abstraction mechanisms for constructing behavioral specifications and state mappings hierarchically, our current configuration of running two separate programs is not ideal. The command files produced by the front end can be