CS 598 (CRN 62819) Topics in Algorithms, Spring 2015

Homework # 6   **Due: May 4, 2015**

Homework is due in class (on paper) on the due date. You can work together in groups of 2-3 if you like. For this assignment, if you work in a group, your group just has to hand in one assignment with all your names on it.

1. **4-cliques.**

    (a) What is the expected number of 4-cliques (a 4-clique is a set of 4 nodes in which all pairs are connected) in $G(n, p)$ as a function of n and p?

    (b) Let X denote the number of 4-cliques. Give a function $f(n)$ such that $E[X] = \Theta(1)$ for $p = f(n)$.

    (c) Using the first-moment method, show that for $p = o(f(n))$, $\Pr(X > 0) = o(1)$, where X and $f(n)$ are as above.

    (d) Using the second-moment method, show that for $p = \omega(f(n))$ [i.e., p growing strictly faster than $f(n)$] we have $\Pr(X = 0) = o(1)$.

2. **Piazza.** On the day after the due date, post your solution to one of the above questions, if a solution to that question has not already been posted. If solutions to all questions have been posted, post your solution to any of the above questions for which your solution is different from the posted solution(s). If all solutions match some posted solution, then post about one of the topics in Chapter 4.