CMU 15-7/381

CSPs

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With thanks to Ariel Procaccia and other prior instructions for slides

Class Scheduling Woes

- 4 more required classes to graduate
  - A: Algorithms
  - B: Bayesian Learning
  - C: Computer Programming
  - D: Distributed Computing
- A few restrictions
  - Algorithms must be taken same semester as distributed computing
  - Computer programming is a prereq for distributed computing and Bayesian learning, so it must be taken in an earlier semester
  - Advanced algorithms and Bayesian Learning are always offered at the same time, so they cannot be taken the same semester
- 3 semesters (semester 1,2,3) when can take classes

Constraint Satisfaction Problems (CSPs)

- *Variables*: $V = \{V_1,..,V_N\}$
- *Domain*: Set of *d* possible values for each variable
- *Constraints*: $C = \{C_1,..,C_K\}$
- A constraint consists of
  - variable tuple
  - list of possible values for tuple (ex.$[(V_2,V_3),\{(R,B),(R,G)\}]$)
  - Or function that describes possible values (ex. $V_2 \neq V_3$)
- Allows useful general-purpose algorithms with more power than standard search algorithms

Overview

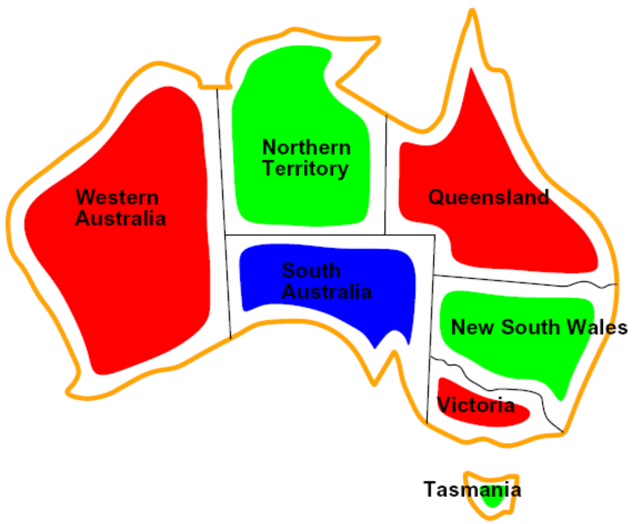
- Real world CSPs
- Basic algorithms for solving CSPs
- Pruning space through propagating information

Overview

- **Real world CSPs**
- Basic algorithms for solving CSPs
- Pruning space through propagating information

Example: Map Coloring

Color a map so that adjacent areas are different colors



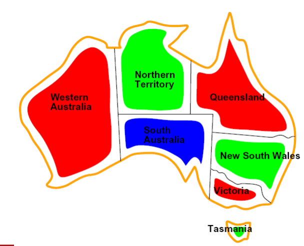
Map Coloring

Variables $WA,\ NT,\ Q,\ NSW$

Domain $\{red, green, blue\}$

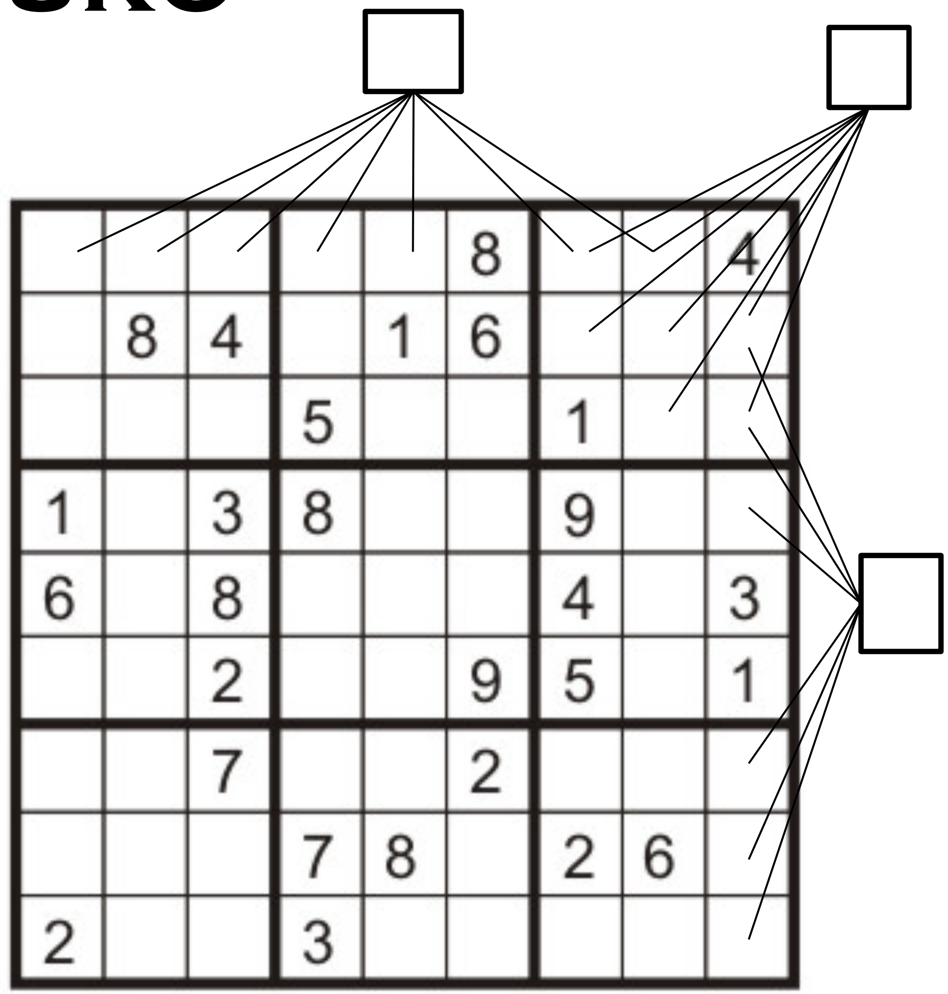
Constraints $(WA, NT) \in \{(red, green), (red, blue), (green, red), \ldots\}$

Solutions
$$\{WA = red, NT = green, Q = red, \\ NSW = green, V = red, SA = blue, \\ T = green\}$$



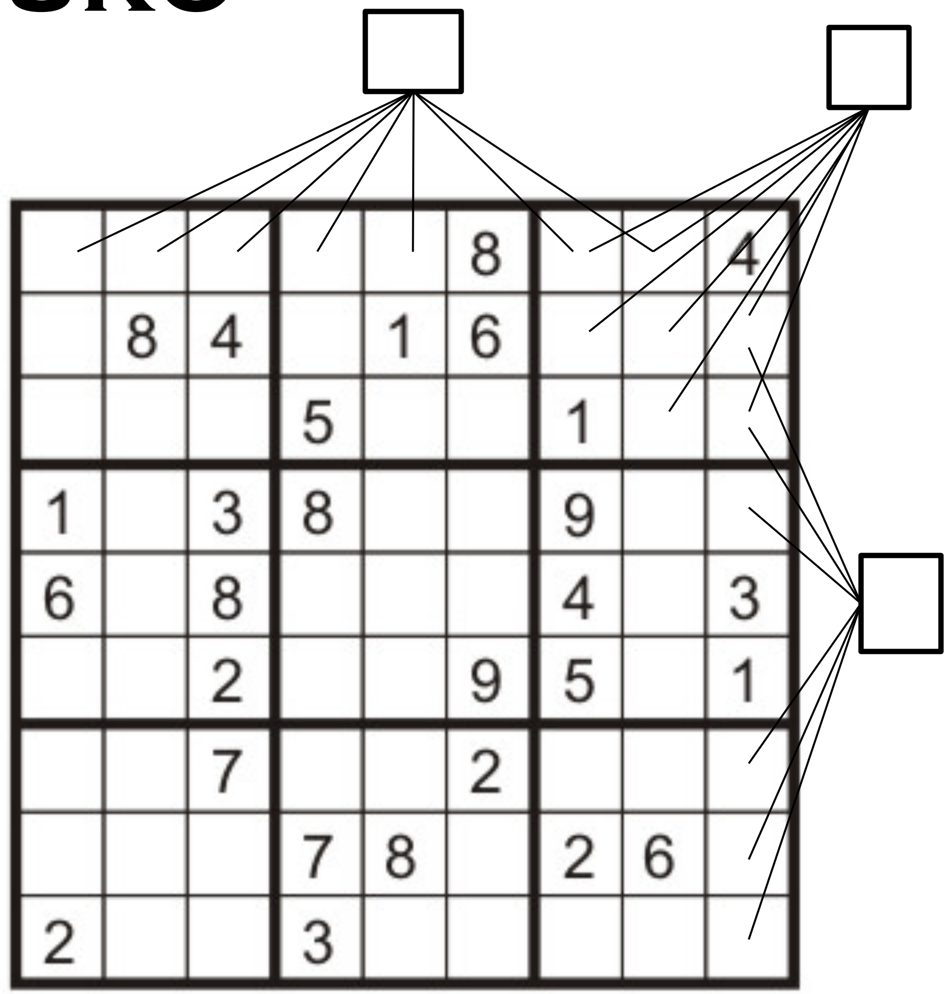
Example: Suduko

- Variables:
- Domain:
- Constraints:



Example: Suduko

- Variables:
  - Each open sqr
- Domain:
  - {1:9}
- Constraints:
  - 9-way all diff col
  - 9-way all diff row
  - 9-way all diff box



Scheduling (Important Ex.)

- Many industries. Many multi-million $ decisions. Used extensively for space mission planning. Military uses.
- People *really care* about improving scheduling algorithms! Problems with phenomenally huge state spaces. But for which solutions are needed very quickly
- Many kinds of scheduling problems e.g.:
  - *Job shop*: Discrete time; weird ordering of operations possible; set of separate jobs.
  - *Batch shop*: Discrete or continuous time; restricted operation of ordering; grouping is important.

Job Scheduling

- A set of N jobs, $J_1, \ldots, J_{n.}$
- Each job j is a seq of operations $O^j_1, \ldots, O^j_{Lj}$
- Each operation may use resource R, and has a specific duration in time.
- A resource must be used by a single operation at a time.
- All jobs must be completed by a due time.
- Problem: assign a start time to each job.

Exercise: Define CSP

- 4 more required classes to graduate: A, B, C, D
- A must be taken same semester as D
- C is a prereq for D and B so must take C earlier than D & B
- A & B are always offered at the same time, so they cannot be taken the same semester
- 3 semesters (semester 1,2,3) when can take classes
- Variables: A,B,C,D
- Domain: {1,2,3}
- Constraints: $A \neq B$, $A=D$, $C < B$, $C < D$

Overview

- Real world CSPs
- **Basic algorithms for solving CSPs**
- Pruning space through propagating information

Why not just do basic search algorithms from last time?

Backtracking

- Only consider a single variable at each point
- Don’t care about path!

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- Order of variable assignment doesn’t matter, so fix ordering

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- Only consider values which do not conflict with assignment made so far

Backtracking

- Only consider a single variable at each point
- Don't care about path!
- Order of variable assignment doesn't matter, so fix ordering
- Only consider values which do not conflict with assignment made so far
- Depth-first search for CSPs with these two improvements is called backtracking search

Backtracking

- Function Backtracking(csp) returns soln or fail
  - Return Backtrack({},csp)
- Function *Backtrack(assignment,csp)* returns soln or fail
  - If assignment is complete, return assignment
  - $V_i \leftarrow$ *select_unassigned_var(csp)*
  - For each val in *order-domain-values(var,csp,assign)*

    If value is consistent with assignment

      Add [V_i = val] to assignment

      Result $\leftarrow$ *Backtrack(assignment,csp)*

      If Result ≠ fail, return result

    Remove [V_i = val] from assignments
  - Return fail

Backtracking Example

- Variables: A,B,C,D
- Domain: {1,2,3}
- Constraints:
  - $A \neq B$, $A=D$, $C < B$, $C < D$
- Variable order: ?
- Value order: ?

Backtracking

- Function Backtracking(csp) returns soln or fail
  - Return Backtrack({},csp)
- Function *Backtrack(assignment,csp)* returns soln or fail
  - If assignment is complete, return assignment
  - V_i ← ***select_unassigned_var(csp)***
  - For each val in ***order-domain-values(var,csp,assign)***

    If value is consistent with assignment

      Add [V_i = val] to assignment

      Result ← *Backtrack(assignment,csp)*

      If Result ≠ fail, return result

    Remove [V_i = val] from assignments
  - Return fail

Think and discuss

- Does the value order used affect how long backtracking takes to find a solution?
- Does the value order used affect the solution found by backtracking?

Example

Variables: A,B,C,D   Domain: {1,2,3}

Constraints: A ≠ B, A=D, C < B, C < D

Variable order: alphabetical   Value order: Descending

- (A=3)

Example

Variables: A,B,C,D   Domain: {1,2,3}

Constraints: A ≠ B, A=D, C < B, C < D

Variable order: alphabetical   Value order: Descending

- (A=3)
- (A=3, B=3) inconsistent with A ≠ B
- (A=3, B=2)
- (A=3, B=2, C=3) inconsistent with C < B
- (A=3, B=2, C=2) inconsistent with C < B
- (A=3, B=2, C=1)
- (A=3, B=2, C=1,D=3) VALID

Example

Variables: A,B,C,D   Domain: {1,2,3}

Constraints: A ≠ B, A=D, C < B, C < D

Variable order: alphabetical   Value order: **Ascending**

- (A=1)

Example

Variables: A,B,C,D   Domain: {1,2,3}

Constraints: A ≠ B, A=D, C < B, C < D

Variable order: alphabetical   Value order: Ascending

- (A=1)
- (A=1,B=1) inconsistent with A ≠ B
- (A=1,B=2)
- (A=1,B=2,C=1)
- (A=1,B=2,C=1,D=1) inconsistent with C < D
- (A=1,B=2,C=1,D=2) inconsistent with A=D
- (A=1,B=2,C=1,D=3) inconsistent with A=D

Example
Variables: A,B,C,D   Domain: {1,2,3}
Constraints: A ≠ B, A=D, C < B, C < D
Variable order: alphabetical   Value order: Ascending

- (A=1)
- (A=1,B=1) inconsistent with A ≠ B
- (A=1,B=2)
- (A=1,B=2,C=1)
- (A=1,B=2,C=1,D=1) inconsistent with C < D
- (A=1,B=2,C=1,D=2) inconsistent with A=D
- (A=1,B=2,C=1,D=3) inconsistent with A=D
- No valid assignment for D, return result = fail
  - Backtrack to (A=1,B=2,C=)
- Try (A=1,B=2,C=2) but inconsistent with C < B
- Try (A=1,B=2,C=3) but inconsistent with C < B
- No other assignments for C, return result= fail
  - Backtrack to (A=1,B=)
- (A=1,B=3)
- (A=1,B=3,C=1)
- (A=1,B=3,C=1,D=1) inconsistent with C < D
- (A=1,B=3,C=1,D=2) inconsistent with A = D
- (A=1,B=3,C=1,D=3) inconsistent with A = D
- Return result = fail
  - Backtrack to (A=1,B=3,C=)
- (A=1,B=3,C=2) inconsistent with C < B
- (A=1,B=3,C=3) inconsistent with C < B
- No remaining assignments for C, return fail
  - Backtrack to (A=1,B=)
- No remaining assignments for B, return fail
  - Backtrack to A
- (A=2)
- (A=2,B=1)
- (A=2,B=1,C=1) inconsistent with C < B
- (A=2,B=1,C=2) inconsistent with C < B
- (A=2,B=1,C=3) inconsistent with C < B
- No remaining assignments for C, return fail
  - Backtrack to (A=2,B=?)
- (A=2,B=2) inconsistent with A ≠ B
- (A=2,B=3)
- (A=2,B=3,C=1)
- (A=2,B=3,C=1,D=1) inconsistent with C < D
- (A=2,B=3,C=1,D=2)   **ALL VALID**

Ordering Matters!

- Function Backtracking(csp) returns soln or fail
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- Function *Backtrack(assignment,csp)* returns soln or fail
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  - For each val in ***order-domain-values(var,csp,assign)***

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    Remove [V_i = val] from assignments
  - Return fail

Min Remaining Values (MRV)

- Choose variable with minimum number of remaining values in its domain
- Why min rather than max?

Min Remaining Values (MRV)

- Choose variable with minimum number of remaining values in its domain
- Most constrained variable
- “Fail-fast” ordering

Least Constraining Value

- Given choice of variable:
  - Choose least constraining value
  - Aka value that rules out the least values in the remaining variables to be assigned
  - May take some computation to find this

- Why least rather than most?

Click! Cost of Backtracking?

- d values per variable
- n variables
- Possible number of CSP assignments?

- **A) $O(d^n)$**
- B) $O(n^d)$
- C) $O(nd)$

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Limitations of Backtracking

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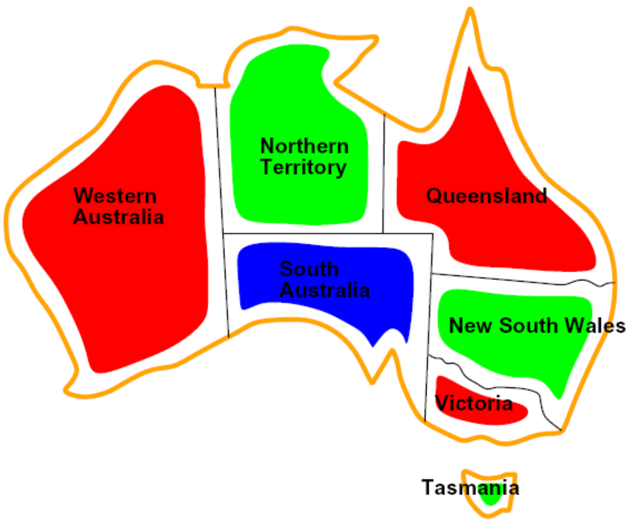
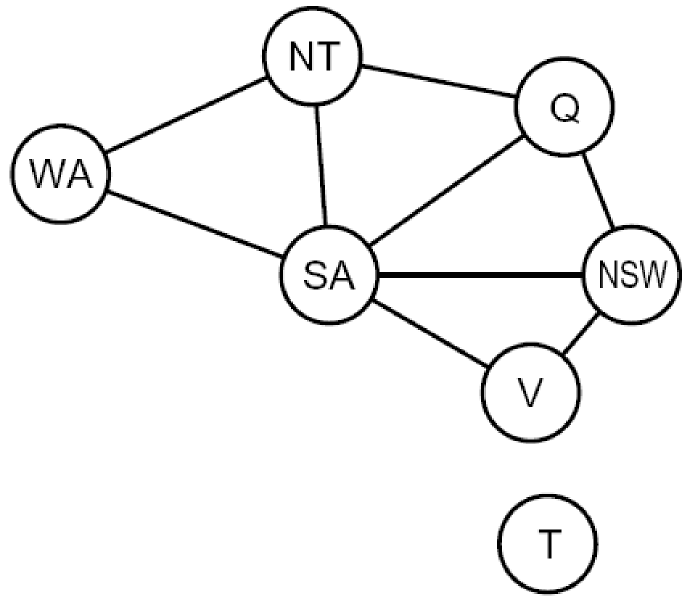
  - Return fail

Constraint Graphs

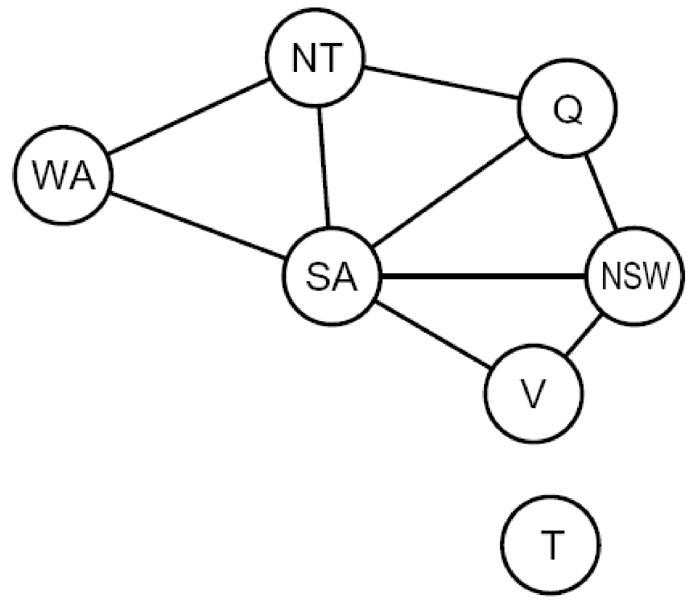
- Nodes are variables
- Arcs show constraints





Propagate Information

- If we choose a value for one variable, that affects its neighbors
- And then potentially those neighbors…
- Prunes the space of solutions



Arc Consistency

- Definition:
  - An “arc” (connection between two variables X → Y in constraint graph) is consistent if:
  - For every value could assign to X
    - There exists some value of Y that could be assigned without violating a constraint

AC-3 (Assume binary constraints)

- Input: CSP
- Output: CSP, possible with reduced domains for variables, or inconsistent
- Local variables: stack, initially stack of all arcs (binary constraints in csp)
- While stack is not empty
- (X_i,X_j) = Remove-First(stack)
- [domainX_i, anyChangeToDomainX_i] = Revise(csp,X_i,X_j)
- if anyChangeToDomainX_i == true
- if size(domainX_i) = 0, return inconsistent
- else
- for each X_k in Neighbors(X_i) except X_j
- add (X_k,X_i) to stack
- Return csp

Have to add in arc for (X_i,X_j) and (X_j,X_i) for i,j constraint

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- **Function Revise(csp,X_i,X_j)** returns DomainXi and anyChangeToDomainX_i
- anyChangeToDomainX_i = false
- for each x in Domain(X_i)
- if no value y in Domain(Xj) allows (x,y) to satisfy constraint between (X_i,X_j)
- delete x from Domain(X_i)
- anyChangeToDomainX_i = true

Have to add in arc for (X_i,X_j) and (X_j,X_i) for i,j constraint

AC-3 Computational Complexity?

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D domain values
C binary constraints

Complexity of revise function? D^2

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Have to add in arc for (X_i,X_j) and (X_j,X_i) for i,j constraint

D domain values
C binary constraints

Complexity of revise function?
D^2

Number of times can put a constraint in stack?

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Have to add in arc for (X_i,X_j) and (X_j,X_i) for i,j constraint

D domain values
C binary constraints

Complexity of revise function?
D^2

Number of times can put a constraint in stack?
D

Total:
CD^3

AC-3 Example

- Variables: A,B,C,D
- Domain: {1,2,3}
- Constraints: $A \neq B$, $C < B$, $C < D$ (subset of constraints from before)

AC-3 Example

- Variables: A,B,C,D
- Domain: {1,2,3}
- Constraints: $A \neq B$, $C < B$, $C < D$ (subset of constraints from before)
- Constraints both ways: $A \neq B$, $B \neq A$, $C < B$, $B > C$, $C < D$, $D > C$

AC-3 Example

- Variables: A,B,C,D
- Domain: {1,2,3}
- Constraints: A ≠ B, C < B, C < D (subset of constraints from before)
- Constraints both ways: A≠ B, B≠ A, C < B, B > C, C < D, D > C
- stack: AB, BA, BC, CB, CD, DC

AC-3 Example

- Variables: A,B,C,D
- Domain: {1,2,3}
- Constraints: A ≠ B, C < B, C < D (subset of constraints from before)
- Constraints both ways: A≠ B, B≠ A, C < B, B > C, C < D, D > C
- stack: AB, BA, BC, CB, CD, DC
- Pop AB:
- *for each x in Domain(A)*

  if no value y in Domain(B) that allows (x,y) to satisfy constraint between (A,B), delete x from Domain(A)
- No change to domain of A

AC-3 Example

- Variables: A,B,C,D
- Domain: {1,2,3}
- Constraints: A ≠ B, C < B, C < D (subset of constraints from before)
- Constraints both ways: A≠ B, B≠ A, C < B, B > C, C < D, D > C
- stack: AB, BA, BC, CB, CD, DC
- Pop AB
- stack: BA, BC, CB, CD, DC

AC-3 Example

- Variables: A,B,C,D
- Domain: {1,2,3}
- Constraints: A ≠ B, C < B, C < D (subset of constraints from before)
- Constraints both ways: A≠ B, B≠ A, C < B, B > C, C < D, D > C
- stack: AB, BA, BC, CB, CD, DC
- Pop AB
- stack: BA, BC, CB, CD, DC
- Pop BA
- *for each x in Domain(B)*

  if no value y in Domain(A) that allows (x,y) to satisfy constraint between (B,A), delete x from Domain(B)
- No change to domain of B

AC-3 Example

- Variables: A,B,C,D
- Domain: {1,2,3}
- Constraints: A ≠ B, C < B, C < D (subset of constraints from before)
- Constraints both ways: A≠ B, B≠ A, C < B, B > C, C < D, D > C
- stack: AB, BA, BC, CB, CD, DC
- stack: BA, BC, CB, CD, DC
- stack: BC, CB, CD, DC
- Pop BC
- *for each x in Domain(B)*

  if no value y in Domain(C) that allows (x,y) to satisfy constraint between (B,C), delete x from Domain(B)
- If B is 1, constraint B >C cannot be satisfied. So delete 1 from B’s domain, B={2,3}
- **Also have to add neighbors of B (except C) back to stack: AB**
- stack: AB, CB, CD, DC

AC-3 Example

Variables: A,B,C,D
Domain: {1,2,3}
Constraints: A ≠ B, C < B, C < D

- stack: AB, BA, BC, CB, CD, DC   A-D = {1,2,3}
- stack: BA, BC, CB, CD, DC   A-D = {1,2,3}
- stack: BC, CB, CD, DC   A-D = {1,2,3}
- stack: AB, CB, CD, DC   B={2,3}, A/C/D = {1,2,3}
- Pop AB
  - For every value of A is there a value of B such that A ≠ B?
  - Yes, so no change

AC-3 Example

Variables: A,B,C,D
Domain: {1,2,3}
Constraints: A ≠ B, C < B, C < D

- stack: AB, BA, BC, CB, CD, DC,   A-D = {1,2,3}
- stack: BA, BC, CB, CD, DC   A-D = {1,2,3}
- stack: BC, CB, CD, DC   A-D = {1,2,3}
- stack: AB, CB, CD, DC   B={2,3}, A/C/D = {1,2,3}
- stack: CB, CD, DC   B={2,3}, A/C/D = {1,2,3}
- Pop CB
  - For every value of C is there a value of B such that C < B
  - If C = 3, no value of B that fits
  - So delete 3 from C's domain, C = {1,2}
  - **Also have to add neighbors of C (except B) back to stack: no change because already in**

AC-3 Example

Variables: A,B,C,D
Domain: {1,2,3}
Constraints: $A \neq B$, $C < B$, $C < D$

- stack: AB, BA, BC, CB, CD, DC, A-D = {1,2,3}
- stack: BA, BC, CB, CD, DC A-D = {1,2,3}
- stack: BC, CB, CD, DC A-D = {1,2,3}
- stack: AB, CB, CD, DC B={2,3}, A/C/D = {1,2,3}
- stack: CB, CD, DC B={2,3}, A/C/D = {1,2,3}
- stack: CD, DC B={2,3}, C = {1,2} A,D = {1,2,3}
- Pop CD
  - For every value of C, is there a value of D such that C < D?
  - Yes, so no change

AC-3 Example

Variables: A,B,C,D
Domain: {1,2,3}
Constraints: $A \neq B$, $C < B$, $C < D$

- stack: AB, BA, BC, CB, CD, DC   A-D = {1,2,3}
- stack: BA, BC, CB, CD, DC   A-D = {1,2,3}
- stack: BC, CB, CD, DC   A-D = {1,2,3}
- stack: AB, CB, CD, DC   B={2,3}, A/C/D = {1,2,3}
- stack: CB, CD, DC   B={2,3}, A/C/D = {1,2,3}
- stack: CD, DC   B={2,3}, C = {1,2} A,D = {1,2,3}
- stack: DC   B={2,3}, C = {1,2} A,D = {1,2,3}
- For every value of D is there a value of C such that D > C?
    - Not if D = 1
    - So D = {2,3}

AC-3 Example

Variables: A,B,C,D
Domain: {1,2,3}
Constraints: $A \neq B$, $C < B$, $C < D$

- stack: AB, BA, BC, CB, CD, DC   A-D = {1,2,3}
- stack: BA, BC, CB, CD, DC   A-D = {1,2,3}
- stack: BC, CB, CD, DC   A-D = {1,2,3}
- stack: AB, CB, CD, DC   B={2,3}, A/C/D = {1,2,3}
- stack: CB, CD, DC   B={2,3}, A/C/D = {1,2,3}
- stack: CD, DC   B={2,3}, C = {1,2} A,D = {1,2,3}
- stack: DC   B={2,3}, C = {1,2} A,D = {1,2,3}
- A = {1,2,3} B={2,3}, C = {1,2} D = {2,3}

Forward Checking

- When assign a variable, make all of its neighbors arc-consistent

Backtracking + Forward Checking

- Function *Backtrack(assignment,csp)* returns soln or fail
  - If assignment is complete, return assignment
  - $V_i \leftarrow$ *select_unassigned_var(csp)*
  - For each val in *order-domain-values(var,csp,assign)*

    If value is consistent with assignment

      Add [V_i = val] to assignment

      Make domains of all neighbors of V_i arc-consistent with [V_i = val]

      Result $\leftarrow$ *Backtrack(assignment,csp)*

      If Result ≠ fail, return result

    Remove [V_i = val] from assignments
  - Return fail

Maintaining Arc Consistency

- Forward checking doesn't ensure all arcs are consistent
- AC-3 detects failure faster than forward checking
- What's the downside? **Computation**

Maintaining Arc Consistency (MAC)

- Function *Backtrack(assignment,csp)* returns soln or fail
  - If assignment is complete, return assignment
  - $V_i \leftarrow$ *select_unassigned_var(csp)*
  - For each val in *order-domain-values(var,csp,assign)*

    If value is consistent with assignment

    Add [V_i = val] to assignment

    Run AC-3 to make all variables arc-consistent with [V_i = val]. Initial stack is arcs (X_j,V_i) of neighbors of V_i that are unassigned, but add other arcs if these vars change domains.

    Result $\leftarrow$ *Backtrack(assignment,csp)*

    If Result $\neq$ fail, return result

    Remove [V_i = val] from assignments
  - Return fail

Sufficient to Avoid backtracking?

- If we maintain arc consistency, we will never have to backtrack while solving a CSP

- A) True
- **B) False**

AC-3 Limitations

- After running AC-3
  - Can have one solution left
  - Can have multiple solutions left
  - Can have no solutions left (and not know it)

AC-3 Limitations

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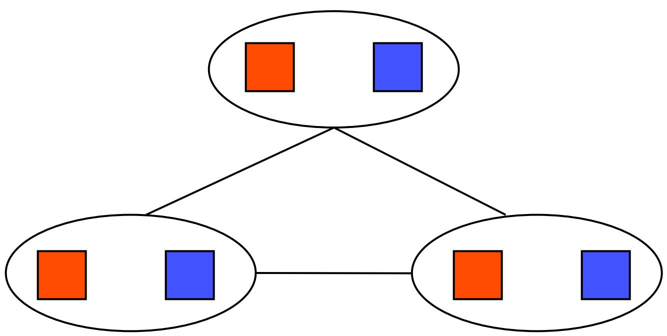


What went wrong here?

Complexity

- CSP in general are NP-hard
- Some structured domains are easier

Constraint Trees

- Constraint tree
  - Any 2 variables in constraint graph connected by <= 1 path
- Can be solved in time **linear in # of variables**

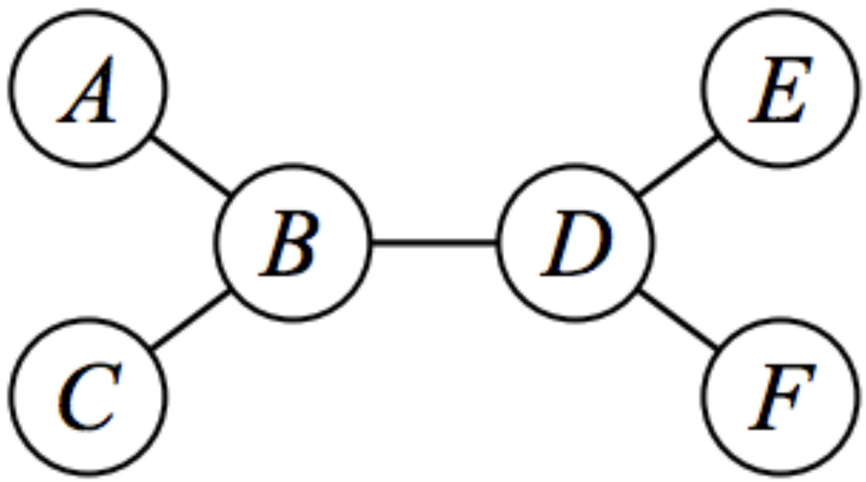
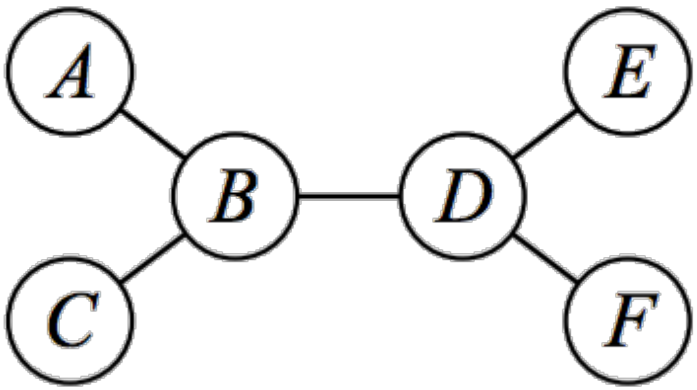
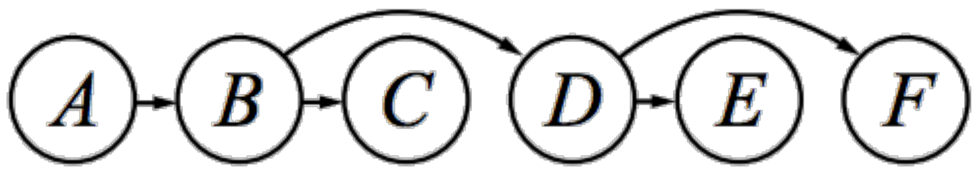


Figure from Russell & Norvig

Algorthm for CSP Trees

1) Choose any var as root and order vars such that every var's parents in constraint graph precede it in ordering



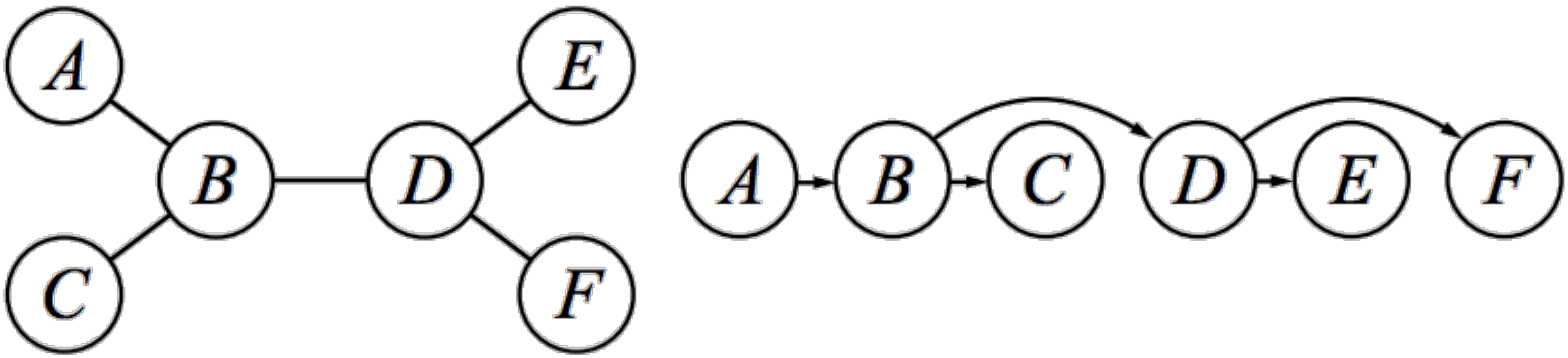


2) Let Xi be the parent of Xj in the new ordering
3) For j=n:2, run arc consistency to arc (Xi,Xj)
4) For j=1:n, assign val for Xj consistent w/val assigned for Xi

Figure from Russell & Norvig

Computational Complexity?

1) Choose any var as root and order vars such that every var's parents in constraint graph precede it in ordering



2) Let Xi be the parent of Xj in the new ordering
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Figure from Russell & Norvig

Summary

- Be able to define real world CSPs
- Understand basic algorithm (backtracking)
  - Complexity relative to basic search algorithms
  - Doesn't require problem specific heuristics
  - Ideas shaping search (LCV, etc)
- Pruning space through propagating information
  - Arc consistency
  - Tradeoffs: + reduces search space, - costs computation
- Computational complexity and special cases (tree)
- Relevant reading: R&N Chapter 6