17-355/17-655/17-819: Program Analysis
Recitation Activity
January 25, 2019 **Andrew ID:** ___________________________

1. Use the big-step operational semantics rules for the WHILE language to write a well-formed derivation with $\langle y := 3; if\ y > 1\ then\ z := y\ else\ z := 2, E\rangle \Downarrow E[y \mapsto 3][z \mapsto 3]$ as its conclusion. Therefore, $\langle y := 3; if\ y > 1\ then\ z := y\ else\ z := 2, E\rangle \Downarrow E[y \mapsto 3][z \mapsto 3]$ is provable. Make sure to indicate which rule you used to prove each premise or conclusion.

2. For homework 2, you will be partially proving that if a statement terminates, then the big- and small-step semantics for WHILE will obtain equivalent results, ie.:

$$\forall \mathrm{S} \in \mathrm{Stmt}.\quad \forall \mathrm{E}, \mathrm{E}' \in \mathrm{Var} \longrightarrow \mathbb{Z}.\quad \langle S, E\rangle \longrightarrow^{*} \langle skip, E'\rangle \Leftrightarrow \langle S, E\rangle \Downarrow E' \qquad (1)$$

You will prove equation (1) by induction on the structure of derivations for each direction of the $\Leftrightarrow$. You may assume that this property holds for arithmetic and boolean expressions, ie. you may assume the following equations hold:

$$\forall \mathrm{a} \in \mathrm{AExp}. \quad \forall \mathrm{n} \in \mathbb{Z}. \quad \langle a, E\rangle \longrightarrow_a^* n \Leftrightarrow \langle a, E\rangle \Downarrow_a n \qquad (2)$$

$$\forall \mathrm{P} \in \mathrm{BExp}. \quad \forall \mathrm{b} \in \{true, false\}. \quad \langle P, E\rangle \longrightarrow_b^* b \Leftrightarrow \langle P, E\rangle \Downarrow_b b \qquad (3)$$

For your proof, you are only required to show (a) the base case(s), and (b) the inductive case for let (using the semantics you developed in question (1) of the homework) and (c) two more representative inductive cases. **For this exercise, you will prove the representative inductive case indicated below for this direction of the proof**:

$$\forall \mathrm{S} \in \mathrm{Stmt}. \quad \forall \mathrm{E}, \mathrm{E}' \in \mathrm{Var} \longrightarrow \mathbb{Z}. \quad \langle S, E\rangle \Downarrow E' \Longrightarrow \langle S, E\rangle \longrightarrow^* \langle skip, E'\rangle$$

Since we are inducting on the structure of the derivation of $\langle S, E\rangle \Downarrow E'$ for this direction of the proof, the representative inductive case you should prove for this exercise is:

$$\frac{\langle P, E\rangle \Downarrow_b false \quad \langle S_2, E\rangle \Downarrow E'}{\langle if\ P\ then\ S_1\ else\ S_2, E\rangle \Downarrow E'}\ big - iffalse$$

So, you will want to show that given $\langle P, E\rangle \Downarrow_b false$ and $\langle S_2, E\rangle \Downarrow E'$, which the inductive hypothesis can be applied to, and $\langle if\ P\ then\ S_1\ else\ S_2, E\rangle \Downarrow E'$, then you can arrive at $\langle if\ P\ then\ S_1\ else\ S_2, E\rangle \longrightarrow^* \langle skip, E'\rangle$.

***Remember to make the justification clear for each step in your proof and you may also assume that:

$$\forall S_1, S_2 \in Stmt. \quad \forall P, P' \in BExp. \quad \forall E \in Var \rightarrow \mathbb{Z}.$$
$$\langle P, E\rangle \longrightarrow_b^* P' \Longrightarrow \langle if\ P\ then\ S_1\ else\ S_2, E\rangle \longrightarrow^* \langle if\ P'\ then\ S_1\ else\ S_2, E\rangle$$

Proof)