

# RECITATION 1

## PROBABILITY

10-701: INTRODUCTION TO MACHINE LEARNING

02/05/2021

### 1 Probability and Statistics

1. What is the probability of rolling a perfect square on a fair 6-sided die?
2. Thomas rolls two 6-sided dice. What is the probability that the top face of at least one die is divisible by 3?
3. Two events, A and B, are considered disjoint (mutually exclusive).  $P(A) = 0.5$ ,  $P(B) = 0.4$ .

What is  $P(A \cup B)$  ?

What is  $P(A \cap B)$  ?

What is  $P(A|B)$  ?

4. Now, instead, the two events A and B are not disjoint, but they are independent.

What is the  $P(A \cup B)$  ?

What is the  $P(A \cap B)$  ?

What is the  $P(A|B)$  ?

5. The chance that it will rain on Monday is  $1/5$ . For each subsequent day, the chance that it rains on the current day given that it rained the previous day is  $1/2$ . What is the probability that it will rain on all of Monday through Friday?
6. A student is looking at her activity tracker (Fitbit/Apple Watch) data and she notices that she seems to sleep better on days that she exercises. They observe the following:

Exercise	Good Sleep	Probability
Yes	Yes	0.3
Yes	No	0.2
No	No	0.4
No	Yes	0.1

What is the  $P(\text{GoodSleep} = \text{Yes} | \text{Exercise} = \text{Yes})$  ?

Why doesn't  $P(\text{GoodSleep} = \text{Yes} \cap \text{Exercise} = \text{Yes}) = P(\text{GoodSleep} = \text{Yes}) \cdot P(\text{Exercise} = \text{Yes})$  ?

- The student merges her activity tracker data with her food logs and finds that the  $P(\text{Eatwell} = \text{Yes} | \text{Exercise} = \text{Yes} \cap \text{GoodSleep} = \text{Yes})$  is 0.25. What is the probability of all three happening on the same day?
- Now suppose that for each subsequent day, the chance that it rains on the current day given that it did not rain the previous day is  $1/5$ . Given that it rained on Wednesday, what is the probability that it rained on Monday?
  - What is  $E[X]$  where  $X$  is a single roll of a fair 8-sided die ( $S = \{1,2,3,4,5,6,7,8\}$ )? What is  $\text{Var}[X]$ ?
  - Imagine that we had a new die where the sides were  $S = \{13,16,19,22,25,28,31,34\}$ . How does  $E[X]$  and  $\text{Var}[X]$  compare to our original die?
  - For any real numbers  $x$  and  $y$  in the interval  $(0,1)$ , the probability of choosing  $y$  is  $y/x$  times the probability of choosing  $x$ . What is the probability of choosing a number between  $1/2$  and  $3/5$ ?
  - Let  $A$  and  $B$  be the product of the values that are rolled on a fair 6-sided die. What is  $E[AB]$ ?
  - If  $x$  be chosen according to the continuous distribution scheme above, What is  $E[x]$ ?
  - Now suppose that the chance of rain is instead dependent on the presence and type of clouds. If there are no clouds, the probability of rain is  $1/10$ . If there are white clouds, the probability is  $1/5$ . Finally, if the clouds are gray, the probability is  $1/2$ . Given that the probability of of no clouds is  $1/2$ , the probability of white clouds is  $2/5$ , and the probability of gray clouds is  $1/10$ , what is the expected number of days from Monday to Friday, inclusive, for which it will rain?
  - Prove that  $E[E[X|Y]] = E[X]$  assuming that the random variables  $X$  and  $Y$  are continuous.