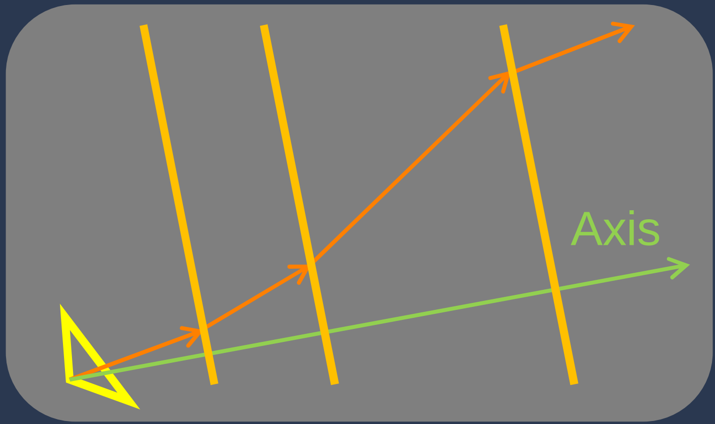
A Theory of Multi-Layer Flat Refractive Geometry



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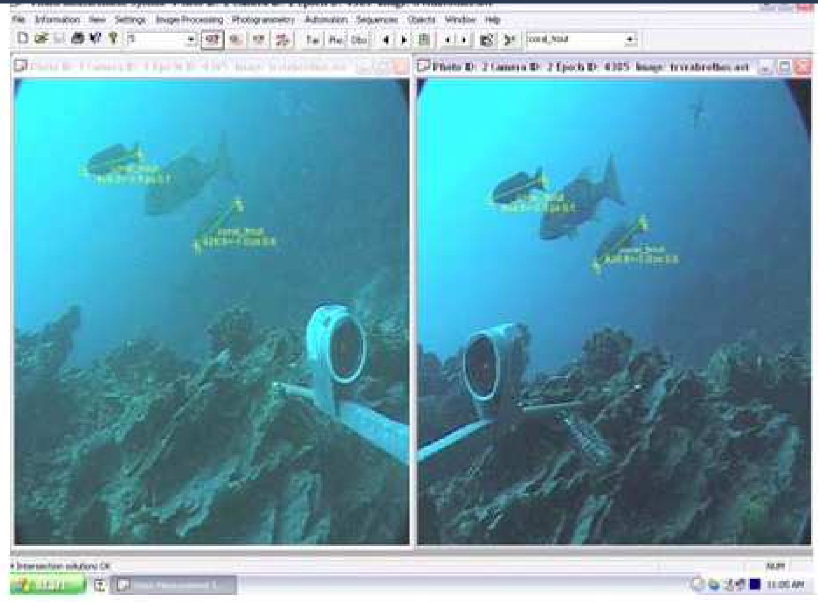
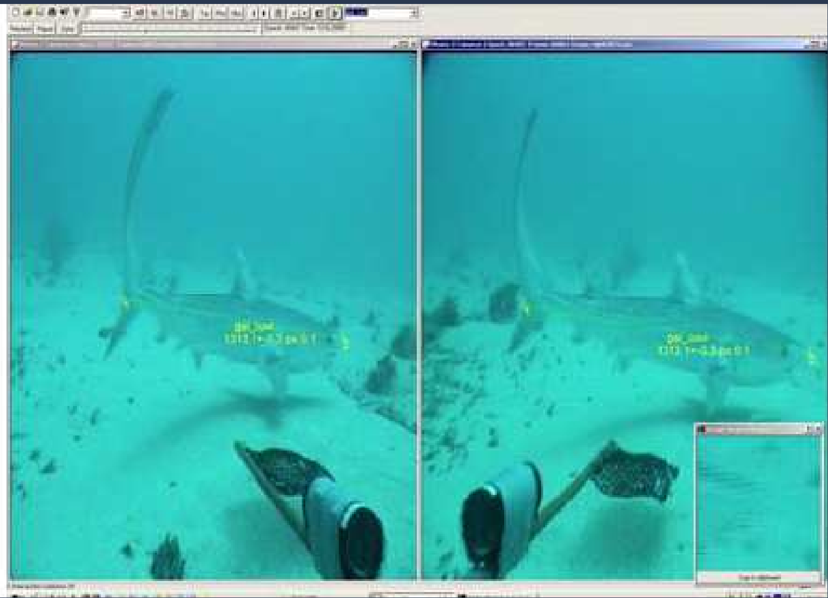
Imaging with Refractions





Source: Shortis et al. SPIE 2007

BP Oil Disaster, 5000 feet under sea level

Deepsea Challenger
submersible

Stereo Cameras, LED lights

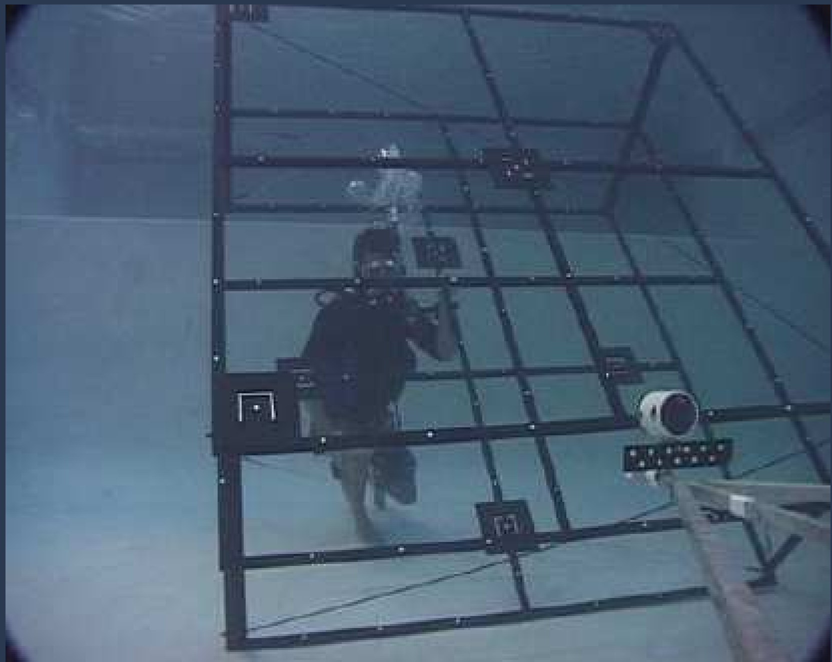

Titanic

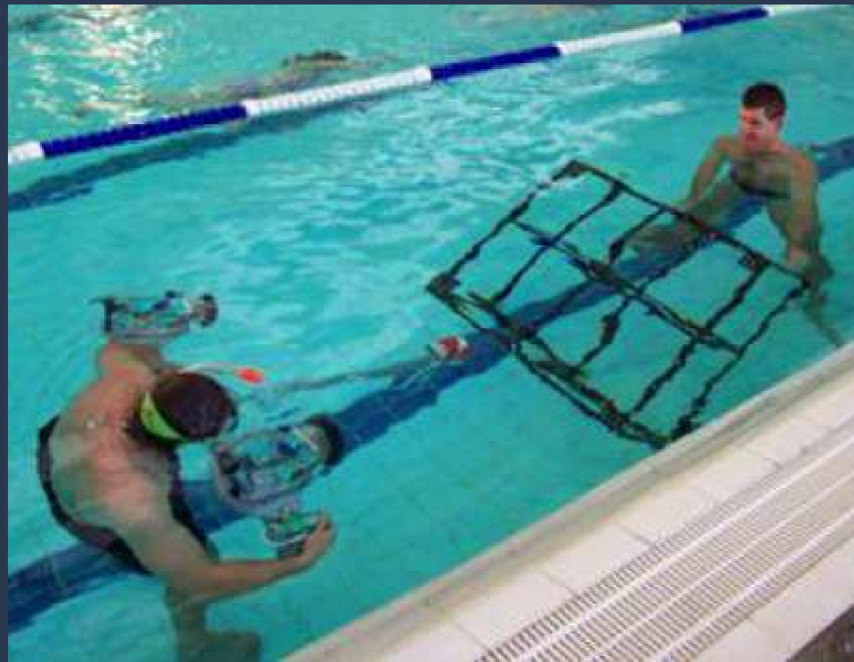

APRO

Imaging through refractions

- Not the same as pinhole imaging
- Pinhole model (central approximation) is not valid

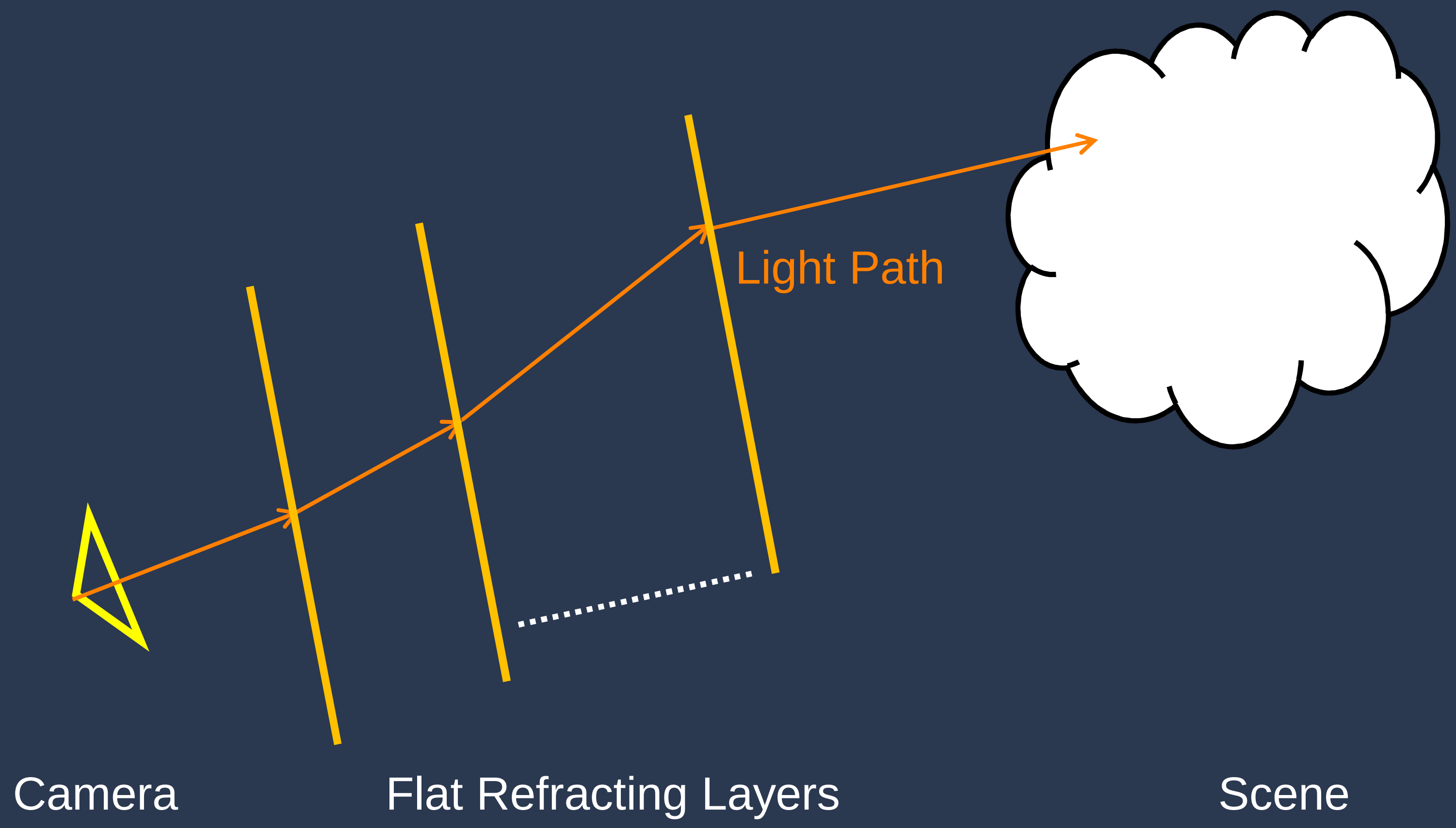
Calibration



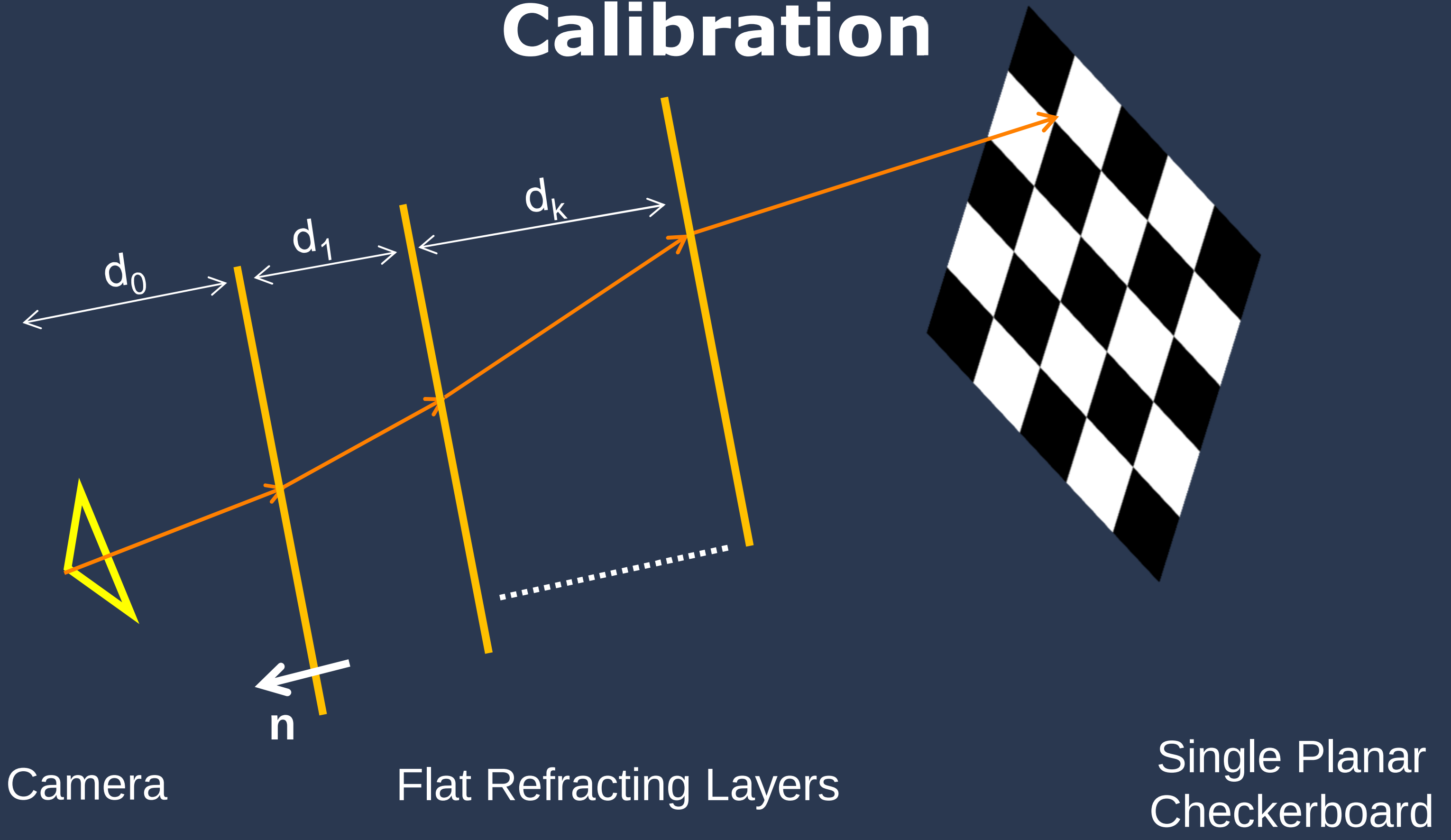


Source: Shortis et al. SPIE 2007

Multi-Layer Flat Refractive Systems

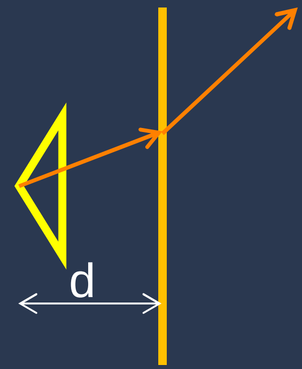


Calibration



- -Unknown Orientation of Layers **n**
- -Unknown Layer Thickness d_0, $d_{1,}$ … d_{k-1}
- -Unknown Refractive Indices μ_0, μ_1, …, μ_{k-1}
- -Unknown Pose of the Checkerboard (R,t)

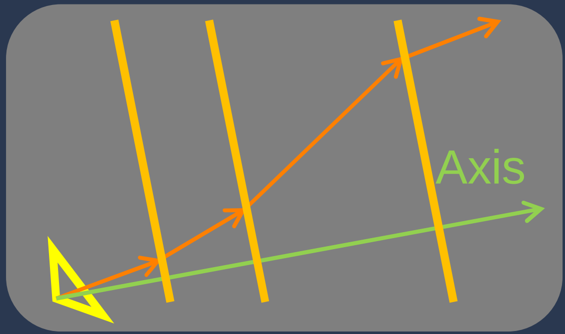
Related Work



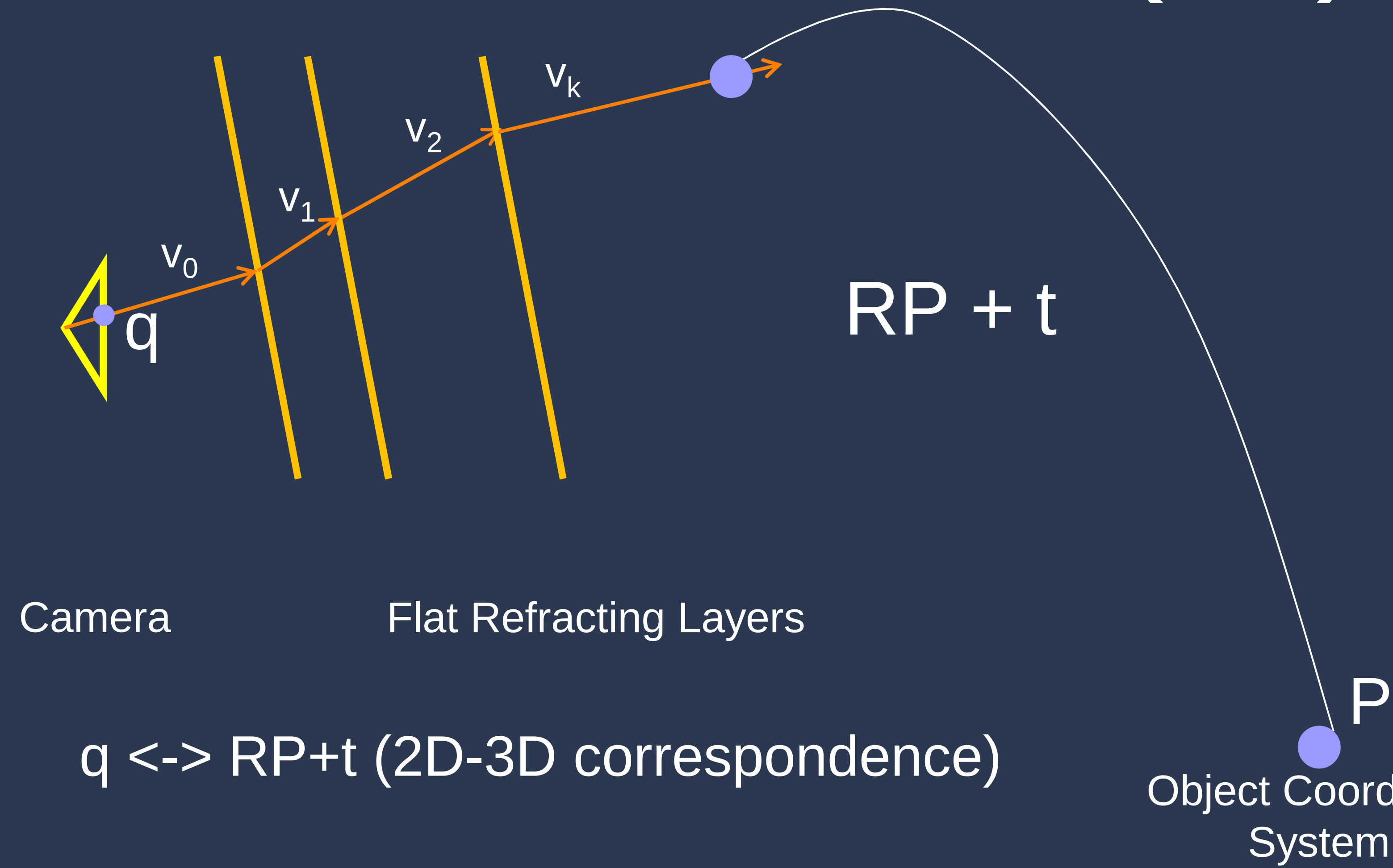
- Treibitz et al. CVPR 2008
  - Singe Refracting Layer
  - Known refractive index
  - Known distance of checkerboard
  - Optimize over one parameter d
    - Known internal camera calibration

- This paper
  - Multiple layers, unknown refractive indices
  - 2K parameters for K layers
  - Unknown Pose of calibrating object (6 parameters)
  - Unknown Orientation of layers (2)
  - 8 + 2K parameters

  - Single layer 10 parameters
  - Two Layers 12 parameters



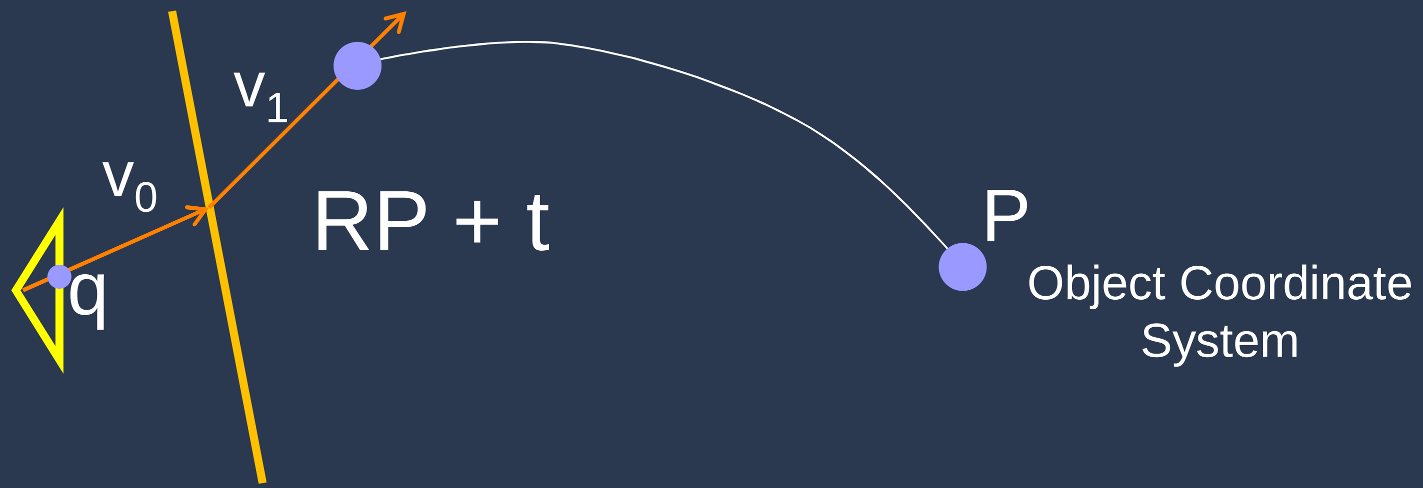
Flat Refraction Constraint (FRC)



q <-> RP+t (2D-3D correspondence)

Transformed 3D point (RP+t) should lie on the outgoing ray v_k

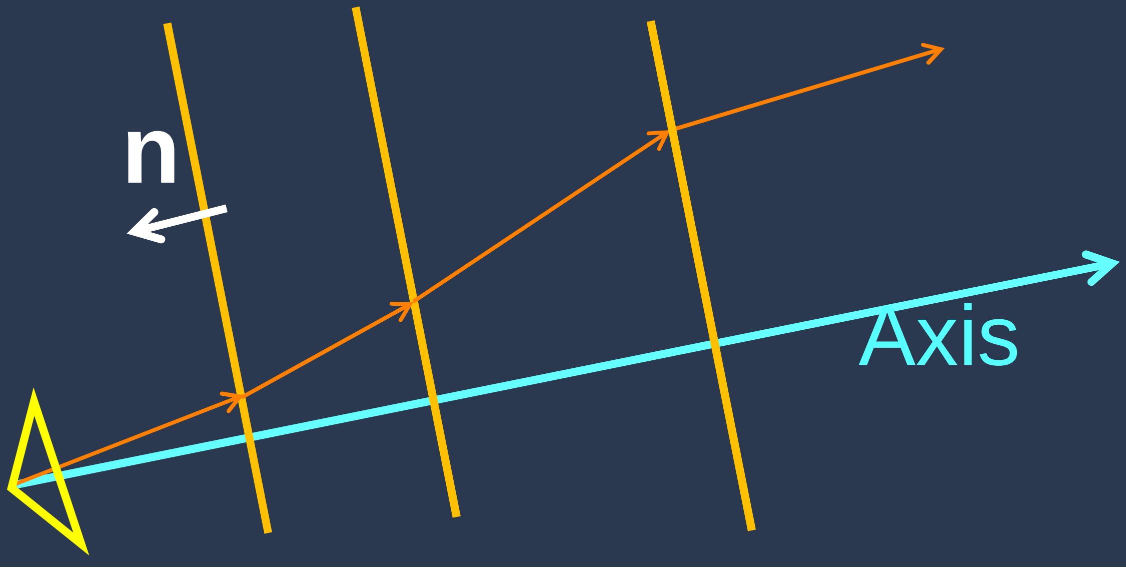
FRC for Single Layer



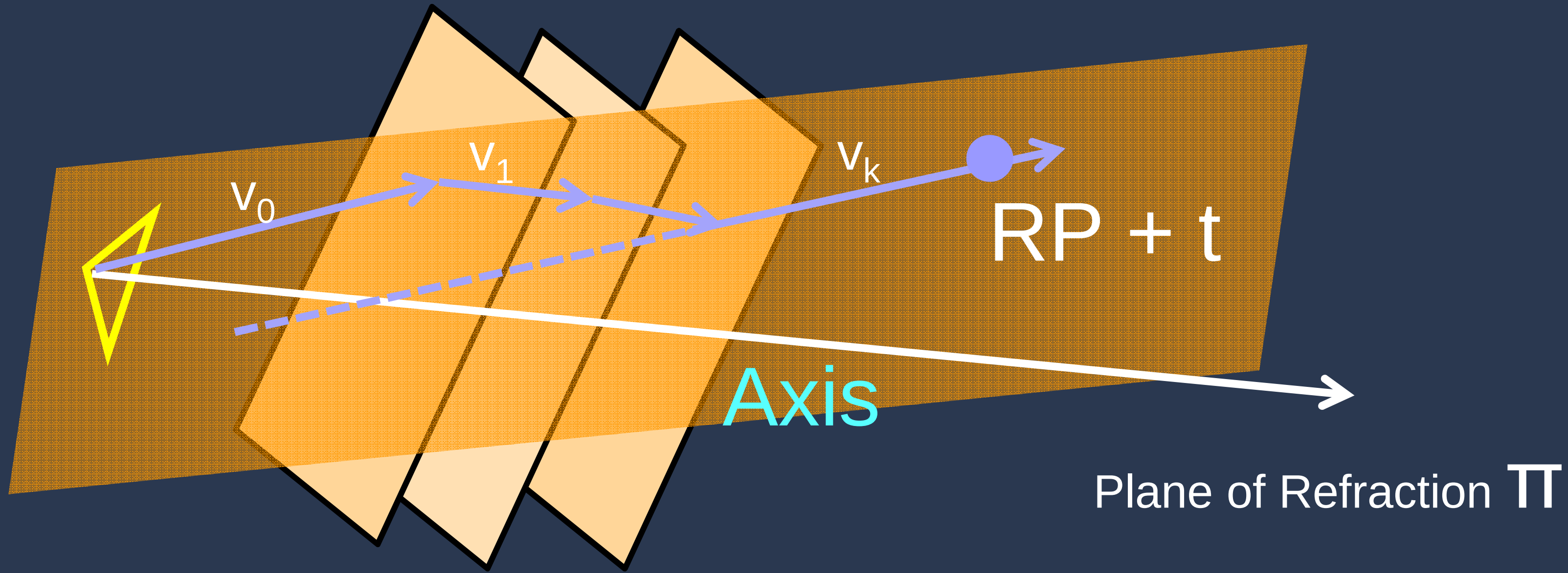
- Non-Linear Equation in 10 unknowns
- Difficult to solve
- Complexity increases with each additional layer

Modeling Flat Refractions

- Pinhole Model is not good
  - Non-single view point camera
  - Well-known in photogrammetry (Kotowski 1988)
  - Treibitz et al. CVPR 2008
- Flat Refraction corresponds to Axial (non-central) camera
  - All outgoing rays pass through an axis
  - Axis: Camera ray parallel to layer orientation **n**



Flat Refraction == Axial Camera



Transformed 3D point (RP + t) should also lie on the plane of refraction

$$(R\mathbf{P} + \mathbf{t})^T (\mathbf{A} \times \mathbf{v}_0) = 0$$

Key Idea: Coplanarity Constraint

- Transformed 3D point (RP+t) should lie on plane of refraction
  - Weaker constraint than FRC
- Axis A, Camera ray v_0

$$(R\mathbf{P} + \mathbf{t})^T(\mathbf{A} \times \mathbf{v}_0) = 0$$

- Independent of number of layers, layer distances and their refractive indices
- Allows estimating axis and pose independently of other calibration parameters

Coplanarity Constraint

$$(R\mathbf{P} + \mathbf{t})^T (\mathbf{A} \times \mathbf{v}_0) = 0$$

$$E = [\mathbf{A}]_\times R \text{ and } \mathbf{s} = \mathbf{A} \times \mathbf{t}.$$

$$\mathbf{v}_0^T E \mathbf{P} + \mathbf{v}_0^T \mathbf{s} = 0$$

- Translation along axis vanishes in **s**
- 5 out of 6 pose parameters can be computed

11 Point Linear Algorithm

$$v_0^T E_{3x3} P + v_0^T s_{3x1} = 0$$

↓

Using 11 2D-3D correspondences, we get 11 by 12 matrix **B**

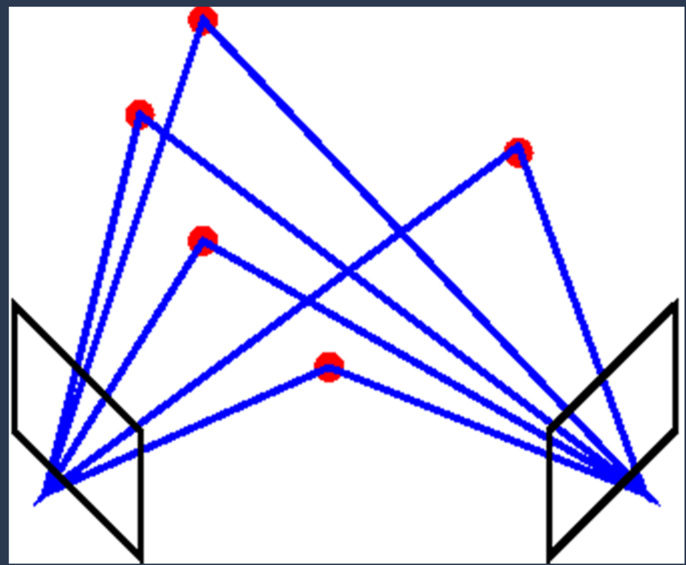
$$\underbrace{\begin{bmatrix} (\mathbf{P}(1)^T \otimes \mathbf{v}_0(1)^T) & \mathbf{v}_0(1)^T \\ \vdots & \vdots \\ (\mathbf{P}(11)^T \otimes \mathbf{v}_0(11)^T) & \mathbf{v}_0(11)^T \end{bmatrix}}_{\mathbf{B}} \begin{bmatrix} E(:) \\ \mathbf{s} \end{bmatrix} = 0$$

↓

SVD based solution

Similarity with 5-point Relative Pose Problem

- $E = [A]_x R$, where A is the axis and R is unknown rotation
- For relative pose between two cameras
  - Essential matrix $E = [t]_x R$, where t is the translation
  - 5-point algorithm [Nister 2004]
- We can map our problem to the 5-point Relative Pose problem



8-Point Axis Estimation Algorithm

$$v_0^T E P + v_0^T s = 0$$

- Using 8 correspondences, we get 8 by 12 matrix **B**
- Solution lies in 4 dimensional sub-space

$$\begin{bmatrix} E(:) \\ \mathbf{s} \end{bmatrix} = \lambda_1 \mathbf{V}_1 + \lambda_2 \mathbf{V}_2 + \lambda_3 \mathbf{V}_3 + \lambda_4 \mathbf{V}_4$$

$$E(:) = \lambda_1 \mathbf{V}_1(1:9) + \lambda_2 \mathbf{V}_2(1:9) + \lambda_3 \mathbf{V}_3(1:9) + \mathbf{V}_4(1:9)$$

Feed subspace vectors to Nister's Solver and obtain λ_i

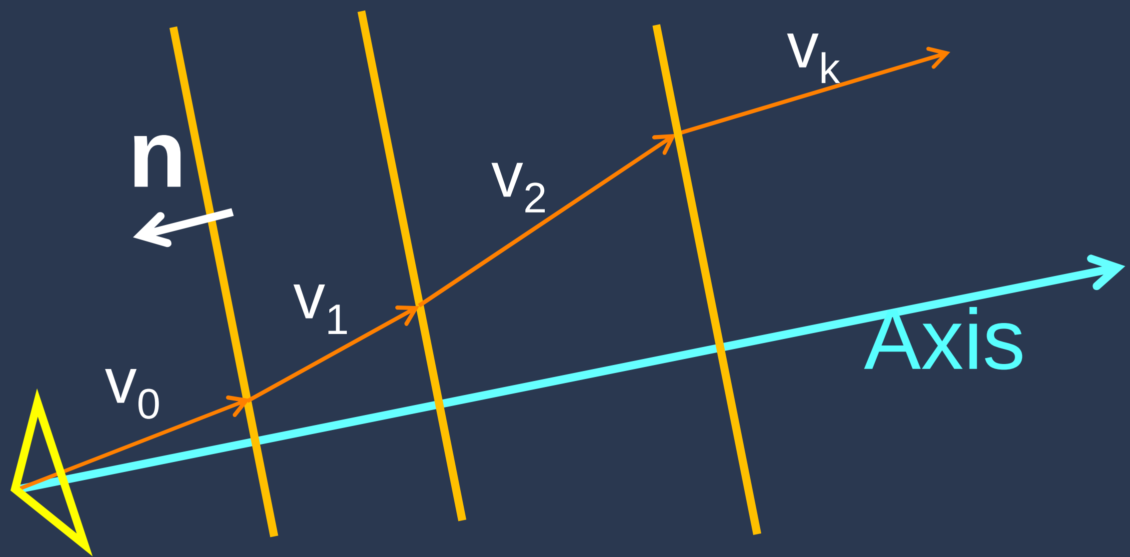
8-Point Axis Estimation Algorithm

- Compute Axis from E as left null-singular vector
  - $A^T E = 0$
- Compute Rotation matrices from E
  - Hartley and Zisserman, Multiview Geometry
  - Twisted pair ambiguity
  - Similar to Relative Pose problem

Obtaining Remaining Calibration Parameters

- Coplanarity Constraint
  - Obtain axis A, rotation R, and $s = A \times t$
- Remaining calibration parameters
  - Translation along axis t_A
  - Layer Thickness d_i, $i = 1$ to k
  - Layer Refractive Indices μ_i, $i = 1$ to k

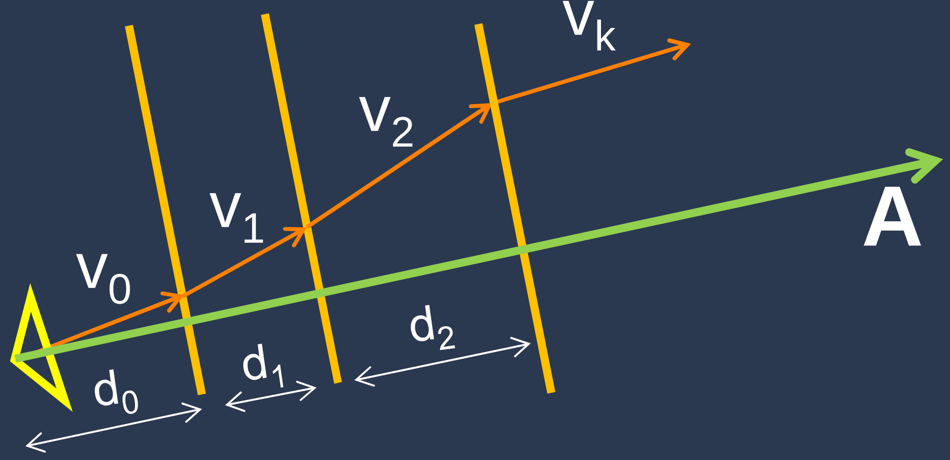
Known Refractive Indices



- Ray directions of v_1,.... v_k can be computed using Snell's Law
- Layer Thicknesses d_i and t_A can be computed linearly

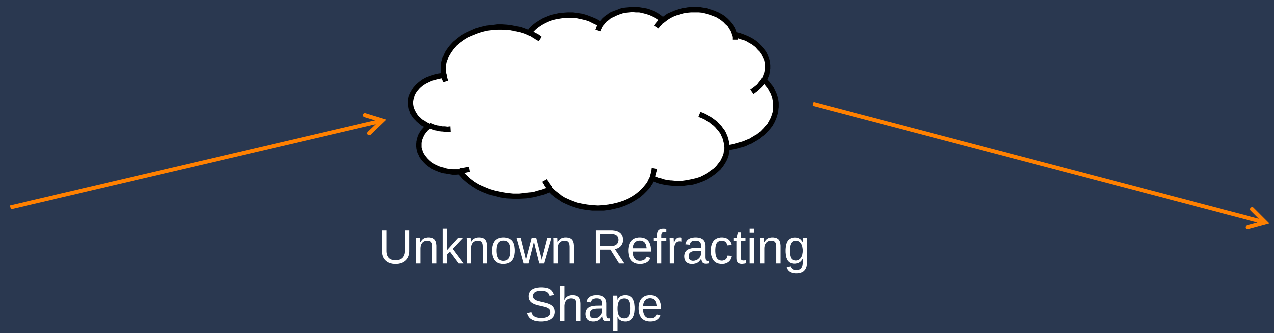
Linear System

$$\begin{pmatrix} v_k \times v_0 & v_k \times v_1 & \cdots & v_k \times v_{k-1} & v_k \times A \end{pmatrix} \begin{pmatrix} d_0 \\ \vdots \\ d_{k-1} \\ t_A \end{pmatrix} = \mathbf{u}$$



Light Path Triangulation

- Steger and Kutulakos, IJCV 2008

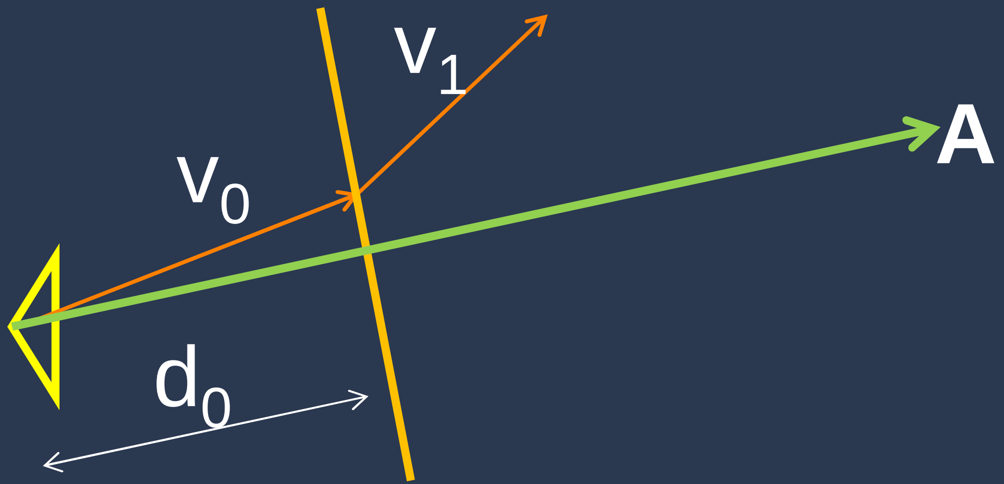


- Triangulation is not possible for more than 2 refractions
- General Shapes

- Theoretically possible for multi-layer flat refractions
  - Partial knowledge of shape
  - Flat layers, parallel to each other

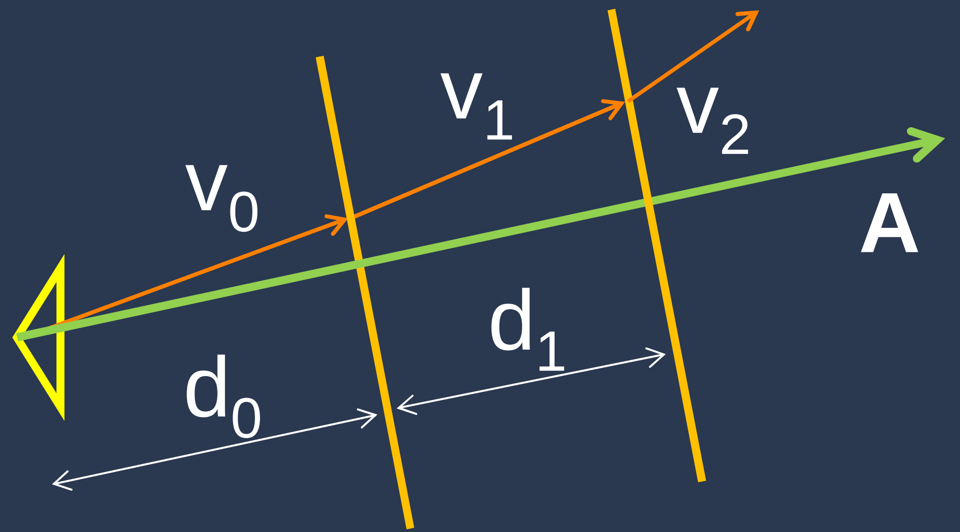
Case 1: Single Layer

$$\begin{pmatrix} v_1 \times v_0 & v_1 \times A \end{pmatrix} \begin{pmatrix} d_0 \\ t_A \end{pmatrix} = \mathbf{u}$$



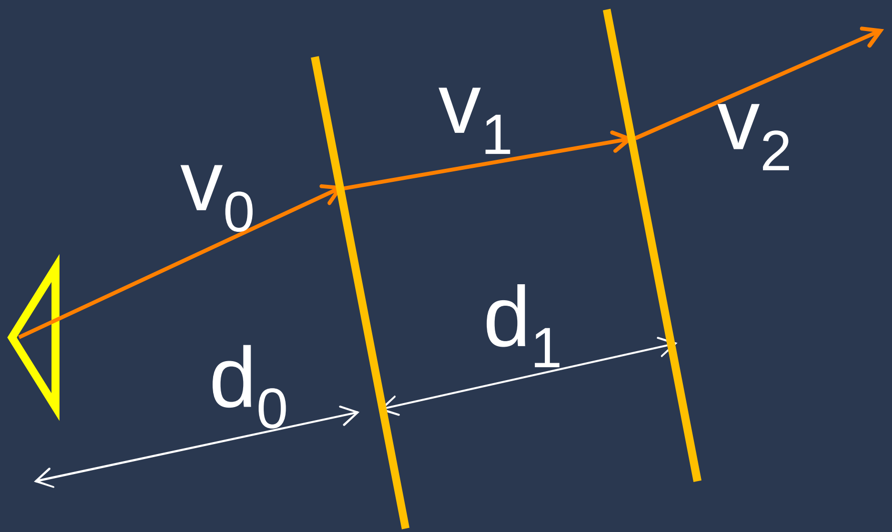
Case 2: Two Layers

$$\begin{pmatrix} v_2 \times v_0 & v_2 \times v_1 & v_2 \times A \end{pmatrix} \begin{pmatrix} d_0 \\ d_1 \\ t_A \end{pmatrix} = \mathbf{u}$$



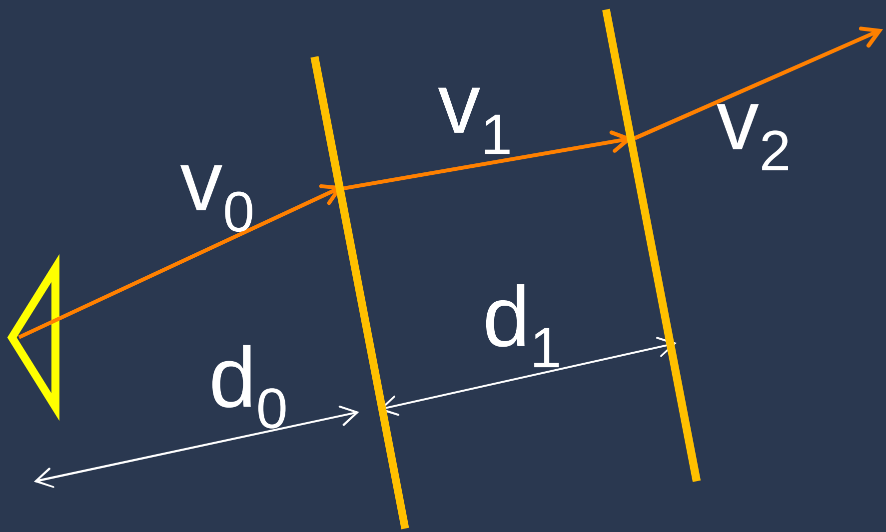
Special Case: Looking through a medium

- Camera and Object are in the same refractive medium
- Example
  - Looking through a thick glass slab
  - (Air – Glass – Air)
  - Final refracted ray v_2 is parallel to camera ray v_0



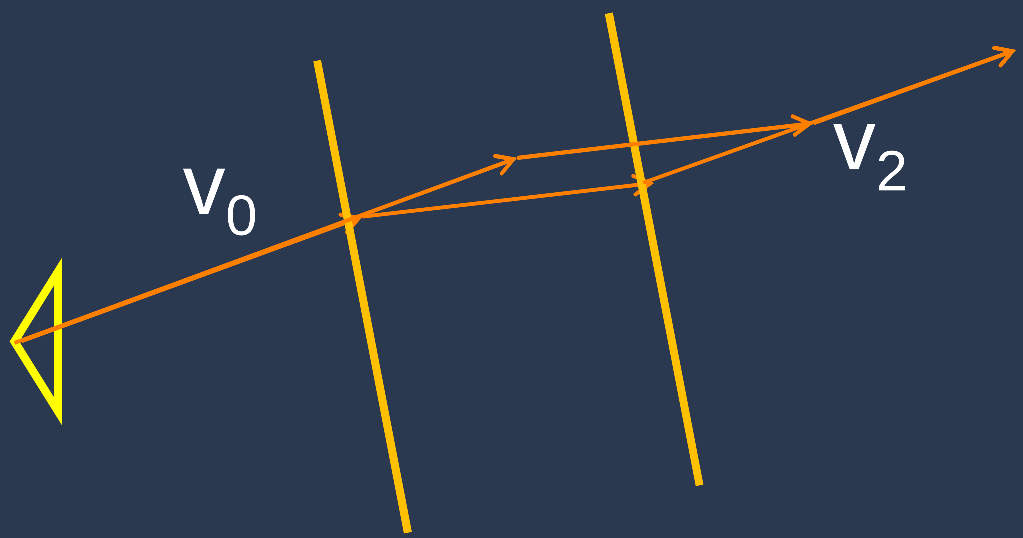
Special Case: Two Layers

$$\left(\begin{array}{ccc} \cancel{v_2 \times v_0}\;0 & v_2 \times v_1 & v_2 \times A \end{array}\right)\left(\begin{array}{c} \cancel{d_0} \\ d_1 \\ t_A \end{array}\right) = \mathbf{u}$$



Special Case: Looking through a medium

- Camera and Object are in the same refractive medium
- Distance to the refractive medium d_0 cannot be estimated
  - Kutulakos and Steger
- Thickness of the medium d_1 can be estimated
- Pose estimation can be done



Multiple Layers

- If two layers i and j have same refractive indices
  - $\mu_i = \mu_j$
- Then only the combined layer thickness $d_i + d_j$ can be estimated

Summary of Calibration

- Step 1: Compute Axis, Rotation and s
  - Using 11 pt or 8 pt algorithm
- Step 2: Compute layer thickness and t_A
  - Solve a linear system

- Unknown Refractive Indices
  - Step 1 remains the same
  - Step 2
    - Solve 6th degree equation for Single Layer
    - Solve 6th degree equation for Air-Medium-Air

  - Too difficult to solve general two layer case

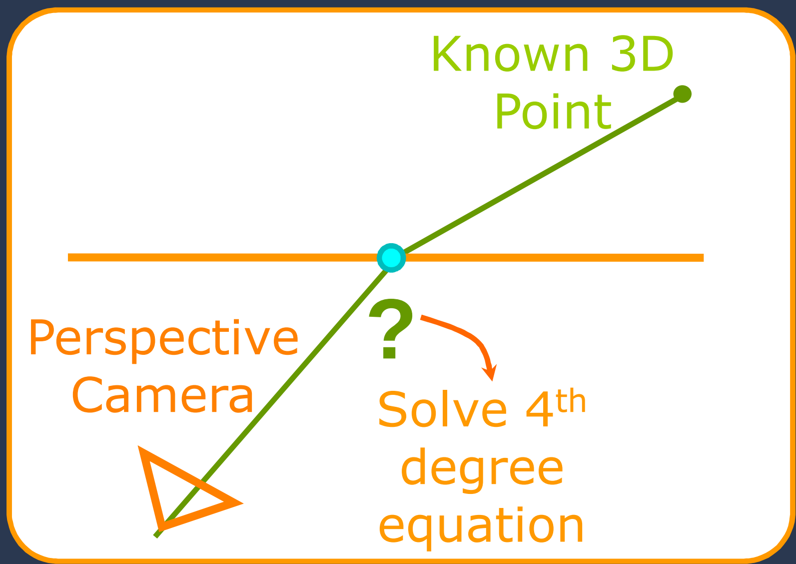
Analytical Forward Projection

- Projection of 3D point onto the image plane?
- Required for minimizing re-projection error
  - bundle-adjustment in SfM
  - Refine calibration parameters

- Perspective projection equations
- $x = P_x/P_z$, $y = P_y/P_z$

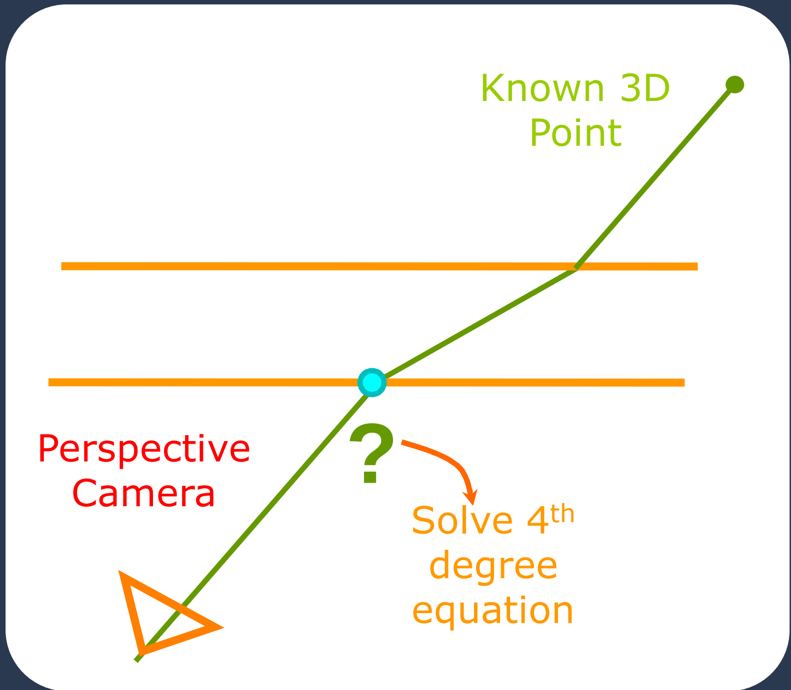
Analytical Forward Projection

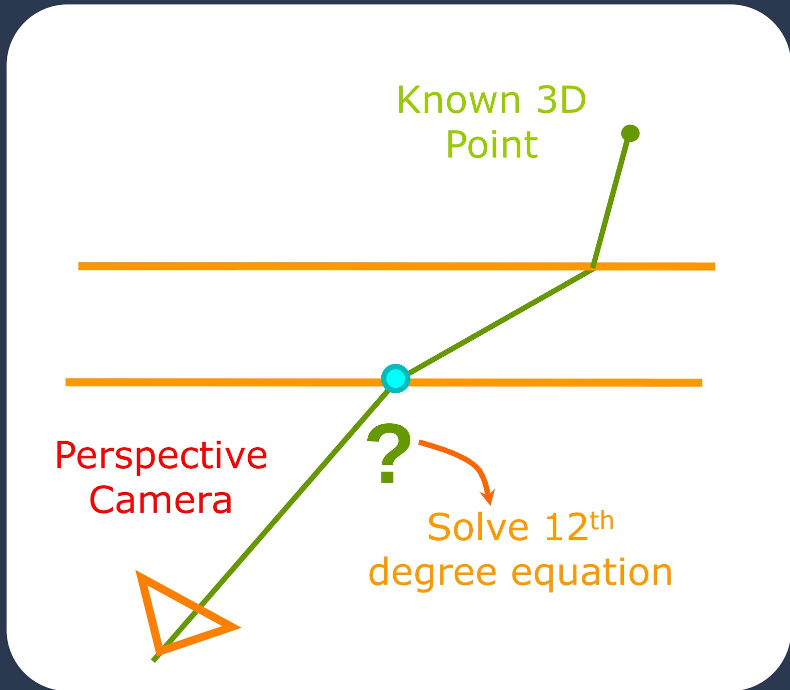
- Single Layer
  - 4^{th} degree equation
  - Glaeser and H.-P.Schrocker. Reflections on refractions, *J. Geometry and Graphics*, 4(1):1–18, 2000



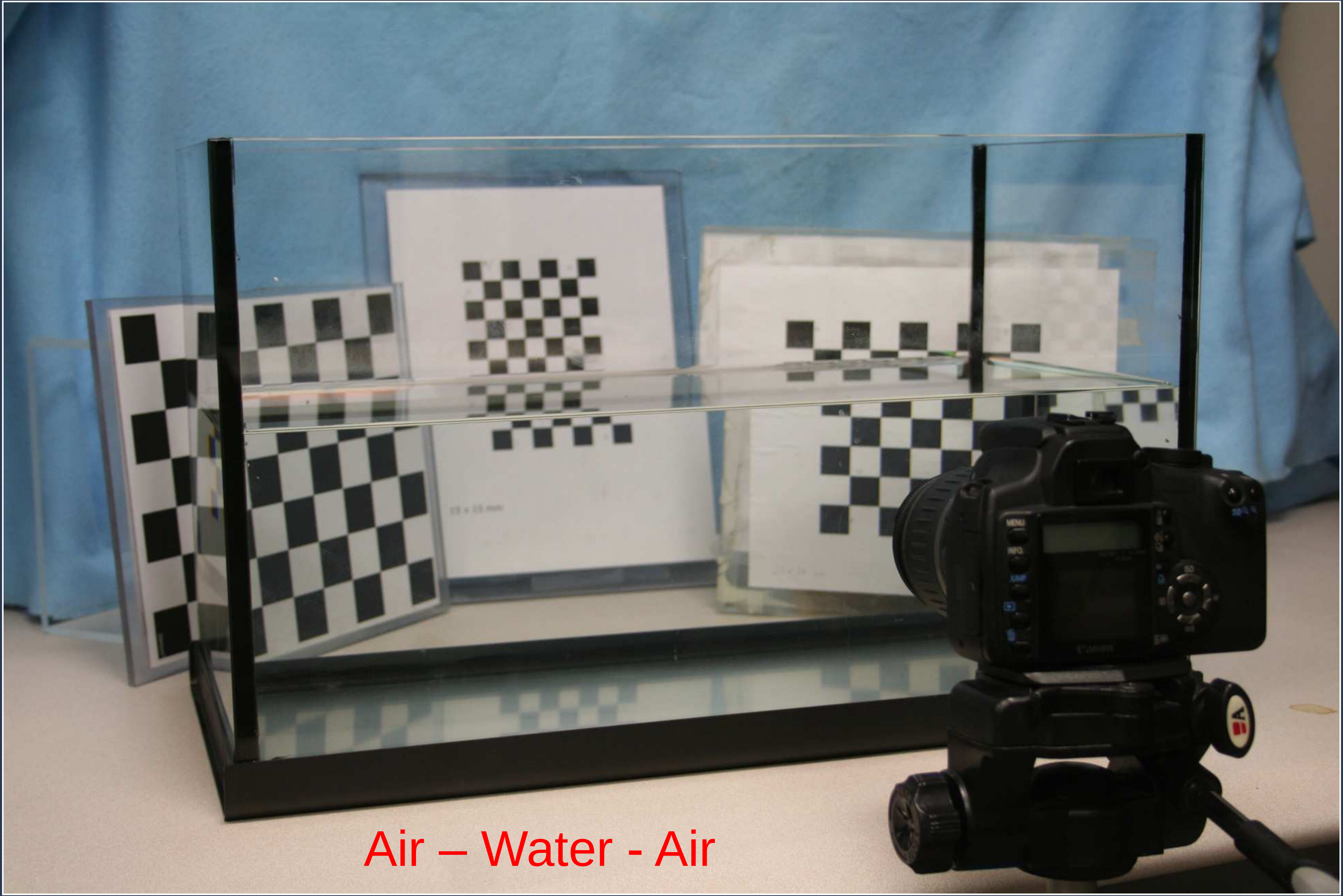
Analytical Forward Projection

- Two Layers
  - Air – Medium – Air: 4^{th} degree equation
  - General Case: 12^{th} degree equation



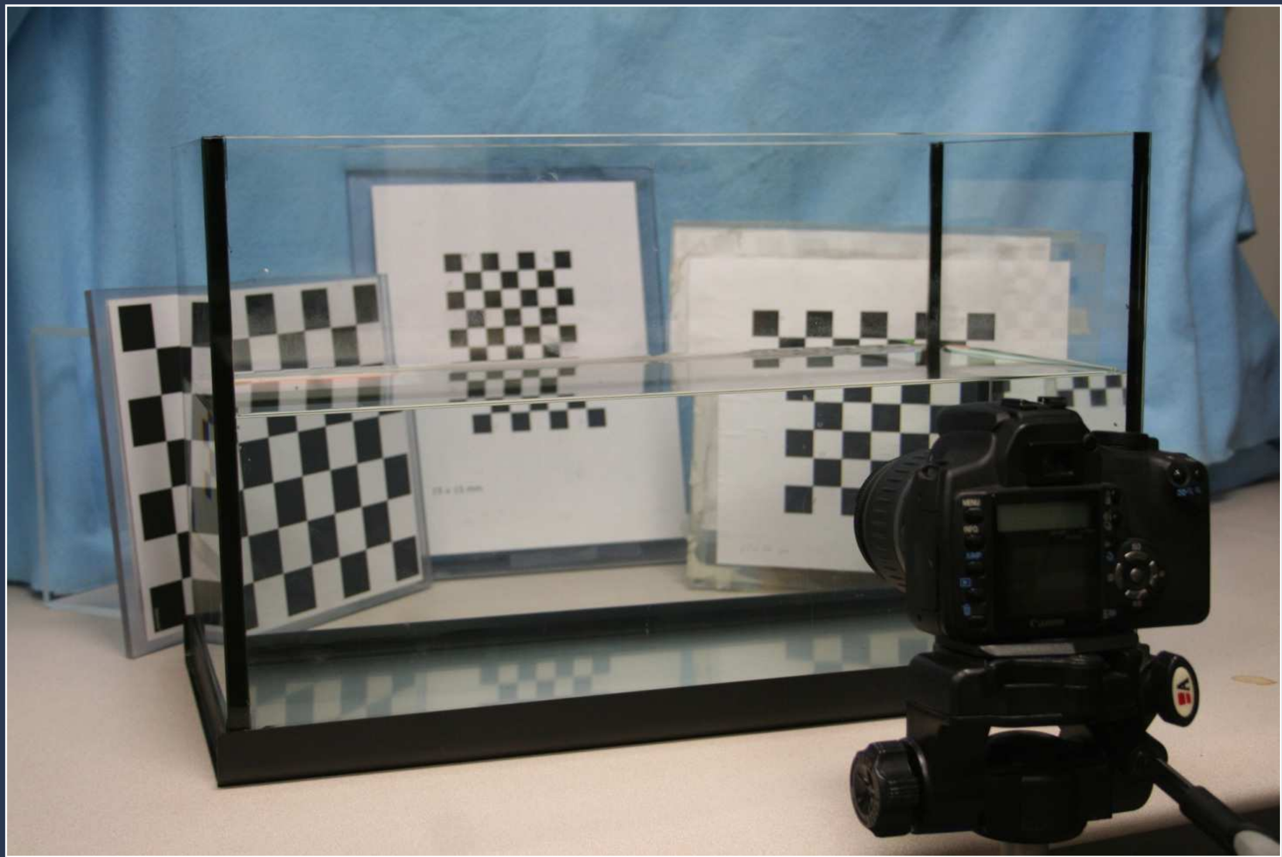


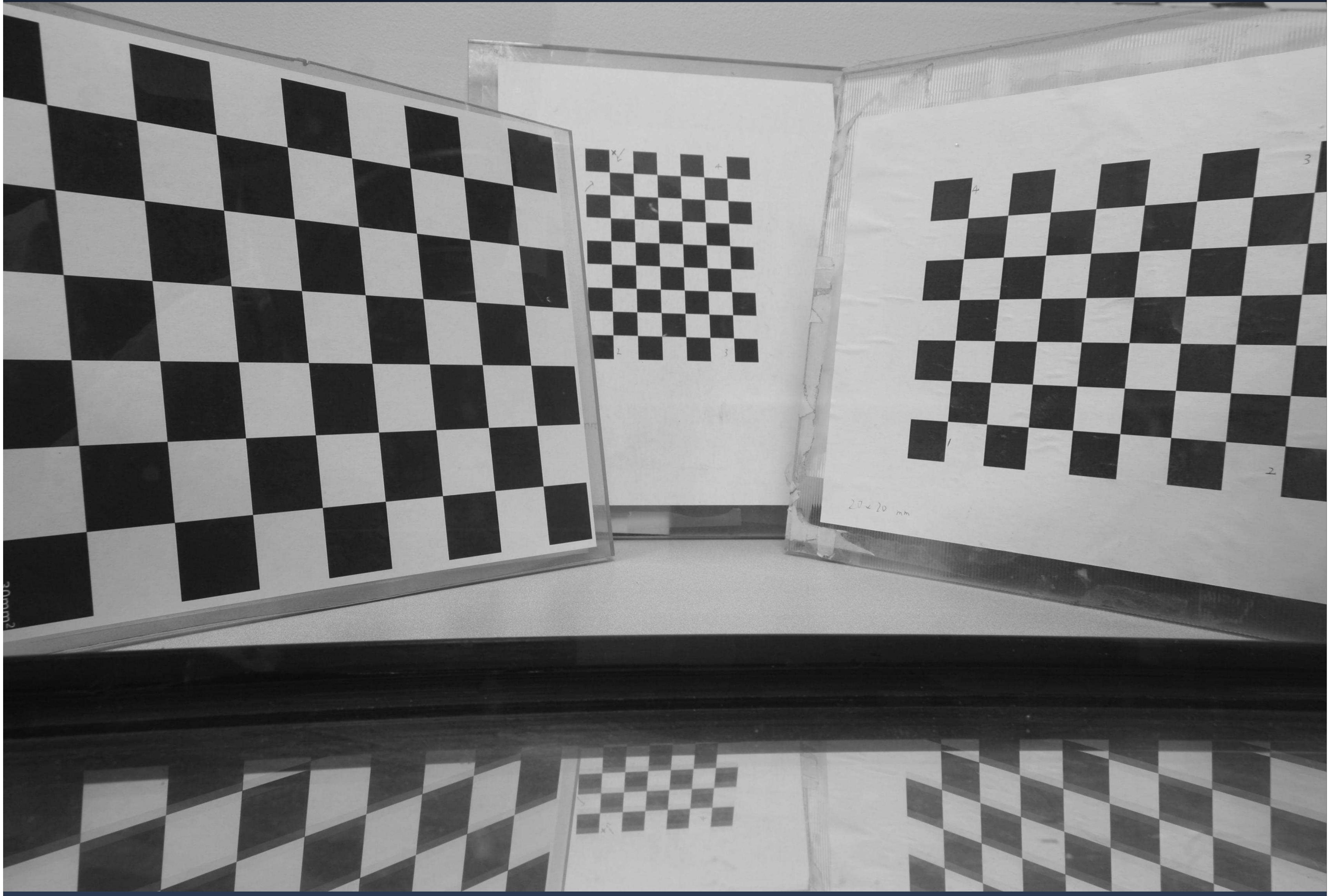
Real Experiment using fish tank



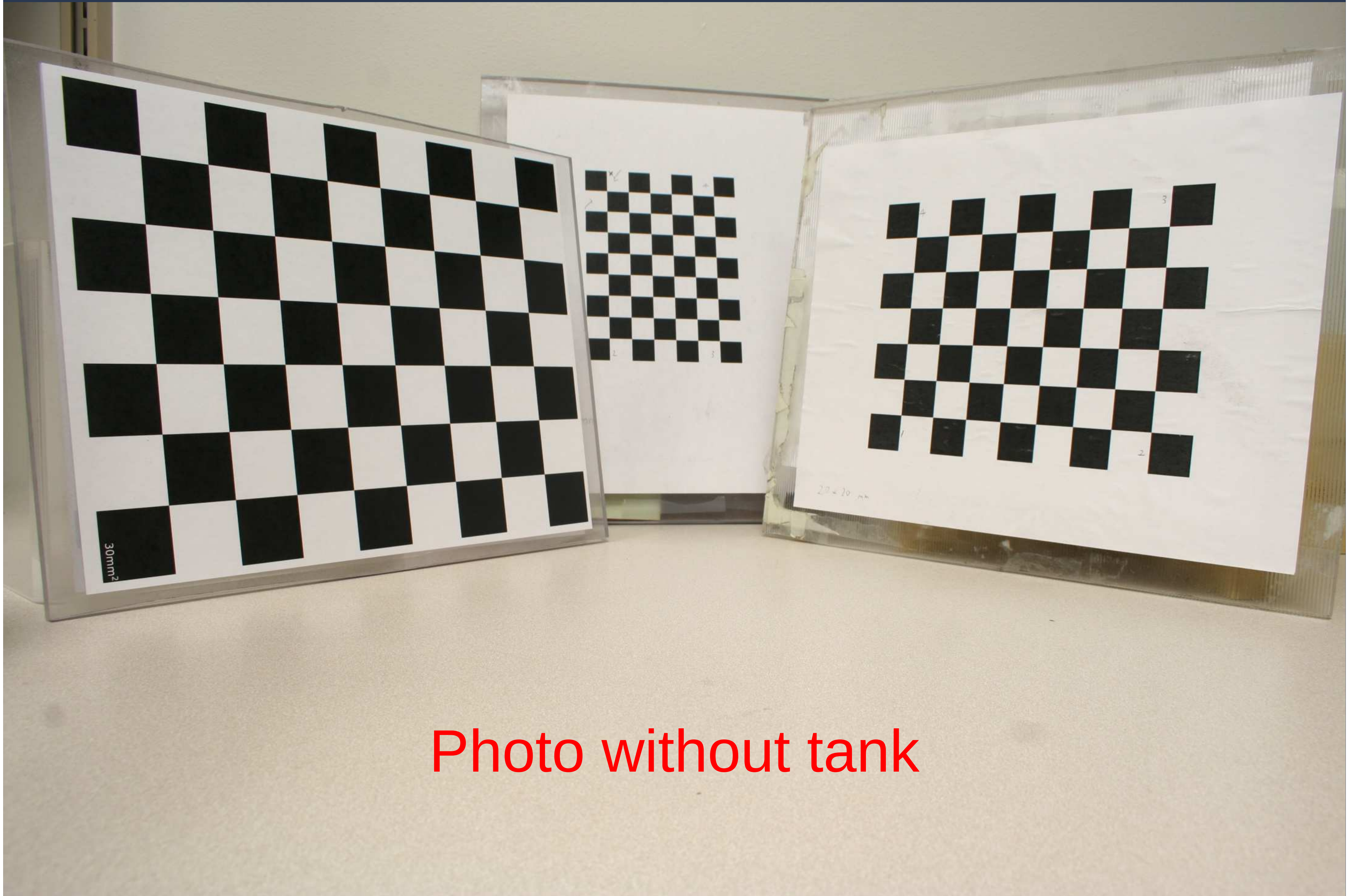
Calibration

- Unknown Thickness of Tank
- Unknown Orientation of Tank
- Unknown Pose of Checkerboards





Captured Photo

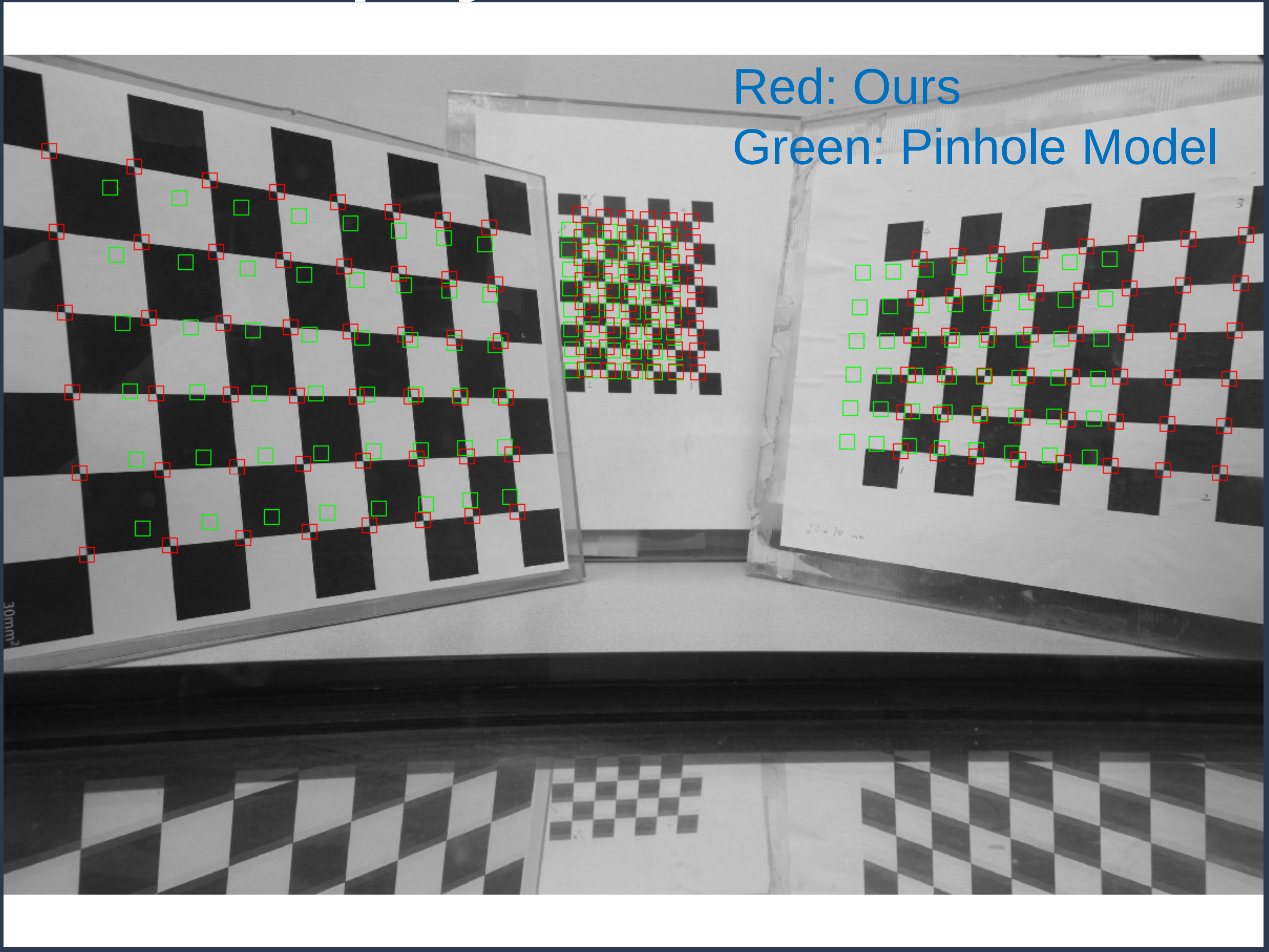

30mm²
Photo without tank

Results

- Thickness of tank measured using ruler = 260 mm

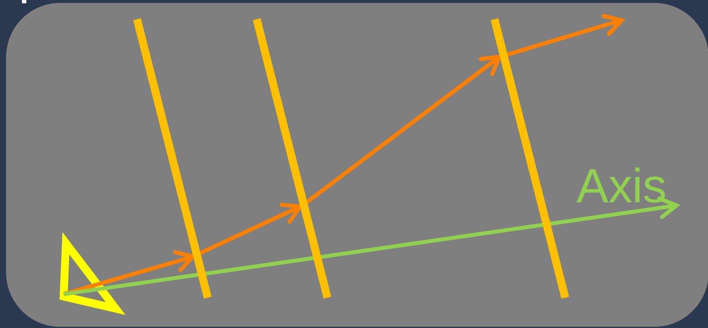
	Estimated Rotation of Checkerboard	Estimated Translation of Checkerboard	Estimated Tank Thickness
Ground truth	131.3, 1.2, 84.0	−237.5,−128.8, 455.8	260
Pinhole Model	130.2, 1.4, 83.8	−217.7,−120.7, 372.1	
Ours (using all planes)	131.3, 1.2, 84.1	−237.1,−128.1, 453.1	255.69
Using Single Plane	131.4, 1.3, 84.0	−239.7,−129.2, 456.3	272.81

Reprojected 3D Points



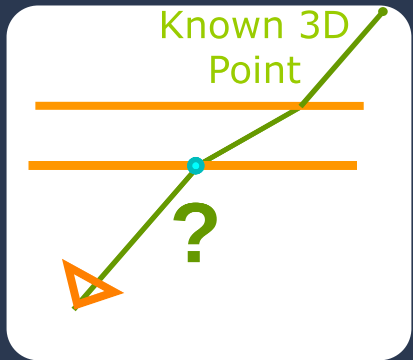
Summary

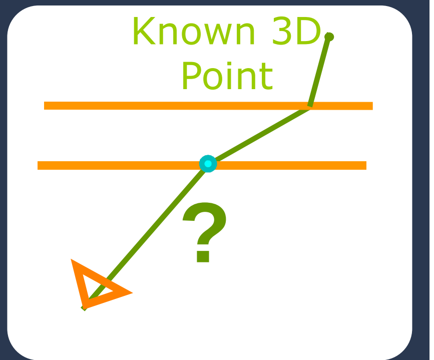
- Multi-Layer Flat Refractions == Axial Camera
  - Pinhole camera model is not a good approximation
  - Calibration algorithm
  - Coplanarity Constraints



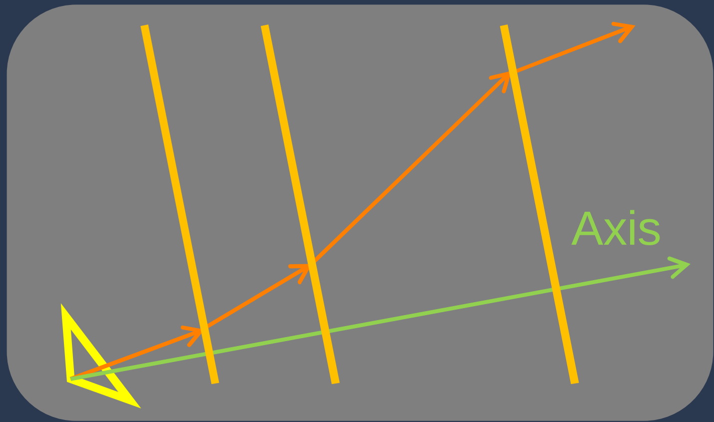
- Applicable to
  - Spherical Ball Refraction (Agrawal et al. ECCV 2010)
  - Catadioptric Cameras (quadric mirrors)
  - Radial distortion correction (Hartley-Kang PAMI 2007)

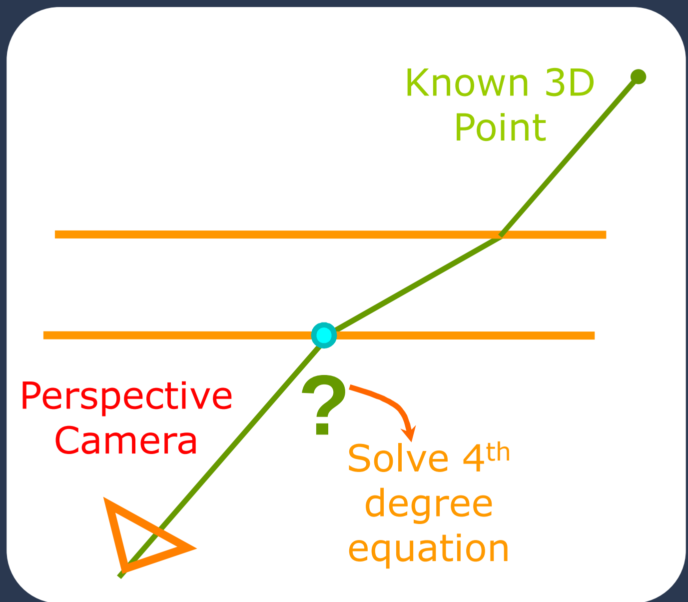
- Analytical Forward Projection

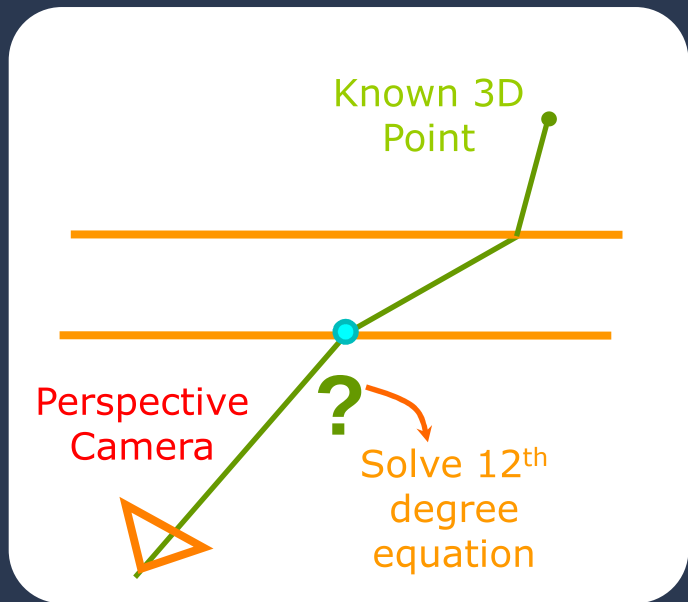




A Theory of Multi-Layer Flat Refractive Geometry







Additional Slides

Related Work

- Calibration of Axial Cameras
  - Ramalingam, Sturm and Lodha, ACCV 2006
    - Requires checkerboard in three positions
  - Tardiff et al. PAMI 2009
    - Models each distortion circle separately
- This paper
  - Calibration using single checkerboard
    - Plane based calibration
  - Global Model

Relationship with Hartley-Kang Algorithm

- Parameter-free radial distortion correction
  - PAMI 2007
- Similar formulation as our coplanarity constraint
  - 8 point algorithm can be applied