#### **Register Allocation**

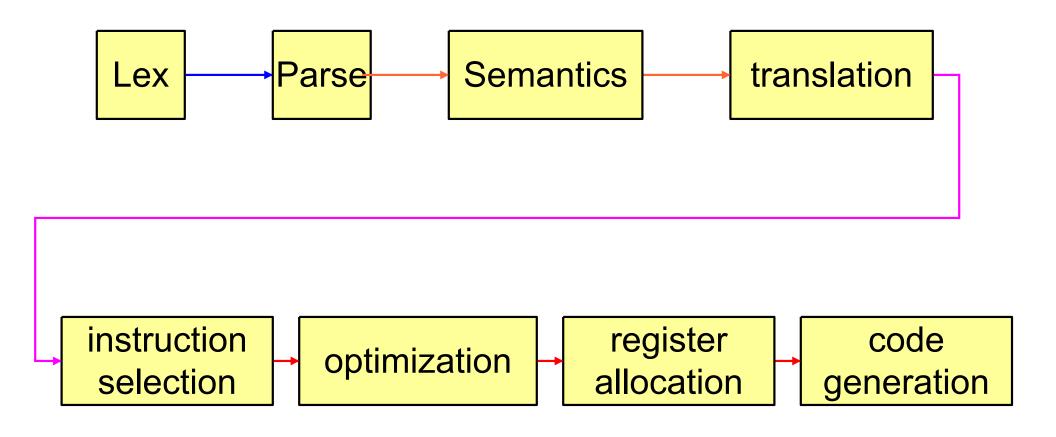
#### 15-411/15-611 Compiler Design

Seth Copen Goldstein



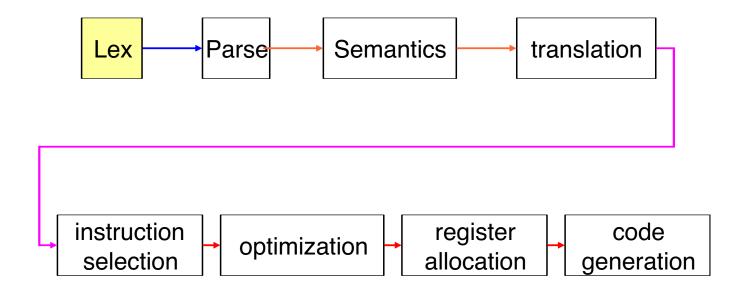
January 16, 2025

### **Cartoon Compiler**



#### **Unusual Order**

- Standard is to start at the start and proceed down the passes: lexing, parsing, ...
- We start with Register Allocation, then do Instruction Selection!



### **Today**

- Intro to language of L1
- briefly: AST, Abstract assembly, Temps
- Register Allocation Overview
- Interference Graph
- Iterated Register Allocation
  - Simplify/Select
  - Coalescing
  - Spilling
- Special Registers

### Simple Source Language

- A language of assignments, expressions, and a return statement.
- Straight-line code
- Basically lab1 subset of C0

## Simple Source Language

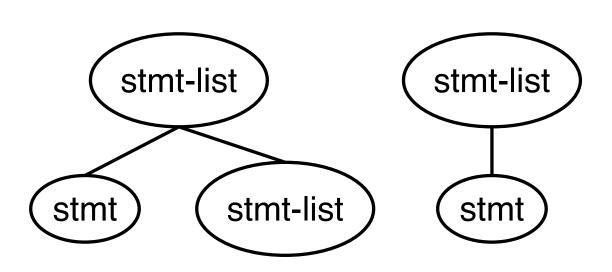
program :=  $S_1$ ;  $S_2$ ; ...  $S_n$ ; sequence of statements

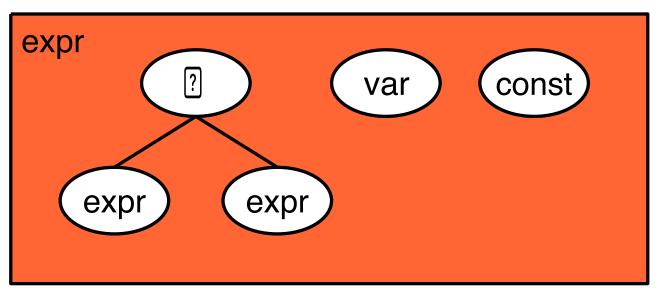
- s := v = e assignment
  - return ereturn
- e = c constant
  - v variable
  - e<sub>1</sub> e<sub>2</sub> binary operation
- ? := + | |\* | / | %

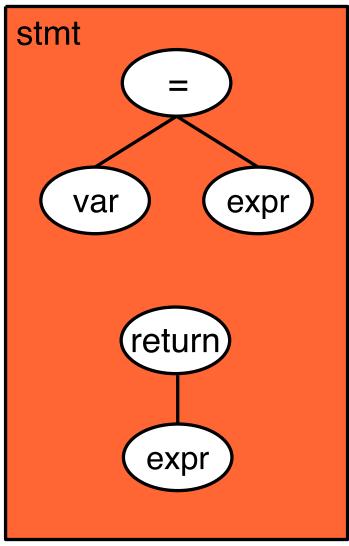
X=4+1

Ambiguity? Semantics?

### **Abstract Syntax Tree**



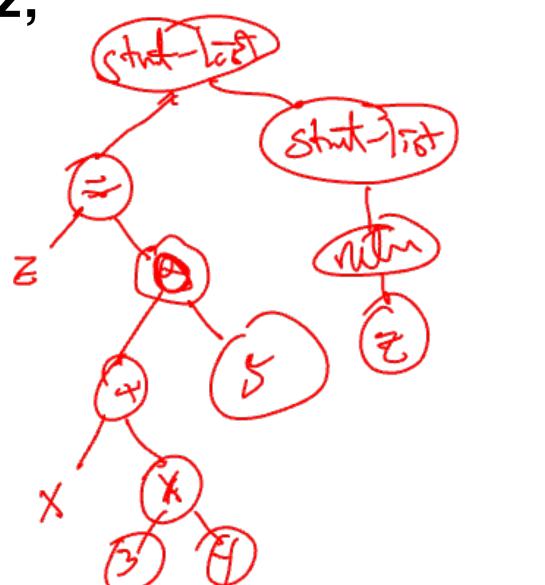




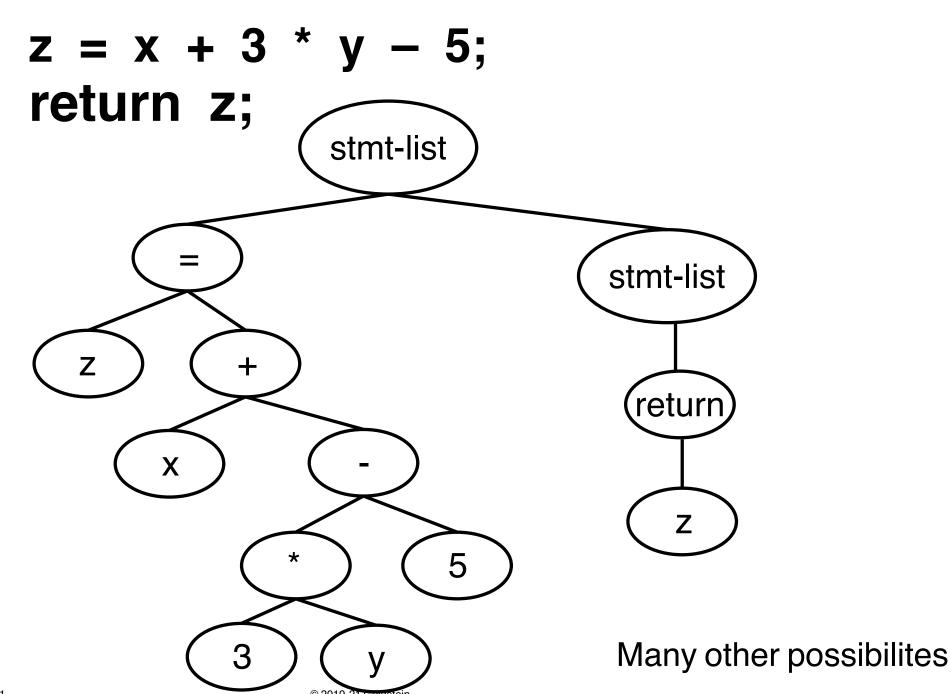
#### **Example**

z = x + 3 \* y - 5;

return z;



#### Possible parse tree



## **Abstract Assembly as IR**

Lowering of AST



- Facilitate
  - Analysis & optimizations
  - Translation to actual assembly

In today's world aka registers

- Features:
  - Unlimited number of "temporaries" -
  - May (or may not) restrict how memory is used
  - Simple operations
  - May (or may not) restrict how constants are used
  - May specify certain "special registers"

### **Abstract Assembly as IR**

#### Features:

- Unlimited number of "temporaries"
- May (or may not) restrict how memory is used
- Simple operations
- May (or may not) restrict how constants are used
- May specify certain "special registers"
- dest ? src1 operator src2
- dest ? operator src1

operator

#### src can be:

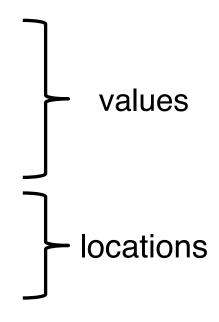
- constant
- temporary
- special register
- memory



### **Abstract Assembly Language**

program :=  $i_1 i_2 ... i_n$  seq of instructions

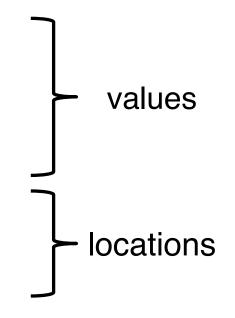
- intermediate constants of some type
- temporary a compiler generated location which holds a value. After compilation it will be mapped to a register or a memory location
- register generally a real register from the target architecture



## **Abstract Assembly Language**

program :=  $i_1 i_2 ... i_n$  seq of instructions

```
i := d ? s move
  I d? s_1? s_2 binop
     return § return
s := c intermediate
       temporary
        register
d := t
? := + | - |* | / | %
```



What is right "level"?

#### Closer to the machine

program :=  $i_1 i_2 ... i_n$  seq of instructions

```
i := d? s move
  I d? s_1? s_2 binop
     return what is in rax
s := c intermediate
     t temporary
     r register
d := t
? := + | - |* | / | %
```

#### **Deep Breath**

- Defined source language using BNF
  - Ambiguity
  - Semantics
- AST
- Abstract assembly
  - Operators
  - L-values and R-values
  - Temps, registers, constants

### **Register Allocation**

 Until register allocation we assume an unlimited set of registers (aka "temps" or "pseudo-registers").

 But real machines have a fixed set of registers.

 The register allocator must assign each temp to a machine register.

### **Register Allocation**

- Map the variables & temps in the abstract assembly to actual locations in the machine
- The locations are either
  - physical registers
  - -(slots in the activation frame



- Essential for modern architectures
  - registers are much faster, consume less power, etc.
  - Some operations require registers
  - Goal: Try and allocate as many of the important variables/temps to registers.
- However, there are only a few registers

#### **Locations**

- Physical registers
- Slots in the activation frame

### **Sub-tasks of Register Allocation**



- Assignment: map temps to particular registers
- Spilling: If we can't assign to a register, assign to a slot in the stack frame and add code to save and restore temp.
- Coalescing: If possible eliminate moves, a? b, and map both a & b to the same location.
- Ensure special cases are handled properly.
  - instructions, e.g., imul, ret, ...
  - ABI, e.g., callee/caller save registers, function arguments.

#### **Register Allocation**

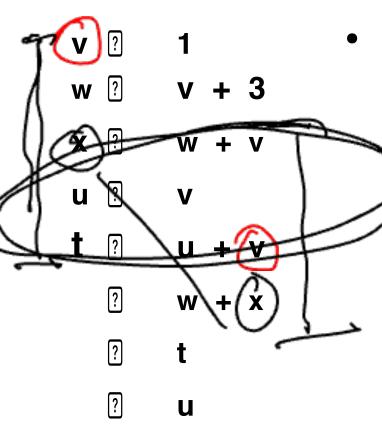
- Given: k registers and code with n registers
   Goal: transform code to use only k registers
- For every instruction we will:
  - Determine which values are in registers
  - -Select a register for each value
- Global register allocation is of course NPcomplete
- Develop heuristics which minimize running time

#### Interference

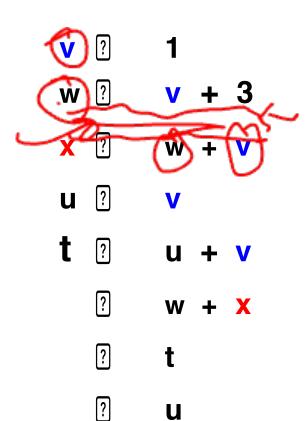
- Consider two temps, t0 and t1.
- If the live ranges for t0 and t1 overlap, we say that they interfere.

- First rule of register allocation:
  - Temps with interfering live ranges may not be assigned to the same machine register.



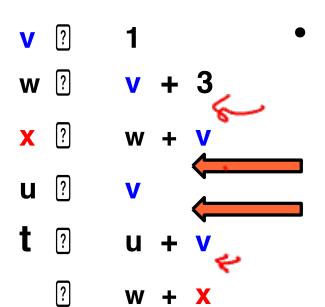


 Two variables, e.g., x & v, need to be in different
 registers if at some point in the program they hold different values.



 Two variables, e.g., x & v, need to be in different registers if at some point in the program they hold different values.

What (if any) program points require x & v to be in different registers? (E.g., where do they "interfere"?)



?

?

t

u

Two variables, e.g., X & V, need to be in different
registers if at some point in
the program they hold different values.

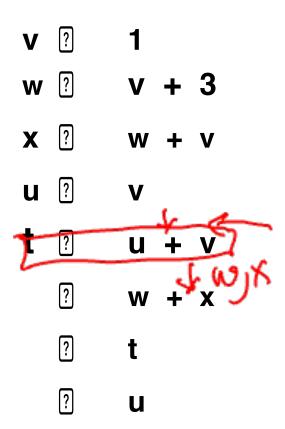
- v ? 1
  w ? v + 3
  x ? w + v
  u ? v
  t ? u + v
- Two variables, e.g., x & v, need to be in different registers if at some point in the program they hold different values.
  - Use liveness information
  - A variable is live at a given point in the program if it is defined and can be used at some later point in the program.

?

?

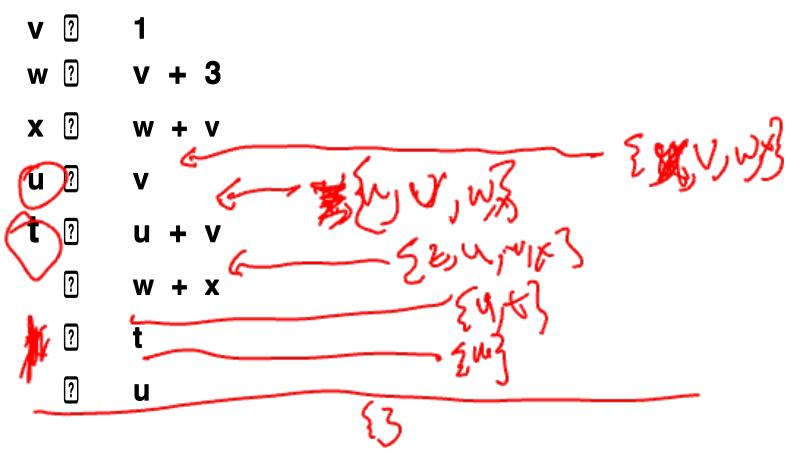
? **u** 

#### Liveness in straight line code



- Work backwards and at each instruction:
- If variable is used on right hand side, it is live-in
- if variable was live before it is still live-in (unless defined on left-hand side)

#### Liveness in straight line code



- Work backwards and at each instruction:
- If variable is used on right hand side, it is live-in
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#### Liveness in straight line code

```
?
V
                     { v }
                                           live-in sets
w ?
                     { w, v }
X ?
                     { w, x, v }
u ?
      V
                       w, u, x, v }
  ?
                       w, t, u, x }
  ?
                     { u, t }
  ?
  ?
```

- Work backwards and at each instruction:
- If variable is used on right hand side, it is live-in
- if variable was live before it is still live-in (unless defined on left-hand side)

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#### Live-out more useful

```
v ?
w ?
X ?
u ?
t ?
  ?
  ?
  ?
```

```
{ V }
{ w, v }
{ w, x, v }
{ w, u, x, v }
{ w, t, u, x }
{ u, t }
{ u }
```

#### Interference and Liveness

```
v ?
                   { V }
w ?
                   { w, v }
X ?
                   { w, x, v }
u ?
      V
                     w, u, x, v }
 ?
                      w, t, u, x
  ?
      W + X
                   { u, t }
  ?
                   { u }
  ?
      u
```

 Two variables that are live at the same point in the program interfere with each other and need to be assigned to different registers.

32

#### **General Plan**

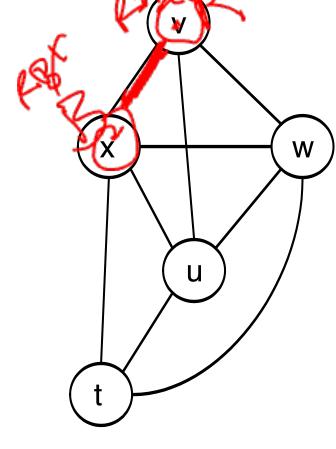
- Construct an interference graph
- Map temps to registers
- Deal with spills
- Generate code to save & restore
- Respect special registers
  - avoid reserved registers
  - Use registers properly
  - respect distinction between callee/caller save registers

### **Interference Graph**

Nodes are temps and registers

• Edge (a,b) indicates a and b "interfere" In other words, a and b cannot be in the

same register.





#### Construct Interference Graph

- Use liveness information
- Each node in the interference graph is a temp
- (u,v) Giff u & v can't be in the same hard register,
   i.e., they interfere

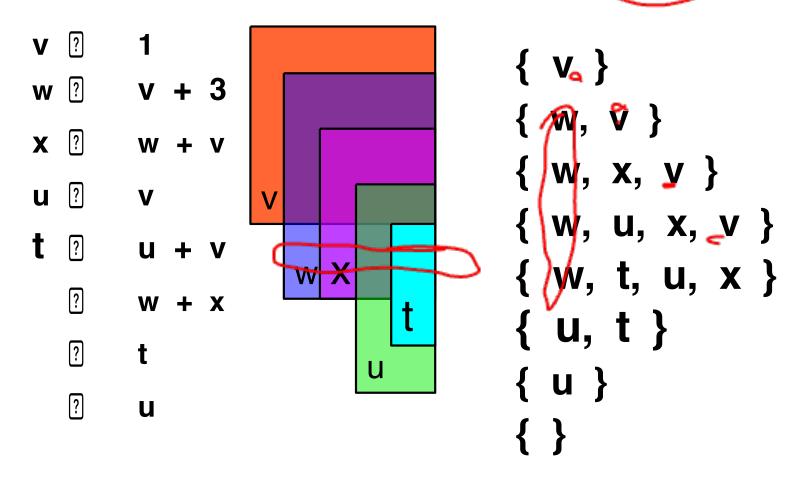
#### Color Graph

Assign to each node a color from a set of k colors,k = I register set I

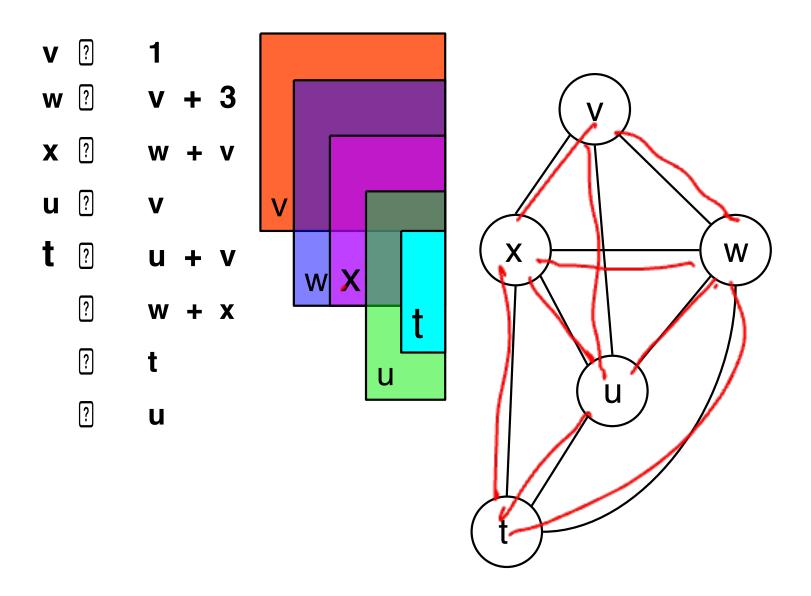
#### Spill

 If can't color graph with k colors then spill some temps into memory. Regenerate asm code and start over.

# An Example (k=4)

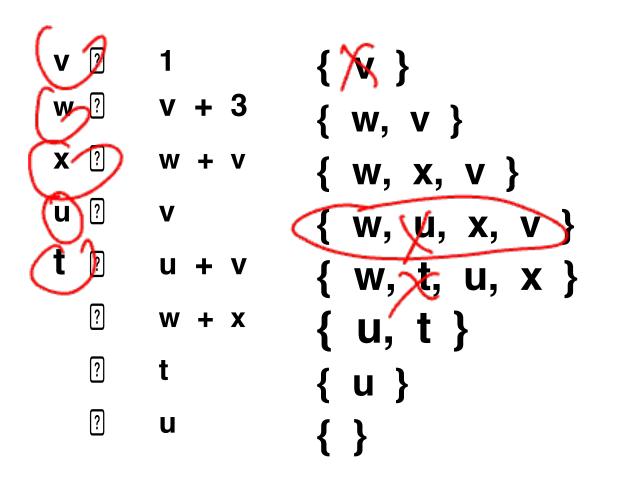


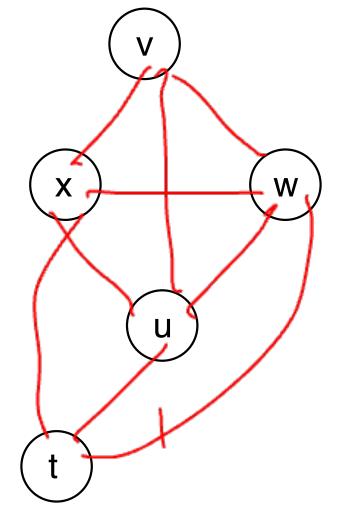
Compute live ranges



Construct the interference graph

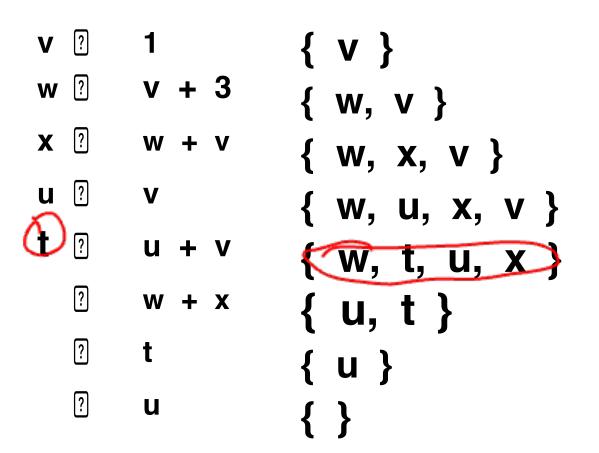
#### **In Practice**

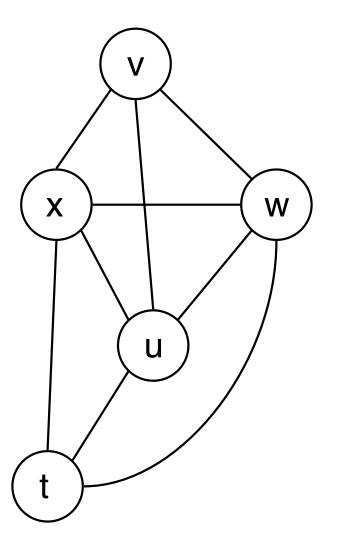




 At point of definition of t, add edges between t and all u ? live-out, t?u

#### In Practice





 At point of definition of t, add edges between t and all u ? live-out, t?u

v ? 1

w ? v + 3

X ? W + V

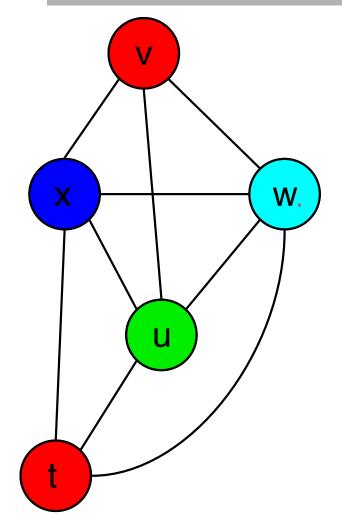
**u** ? **v** 

t ? u + v

? **t** 

? **u** 

#### Voila, registers are assigned!



A greedy Coloring

#### A Special Interference Edge

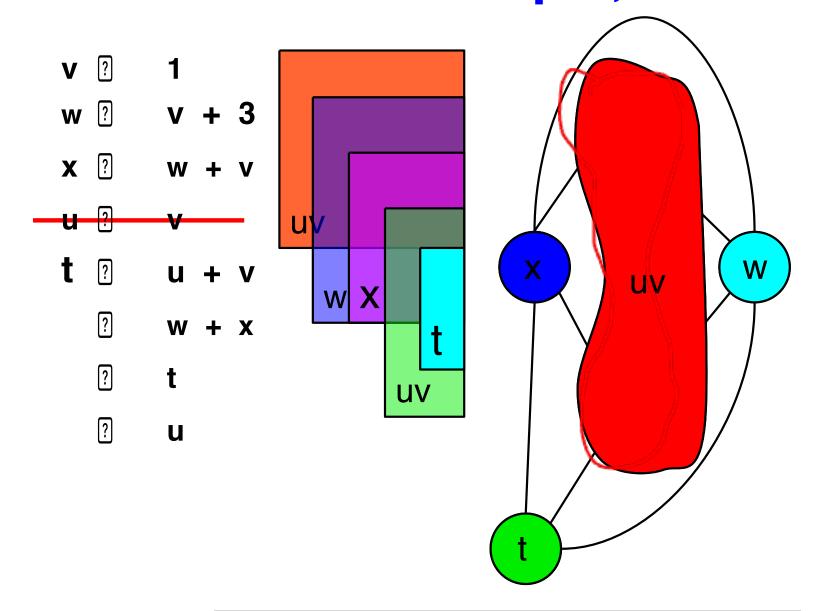
```
{ w, v }
         { w, x, v }
          { w, u, x(v) } v
                                                W
   w + x { u, t }
          { u }
?
   u
```

u & v are special. They interfere, but only through a move!

#### Interference and Coalescing

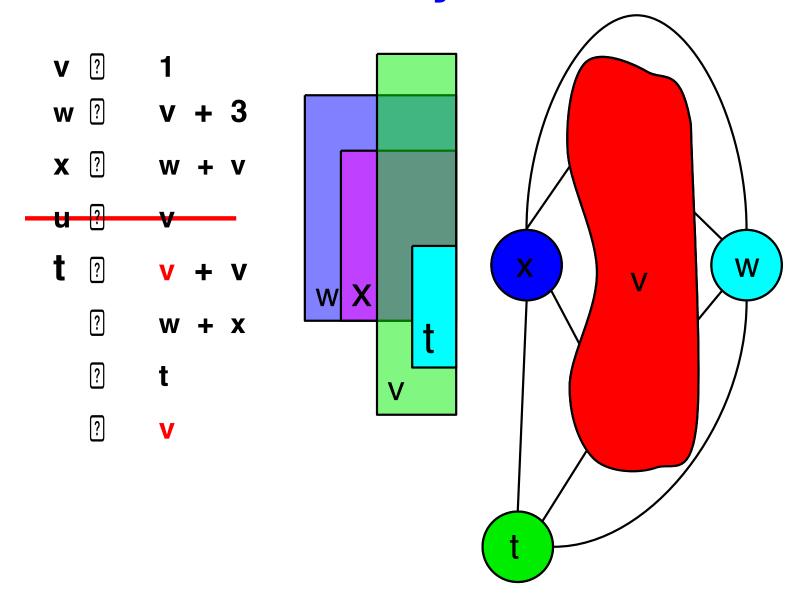
```
v ?
                    { V }
w ?
                    { w, v }
X ?
                    { w, x, v }
u ?
                    { w, u, x, v }
t ?
                    { w, t, u, x }
  ?
      W + X
                    { u, t }
  ?
      t
                    { u }
  ?
      u
```

We would like to eliminate the move u ? v by having u and v share a register (i.e, coalescing)

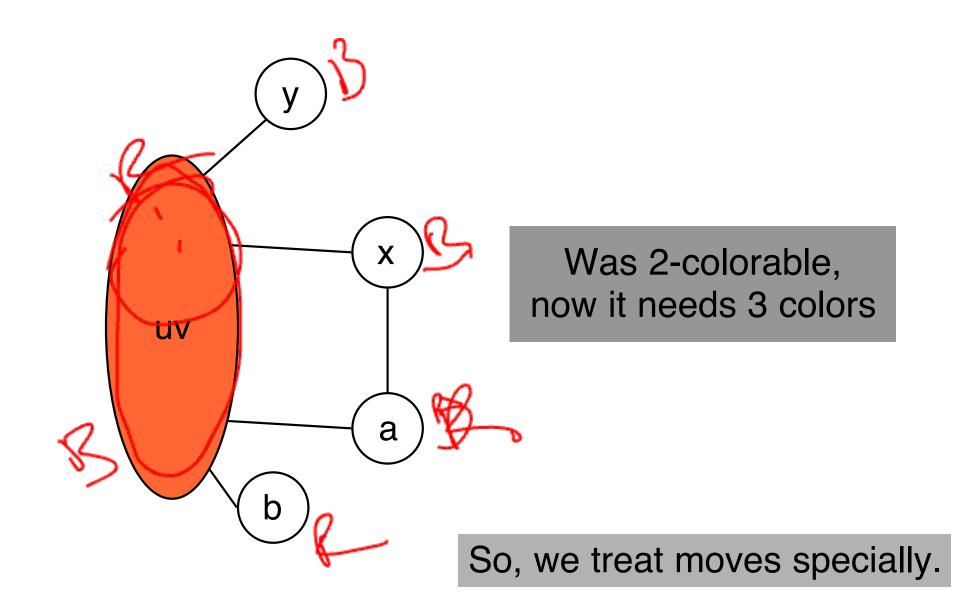


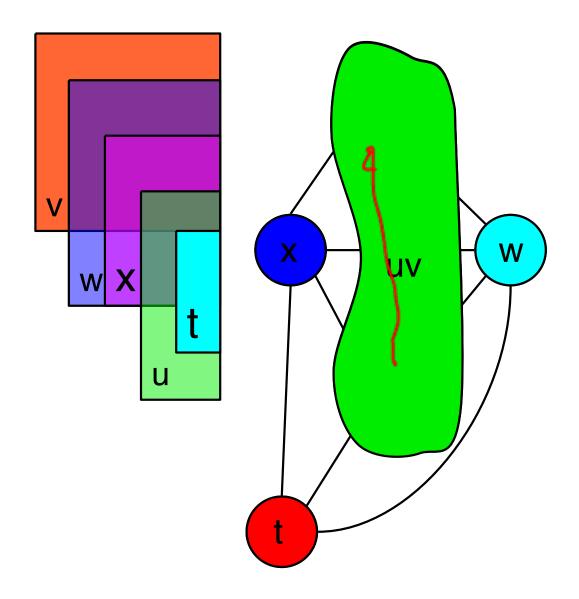
Rewrite the code to coalesce u & v

#### Another way to think about it



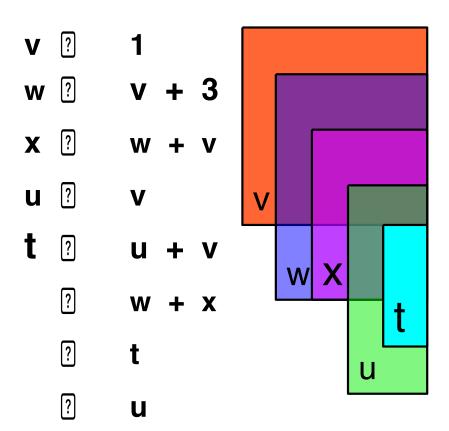
## Is Coalescing always good?



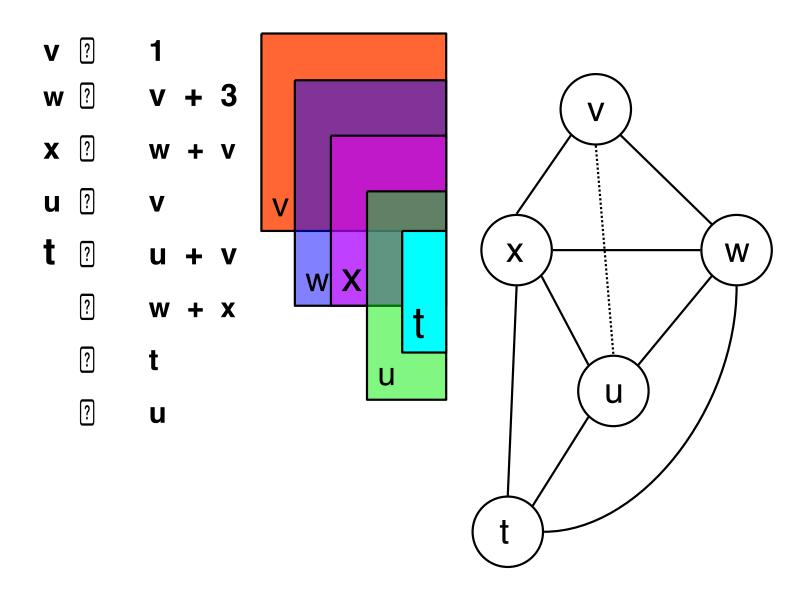


Interference from moves become "move edges."

v ? 1
w ? v + 3
x ? w + v
u ? v
t ? u + v
? w + x
? t
? u



Compute live ranges



Construct the interference graph

 v
 ?
 1

 w
 ?
 v
 +
 3

 x
 ?
 w
 +
 v

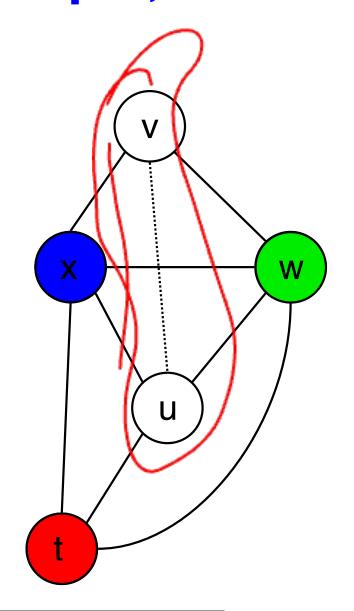
 u
 ?
 u
 +
 v

 ?
 w
 +
 x

 ?
 t
 t
 t

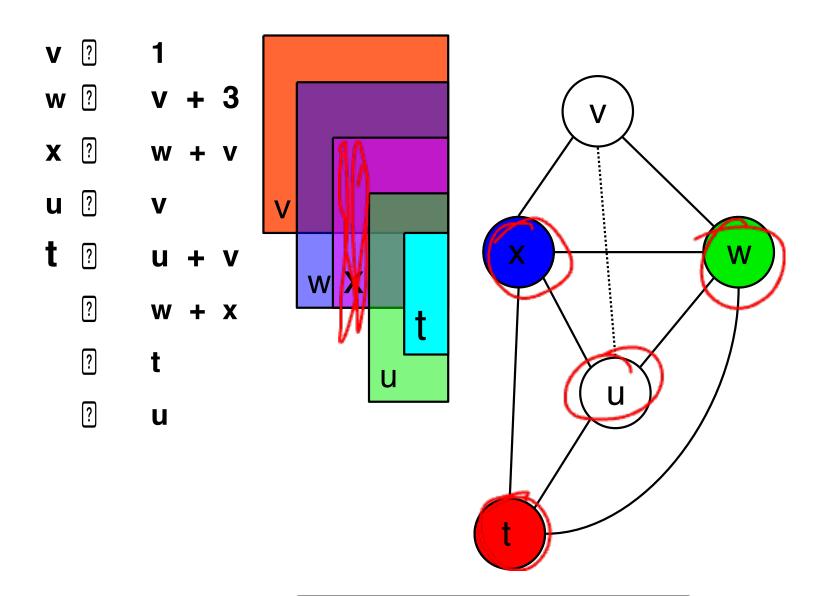
u

?



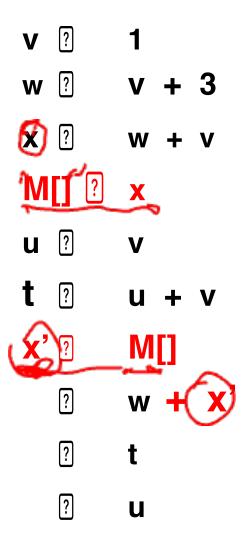
50

So, we need to spill



What to spill? Why?

Choose x and Rewrite program



recalculate live ranges

```
      v
      ?
      1

      w
      ?
      v
      +
      3

      x
      ?
      w
      +
      v

      M[]
      ?
      x
      -
      -

      u
      ?
      v
      -
      -
      -
      v

      t
      ?
      M[]
      -
      v

      x
      ?
      M[]
      -
      -
      -
```

? w + x'

?

? **u** 

{ }

recalculate live ranges

```
v ?
w ?
X ?
M[] ?
u ?
t ?
       M[]
X'?
      w + x'
  ?
  ?
       u
```

```
{ V }
{ w, v }
 W, V€ €
{ w, v }
{ w, u, v }
{ w, t, u }
{ u, t }
{ u }
```

v ? 1

w ? v + 3

X? W + V

M[] ? x

u ? v

t ? u + v

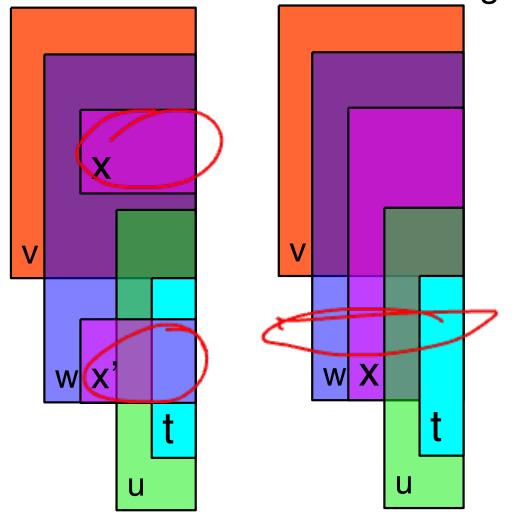
x'? M[]

? w + X'

? **t** 

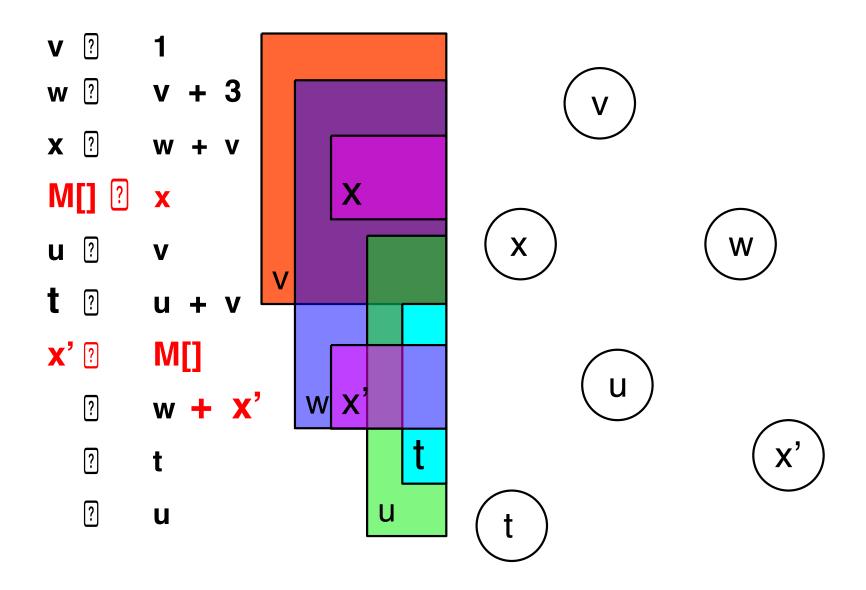
? **u** 

recalculate live ranges

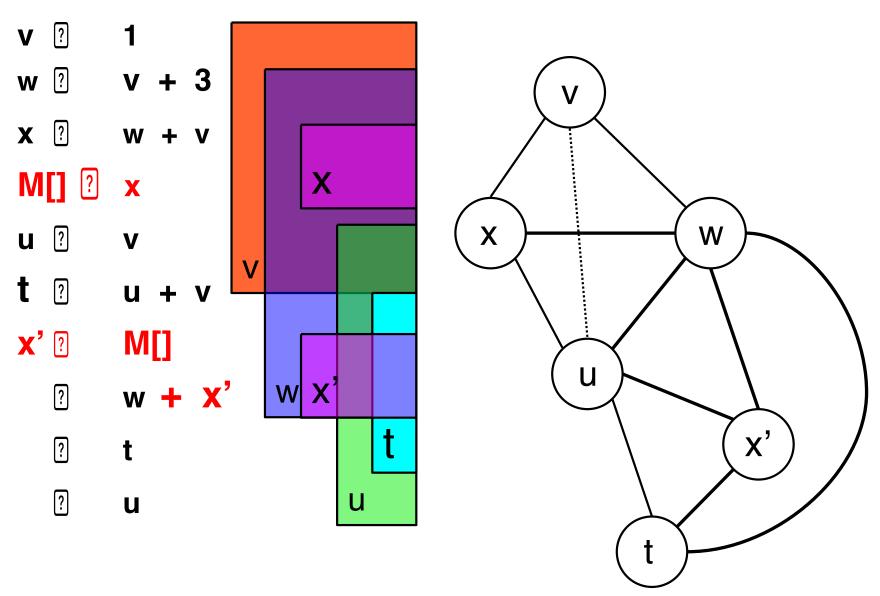


Spilling reduces live ranges, which decreases register pressure.

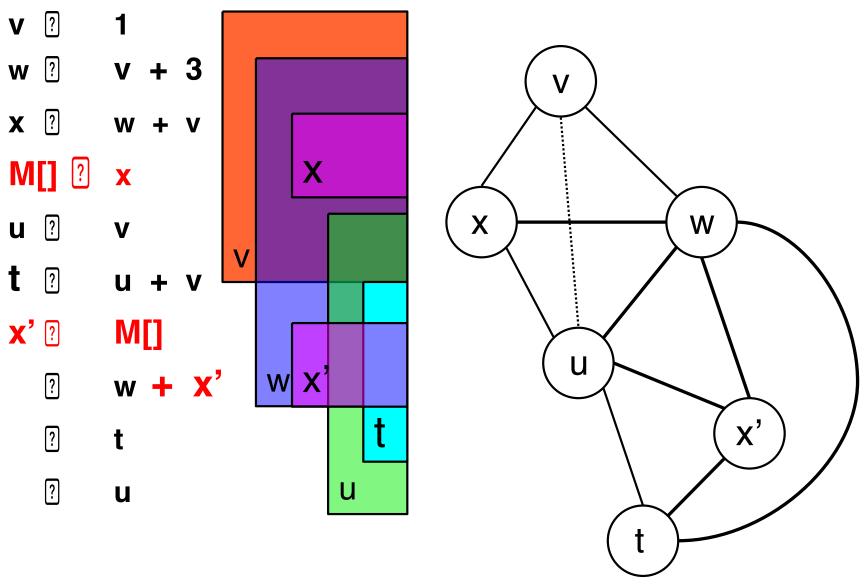
15-411/611 © 2019-21 Go



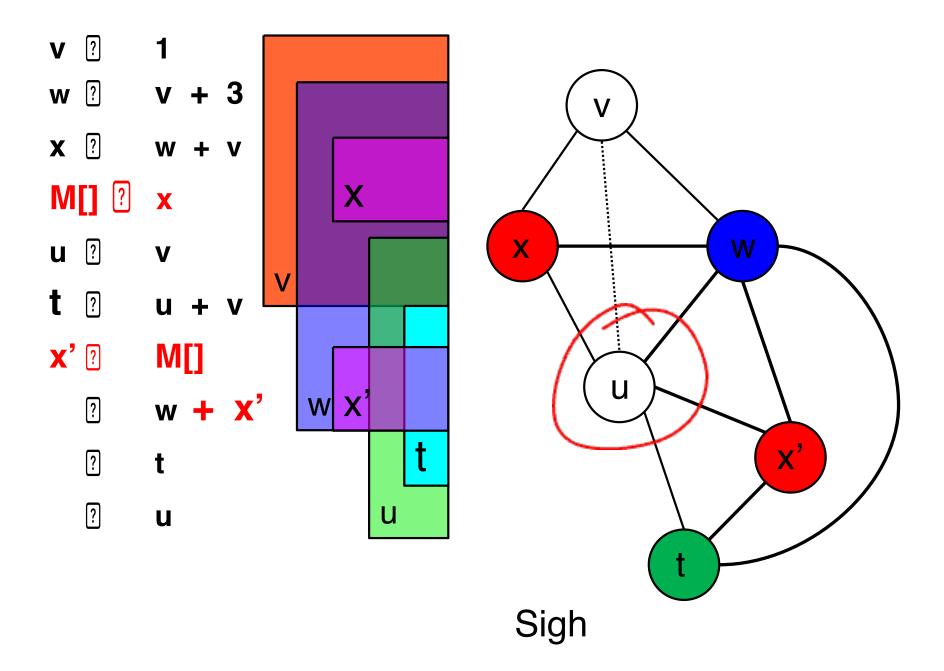
Recalculate interference graph



Recalculate interference graph

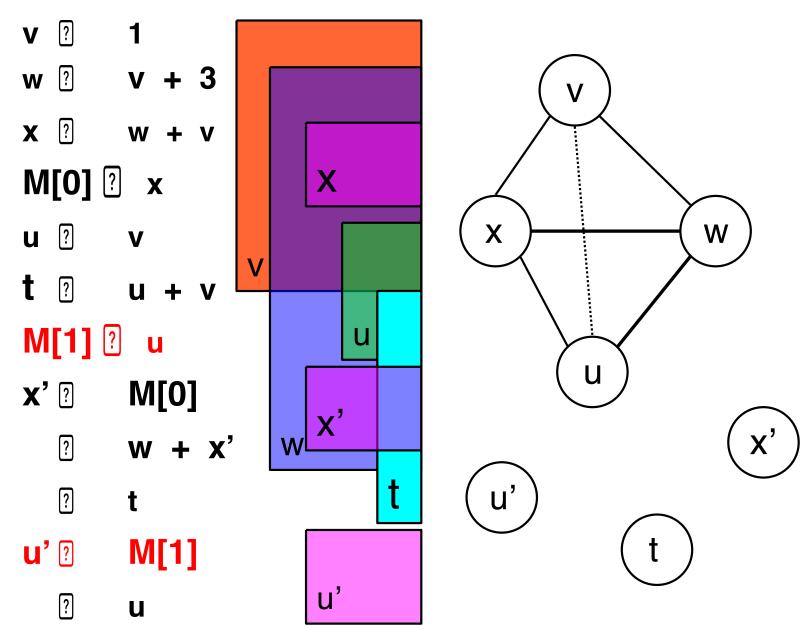


Recolor Graph

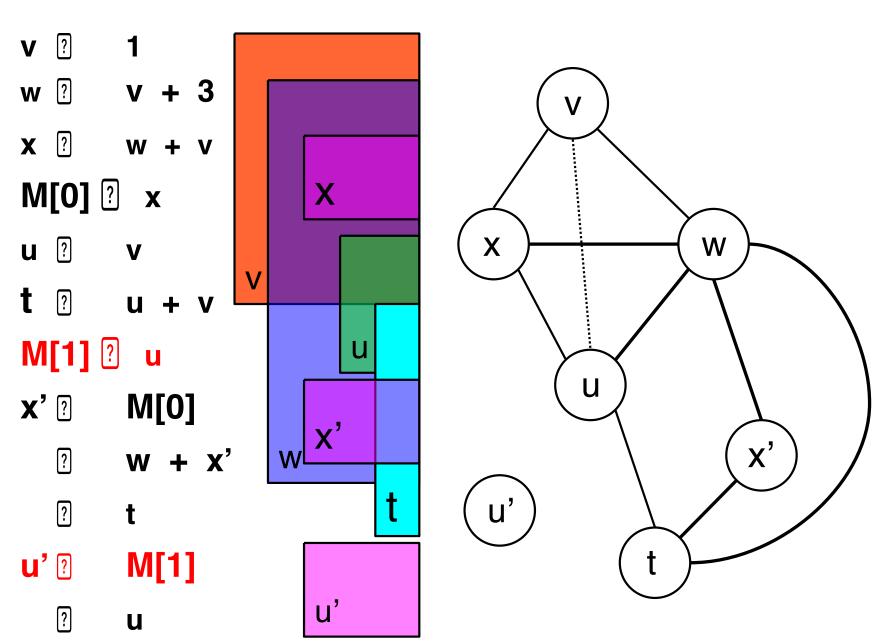


**v** ? **w** ? **X** ? M[0] ? x **u** ? **t** ? u + vM[1] ? u M[0]**X**'? W + X'? **M**[1] **u**'? ? u

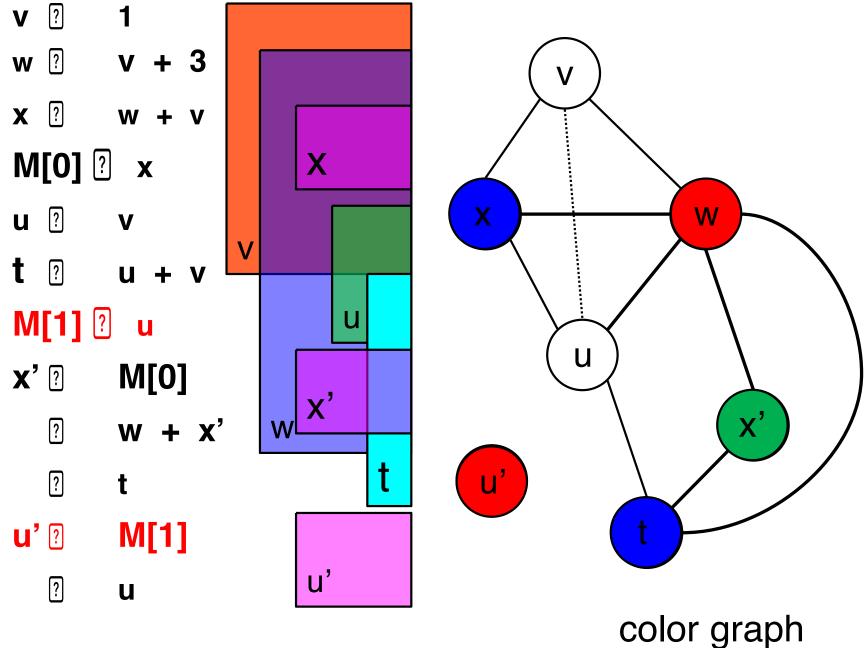
respill



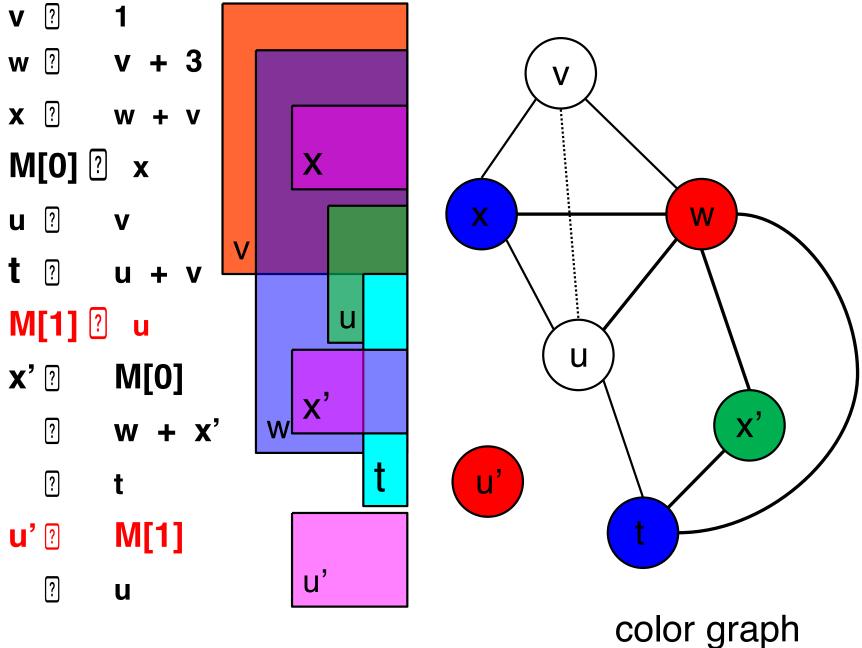
construct new interference graph



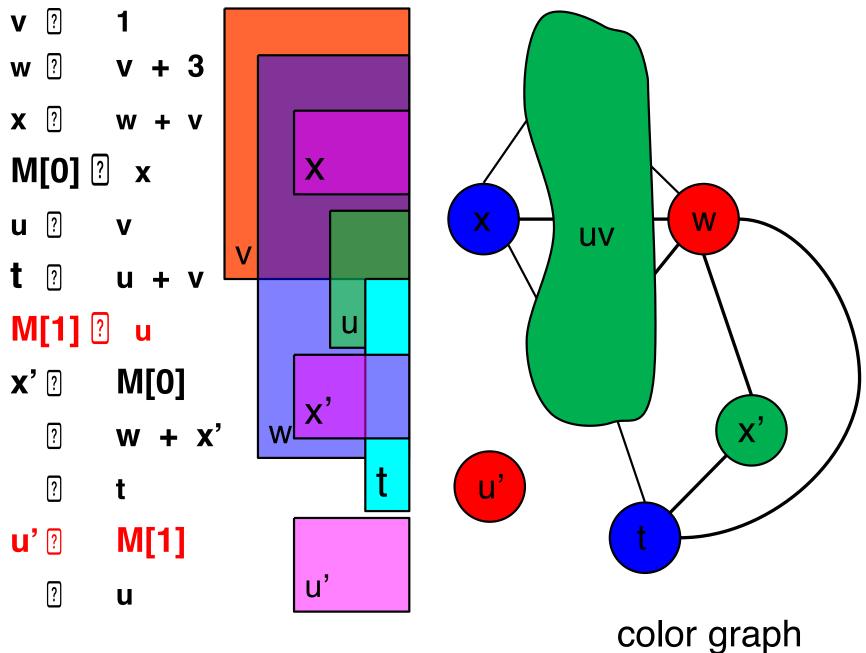
construct new interference graph



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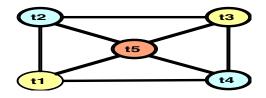
#### **Graph coloring**

- Once we have an interference graph, we can attempt register allocation by searching for a K-coloring
- This is an NP-complete problem (for K>2)
- But a linear-time simplification algorithm (by Kempe, 1879) tends to work well in practice

#### Kempe's observation

- Given a graph G that contains a node n with degree less than K, the graph is Kcolorable iff G with n removed is Kcolorable
  - This is called the "degree<K" rule</li>
- So, let's try iteratively removing nodes with degree<K</li>
- If all nodes are removed, then G is definitely K-colorable

#### Doesn't always help...

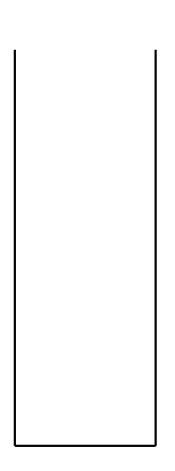


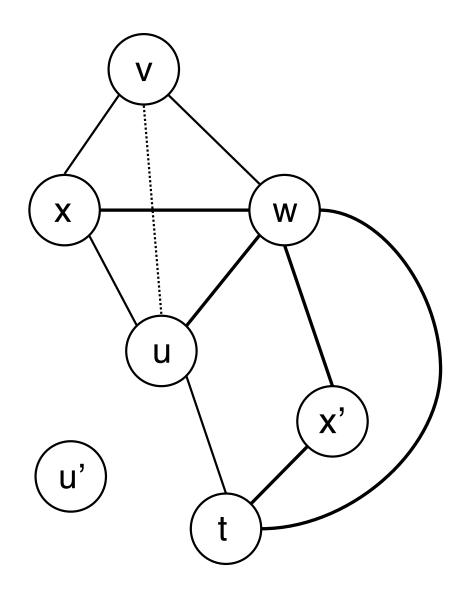
This graph is 3-colorable, but has no nodes with degree < 3

## Kempe's algorithm

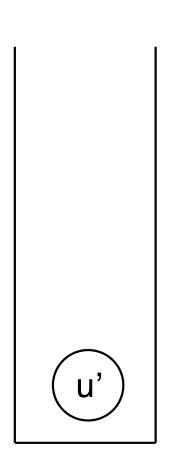
- First, iteratively remove degree<K nodes, pushing each onto a stack
- If all get removed, then pop each node and rebuild the graph, coloring as we go
- If we get stuck (i.e., no degree<K nodes),</li>
   then remove any node and continue

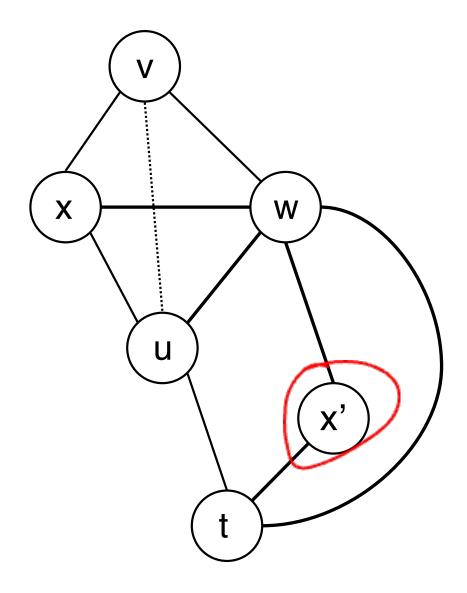
# Example, k=3



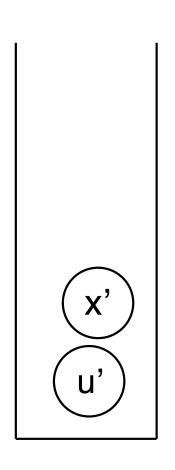


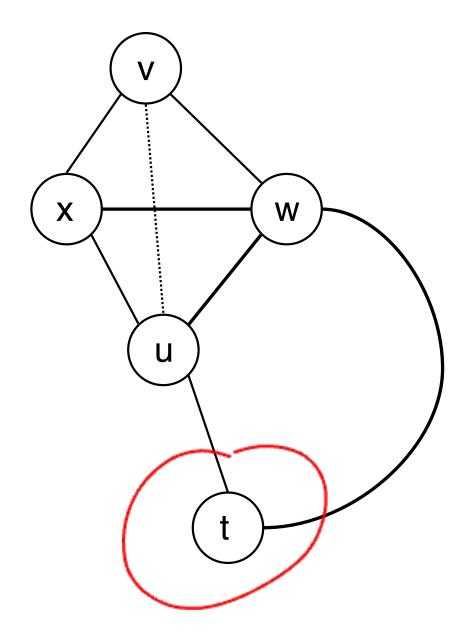
# Example, k=3

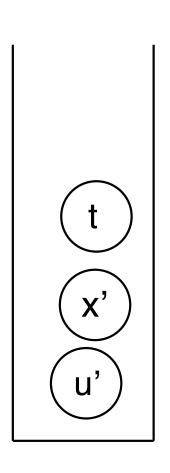


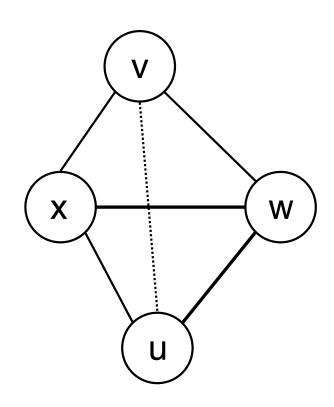


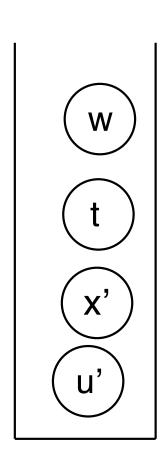
# Example, k=3

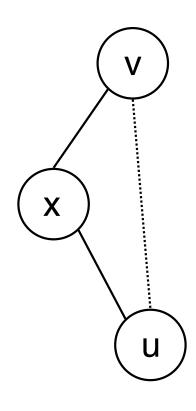


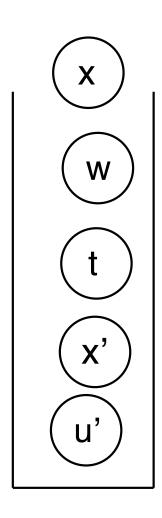


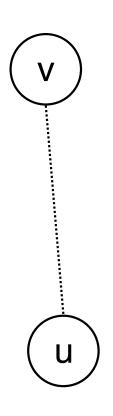


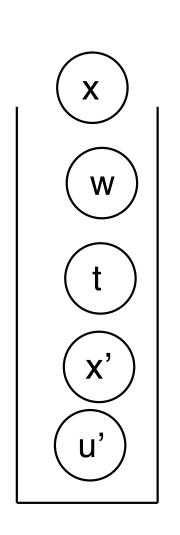


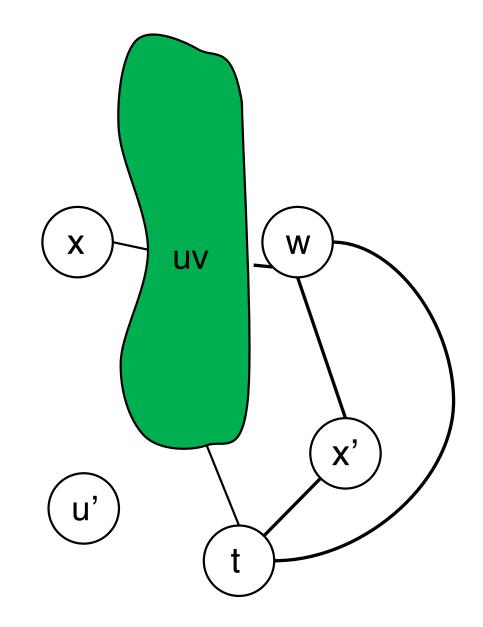


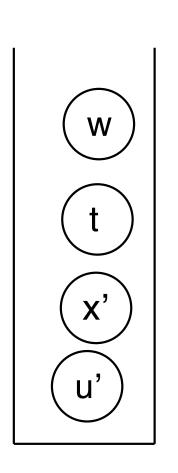


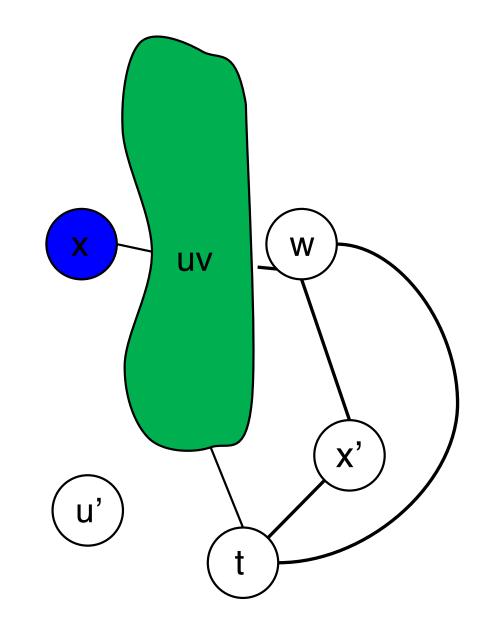


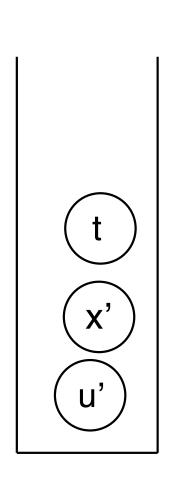


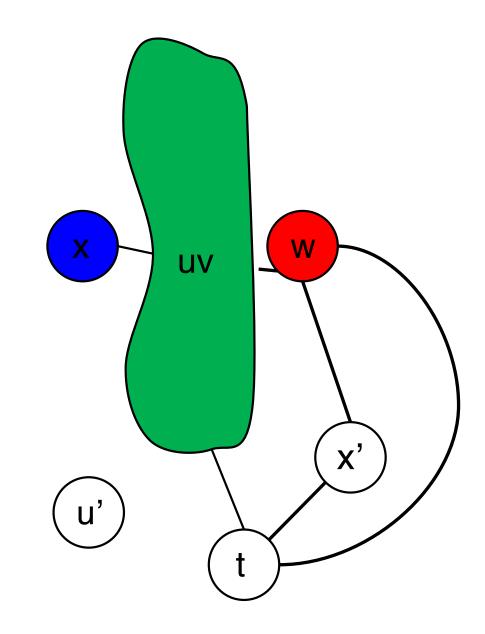


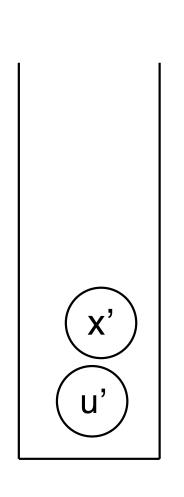


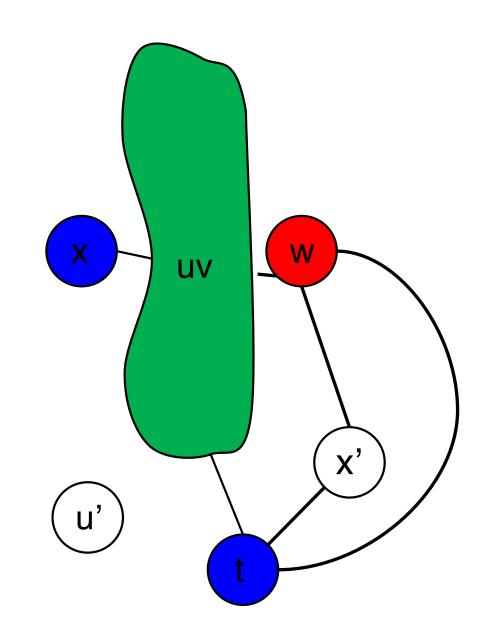


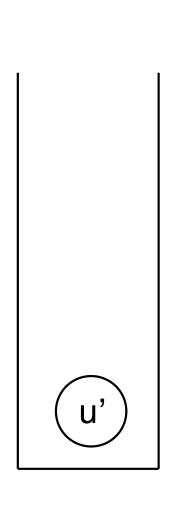


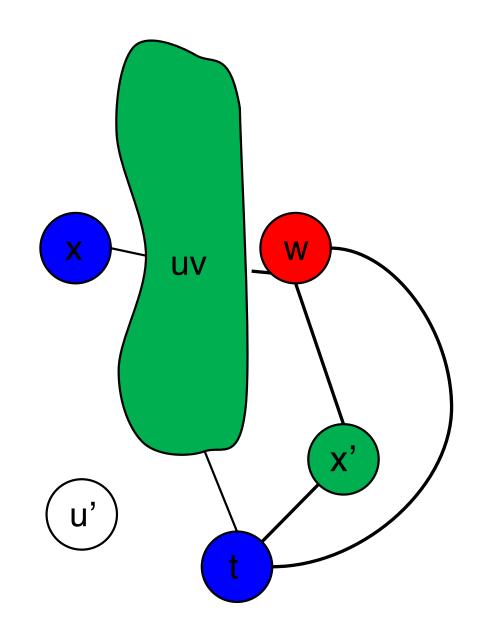


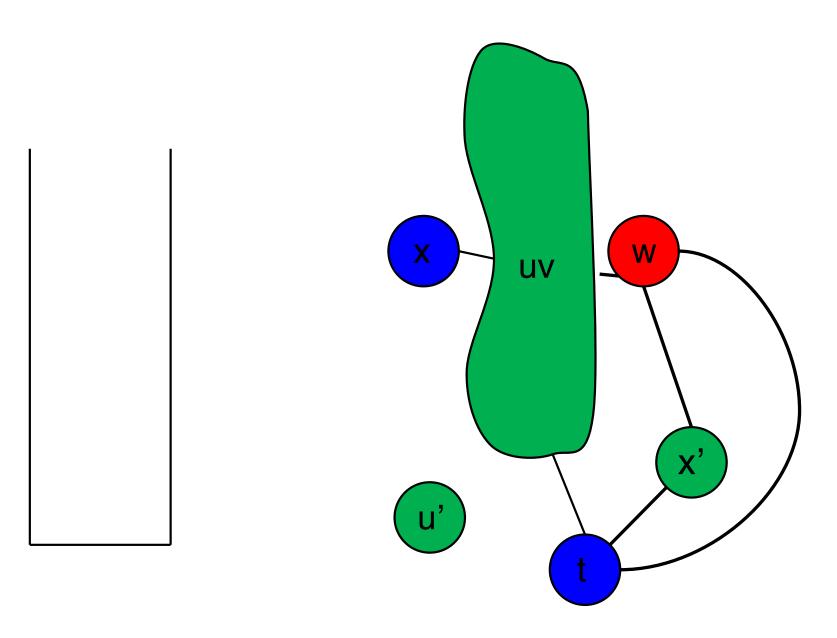






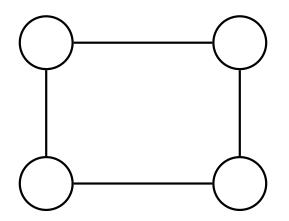


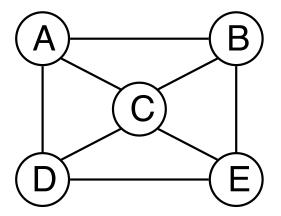




Voila!

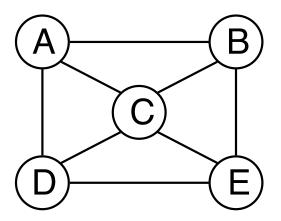
### Alg not perfect





What should we do when there is no node of degree < k?

### **Optimisitic Coloring**



#### Chaitin's allocator

- Build: construct the interference graph
- Simplify: node removal, a la Kempe
- Spill: if necessary, remove a degree≥K node, marking it as a potential spill
- Select: rebuild the graph, coloring as we go
  - if a potential spill can't be colored, mark it as an actual spill and continue
- Start over: if there are actual spills, generate spill code and then start over

#### **Choosing potential spills**

- When choosing a node to be a potential spill, we want to minimize its performance impact
- Can attempt to compute a spill cost for each temp
  - by estimating performance cost
  - or by using actual profile information
- More on this later...

#### **Choosing Potential Spills**

- When choosing a node to be a potential spill, we want to minimize its performance impact
- What should we choose to spill?
  - Something that will eliminate a lot of interference edges
  - Something that is used infrequently
  - Something that is NOT used in loops
  - Maybe something that is live across a lot of calls?

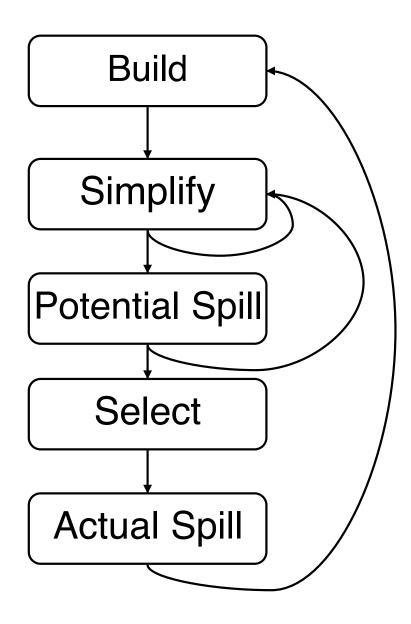
### **Setting Up For Better Spills**

- We want temps not-live across procedures to be allocated to caller-save registers. Why?
- We want temps live across many procs to be in callee-save registers
- We prefer to use callee-save registers last.
- We want live ranges of precolored nodes to be short!

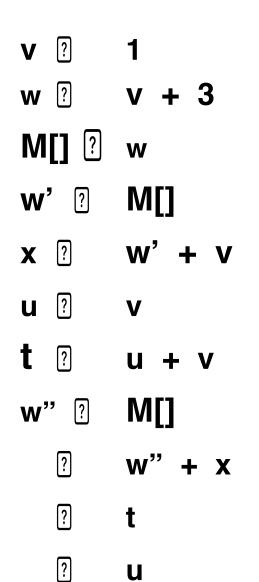
### K-coloring a graph

- Lets say we have a node, n, s.t. n <sup>2</sup> < k and let G' = G {n}, then if G' can be k-colored, then G can be k-colored.</li>
- Proof?
- This suggests the following optimistic heuristic:
- While IGI > 0
  - choose some n with degree < k</p>
  - push n on stack
  - remove n from G
- While ISI > 0
  - pop n from S
  - color with a legal color

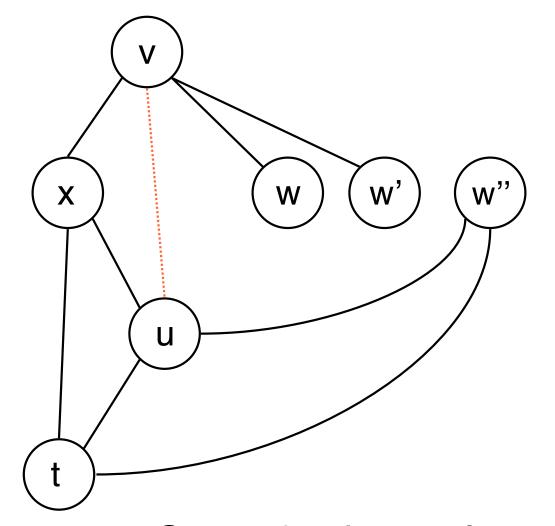
#### **Where We Are**



#### Coalescing

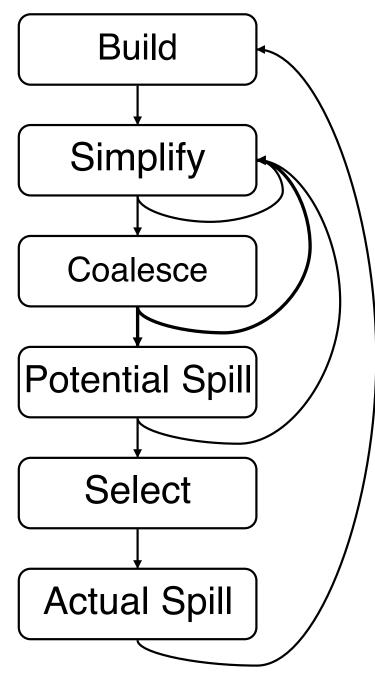


u



Can u & v be coalesced? Should u & v be coalesced?

#### **Where We Are**



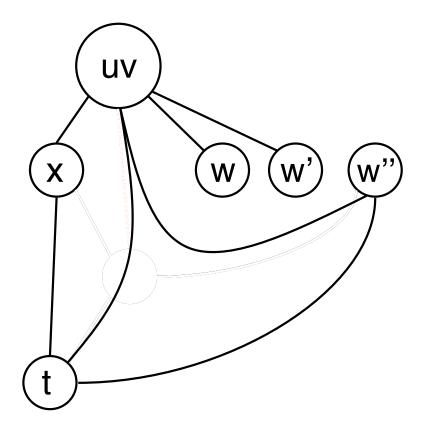
91

#### Coalescing

- Conservative or Aggressive?
- Aggressive:
  - coalesce even if potentially causes spill
  - Then, potentially undo
- Conservative:
  - coalesce if it won't make graph uncolorable
  - How to detect?

#### **Briggs**

- Can coalesce a and b if
   (# of neighbors of ab with degree k) < k</p>
- Why?
  - Simplify removes all nodes with degree < k</li>
  - # of remaining nodes < k</p>
  - Thus, ab can be simplified

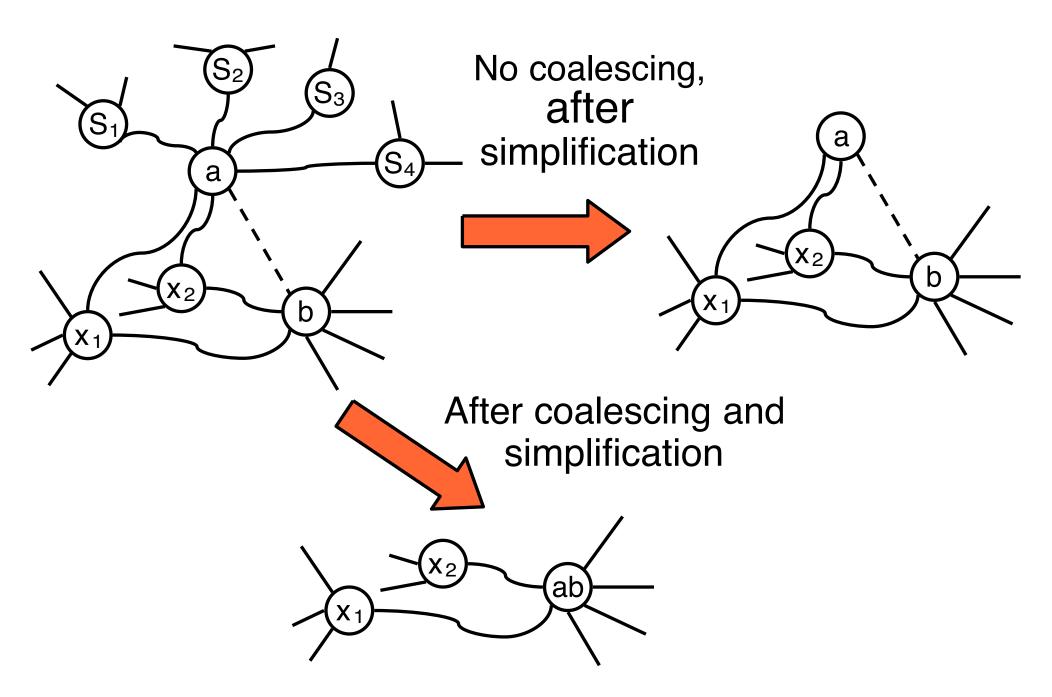


#### **Preston**

- Can coalesce a and b if foreach neighbor t of a
  - -t interferes with b, or,
  - -degree of t < k

- Why?
  - let S be set of neighbors of a with degree < k</p>
  - If no coalescing, simplify removes all nodes in S, call that graph G<sup>1</sup>
  - If we coalesce we can still remove all nodes in S, call that graph G<sup>2</sup>
  - − G<sup>2</sup> is a subgraph of G<sup>1</sup>

#### **Preston**



### Why Two Methods?

- With Briggs one needs to look at: neighbors of a & b
- With Preston, only need to look at neighbors of a.
- As we will see, we will need to insert "hard" registers into graph and they have LOTS of neighbors
  - RAX, RCX, RDI, ...
  - Called hard registers
  - aka precolored nodes

#### **Briggs and Preston**

- With Briggs one needs to look at: neighbors of a & b
- With Preston, only need to look at neighbors of a.
- Briggs
   Used when a and b are both temps
- Preston
   Used when either a or b is precolored

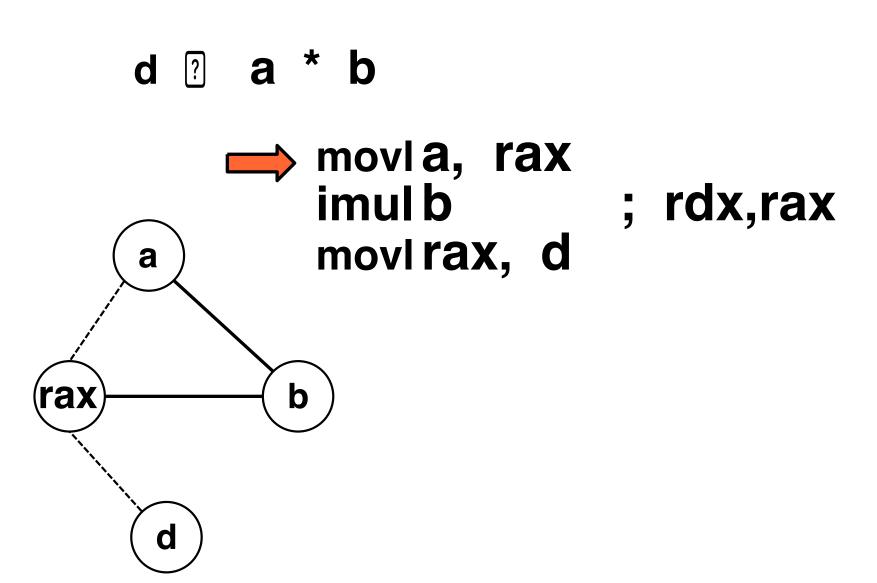
Instructions with register requirements

d ? a \* b

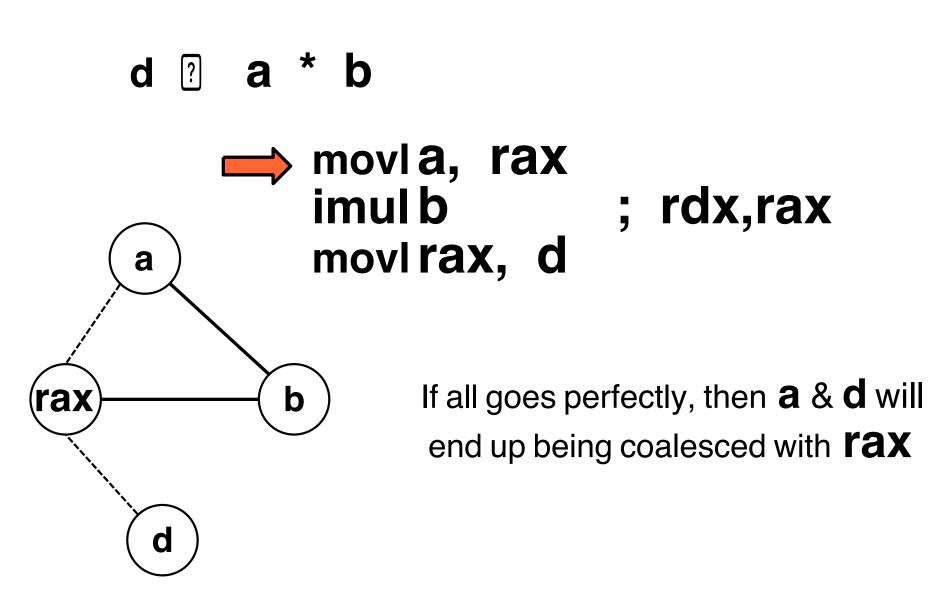
ret x

- Callee-save registers
  - x86-64: RDI, RSI, RDX, RCX, R8, R9 must be saved by callee if callee wants to use them.

Instructions with register requirements



Instructions with register requirements



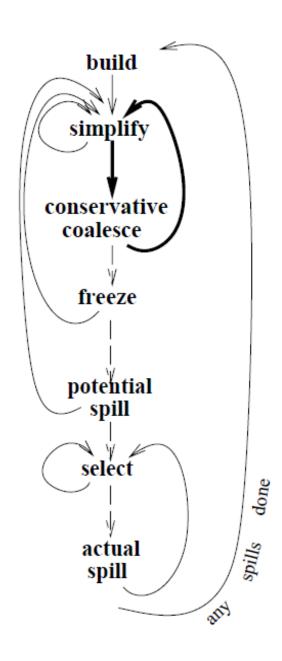
Instructions with register requirements

#### **Preserving Callee-registers**

- Move callee-reg to temp at start of proc
- Move it back at end of proc.
- What happens if there is no register pressure?
- What happens if there is a lot of register pressure?

```
prologue: define r
t1 ? r
...
epilogue: r ? t1
use r
```

### **Iterated Register Coloring**



#### In practice

- Iterated Register Coloring does a good job
- Building Interference Graph is Expensive
  - Calculating live ranges
  - graph is  $O(n^2)$
  - Need quick test for interference
  - Need quick test for neighbors
- Coalescing is important
  - Many passes generate extra temps and moves
  - Aggressive requires fix-up (e.g., live range splitting)
- Spilling has biggest impact on generated code