Dataflow Analysis

15-411/15-611 Compiler Design

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Today

- Full Employment Theorem
- Local Optimization: Value Numbering
- Dataflow Analysis
  - reaching definitions
  - liveness
  - available expressions
  - very busy expressions
- Framework

Optimizations

- Register Allocation
- Common subexpression elimination
- Constant Propagation
- Copy propagation
- Dead-code elimination
- Loop optimizations
  - Hoisting
  - Induction variable elimination
- And, many many more.

How many?

Infinitely Many!

- Lets assume there existed
  optc: A perfect optimizing compiler
- optc(P) $\Rightarrow$ the smallest possible P’ which has equivalent behavior to P.
- Then, ?

Infinitely Many!

- Lets assume there existed
  optc: A perfect optimizing compiler
- optc(P) $\Rightarrow$ the smallest possible P' which has equivalent behavior to P.
- Then, if P never halts, it should produce:

```
L1: jmp L1
```

- So, instead we build "optimizing compilers" and there is always a job for a compiler writer to invent new optimizations!

Approach to Optimization

- Three parts to any optimization:
  - Determine if an optimization is legal
  - Determine if an optimization is profitable
  - Implement optimization
- Consider also compilation time

Scope of Optimization

- Local

  Within a basic block

- Global

  Within a function, across basic blocks

- Interprocedural

  The entire program (file), across functions and basic blocks.

- Whole progam

  Across all the files

Value Numbering

- (Originally) A local optimization for:
  - common sub-expression elimination
  - constant folding
  - constant propagation
  - dead-code removal
- A long history
- An ancestor to SSA

Local Value Numbering

- Goal: recognize that $X \oplus Y$ is redundant and eliminate redundant operation
- Idea: Assign a value number, V(exp), to each expression such that:

  $V(exp) = V(X \oplus Y)$ iff

  $X \oplus Y$ has same value as exp (on all paths)

Beware:

$a \leftarrow x + y$
$b \leftarrow x + y$
$x \leftarrow 5$
$c \leftarrow x + y$

Value Numbering Example

```
void quicksort(int m, int n) {
   int i = m-1;
   int j = n;
   int v;
   int x;
   if n <= m then return;
   v = a[n];
   while (true) {
        i = i+1;
        while (a[i] < v) i = i+1;
        j = j-1;
        while (a[j] > v) j = j-1;
        if  i>=j break;
        x = a[i]; a[i] = a[j]; a[j] = x
   }
   x = a[i]; a[i] = a[n]; a[n] = x;
   quicksort(m,j);
   quicksort(i+1), n);
}
```

Example: CSE

```
void quicksort(int m, int n) {
   int i = m-1;
   int j = n;
   int v;
   int x;
   if n <= m then return;
   v = a[n];
   while (true) {
      i = i+1;
      while (a[i] < v) i = i+1;
      j = j-1;
      while (a[j] > v) j = j-1;
      if  i>=j break;
      x = a[i]; a[i] = a[j]; a[j] = x
   }
   x = a[i]; a[i] = a[n]; a[n] = x;
   quicksort(m,j);
   quicksort(i+1), n);
}
```

```
B1: i     = m - 1
    j     = n
    t1    = 4*n
    v     = a[t1]

B2: i     = i + 1
    t2    = 4 * i
    t3    = a[t2]
    cjump t3<v  B2, B3

B3: j     = j - 1
    t4    = 4 * j
    t5    = a[t4]
    cjump t5>v  B3, B4

B4: cjump i>=j  B6, B5
```

Example: CSE

```
void quicksort(int m, int n) {
   int i = m-1;
   int j = n;
   int v;
   int x;
   if n <= m then return;
   v = a[n];
   while (true) {
      i = i+1;
      while (a[i] < v) i = i+1;
      j = j-1;
      while (a[j] > v) j = j-1;
      if  i>=j break;
      x = a[i]; a[i] = a[j]; a[j] = x
   }
   x = a[i]; a[i] = a[n]; a[n] = x;
   quicksort(m,j);
   quicksort(i+1), n);
}
```

```
B5: t6     = 4*i
    x      = a[t6]
    t7     = 4 * i
    t8     = 4 * j
    t9     = a[t8]
    a[t7]  = t9
    t10    = 4*j
    a[t10] = x
    jump   B2

B6: t11    = 4*i
    x      = a[t11]
    t12    = 4 * i
    t13    = 4 * n
    t14    = a[t13]
    a[t12] = t14
    t15    = 4*n
    a[t15] = x
```

Example: CSE

```
B1:   i      = m - 1
      j      = n
      t1     = 4*n
      v      = a[t1]

B2:   i      = i + 1
      t2     = 4 * i
      t3     = a[t2]
      cjump  t3<v   B2, B3

B3:   j      = j - 1
      t4     = 4 * j
      t5     = a[t4]
      cjump  t5>v   B3, B4

B4:   cjump  i >= j B6, B5
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```
B5:   t6     = 4*i
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      t13    = 4 * n
      t14    = a[t13]
      a[t12] = t14
      t15    = 4*n
      a[t15] = x
```

Local CSE

```
B1:    i      = m - 1
       j      = n
       t1     = 4*n
       v      = a[t1]

B2:    i      = i + 1
       t2     = 4 * i
       t3     = a[t2]
       cjump  t3<v   B2, B3


B3:    j      = j - 1
       t4     = 4 * j
       t5     = a[t4]
       cjump  t5>v   B3, B4

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```
B5:    t6     = 4*i
       x      = a[t6]
       t7     = 4 * i
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       t9     = a[t8]
       a[t7]  = t9
       t10    = 4*j
       a[t10] = x
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B6:    t11    = 4*i
       x      = a[t11]
       t12    = 4 * i
       t13    = 4 * n
       t14    = a[t13]
       a[t12] = t14
       t15    = 4*n
       a[t15] = x
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Local CSE

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B1:     i       = m - 1
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        t1      = 4*n
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        t2      = 4 * i
        t3      = a[t2]
        cjump   t3<v   B2, B3


B3:     j       = j - 1
        t4      = 4 * j
        t5      = a[t4]
        cjump   t5>v   B3, B4

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```
B5:     t6      = 4*i
        x       = a[t6]
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        t10     = 4*j
        a[t8]   = x
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Local CSE

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B1:    i      = m - 1
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       v      = a[t1]

B2:    i      = i + 1
       t2     = 4 * i
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B3:    j      = j - 1
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       t5     = a[t4]
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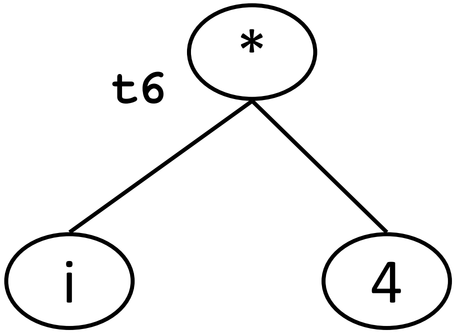
```
B5:    t6     = 4*i
       x      = a[t6]

       t8     = 4 * j
       t9     = a[t8]
       a[t6]  = t9

       a[t8]  = x
       jump   B2

B6:    t11    = 4*i
       x      = a[t11]

       t13    = 4 * n
       t14    = a[t13]
       a[t11] = t14

       a[t13] = x
```

How do we find this?

Build a DAG

```
B5: t6      = 4*i
    x       = a[t6]
    t7      = 4 * i
    t8      = 4 * j
    t9      = a[t8]
    a[t7]   = t9
    t10     = 4*j
    a[t10] = x
    jump    B2
```

- For each var & constant not seen before create a leaf
- For each op, create a node and label with lvalue



Build a DAG

- For each var & constant not seen before create a leaf
- For each op, create a node and label with lvalue

```
B5: t6     = 4*i
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    jump   B2
```

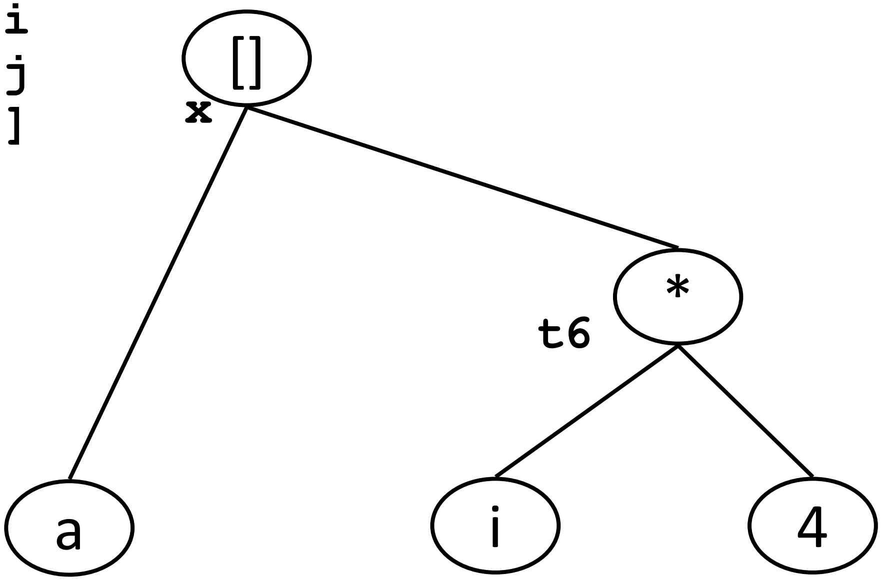

Build a DAG

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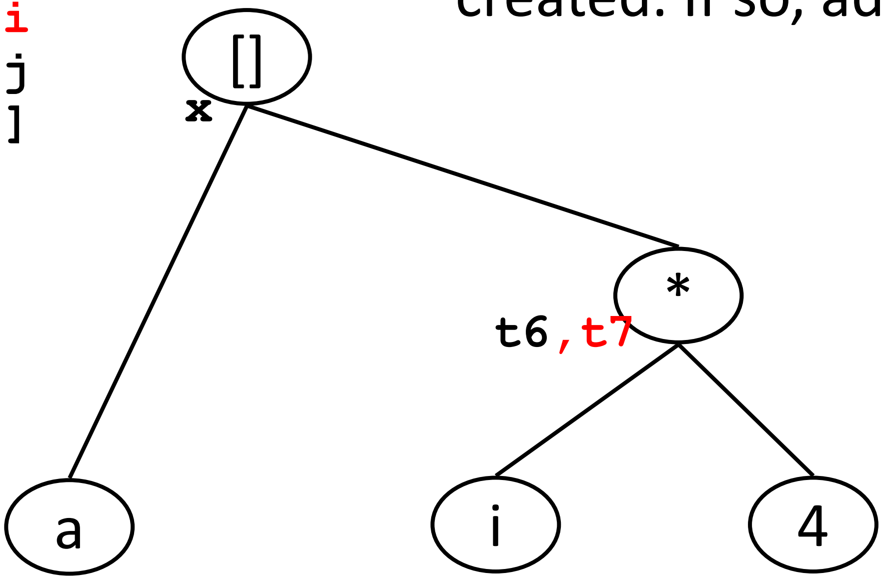
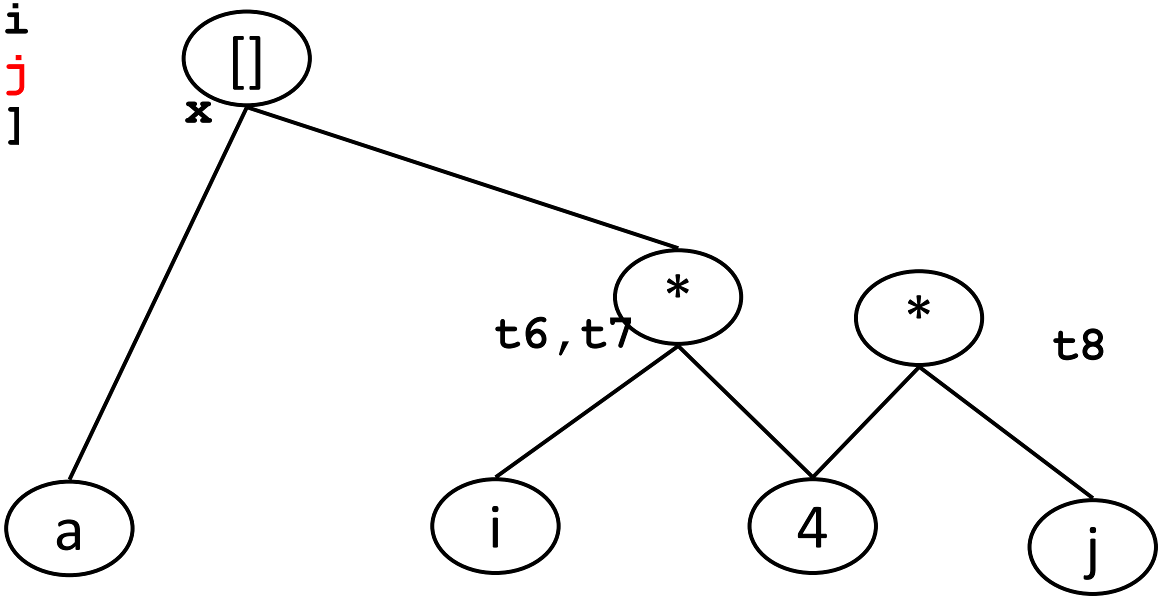
    jump    B2
```



- If you have seen all rvalues before see if an interior node with same “op” and operands has already been created. If so, add a label.

Build a DAG

```
B5: t6      = 4*i
    x       = a[t6]
    t7      = 4 * i
    t8      = 4 * j
    t9      = a[t8]
    a[t7]   = t9
    t10     = 4*j
    a[t10]  = x
    jump    B2
```



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    t7     = 4 * i
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```

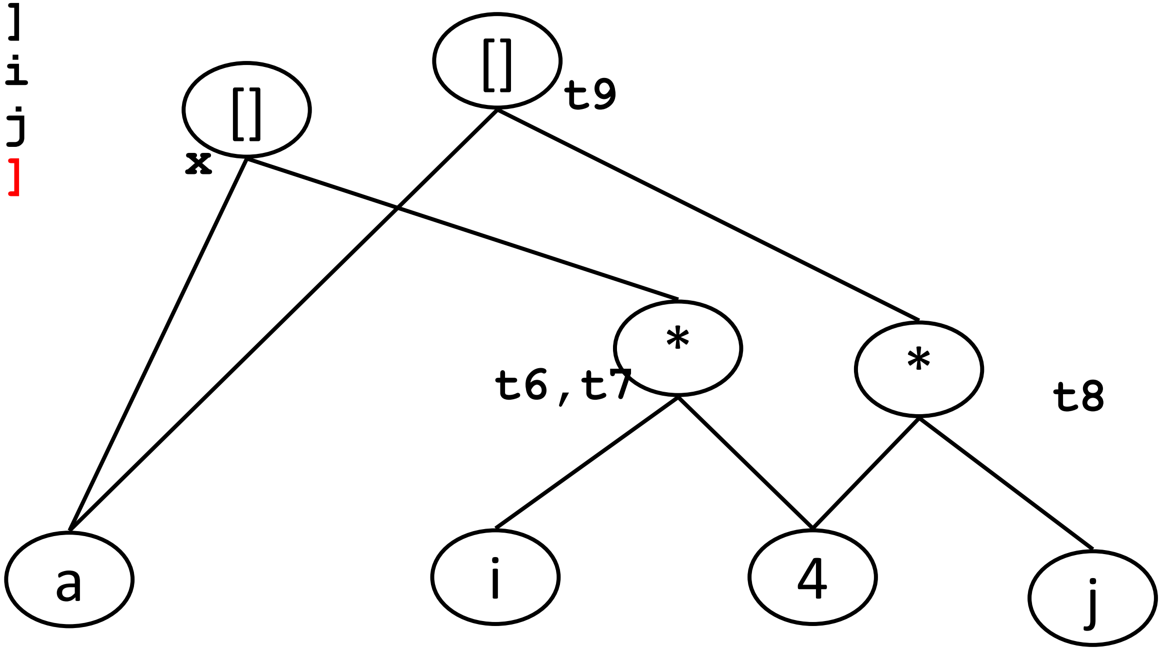

Build a DAG

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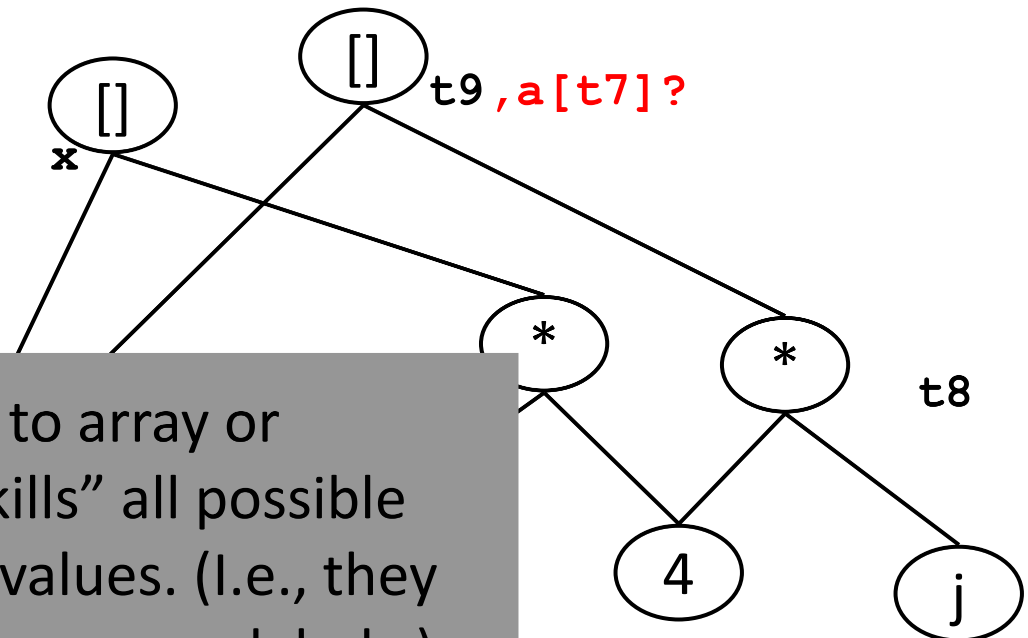


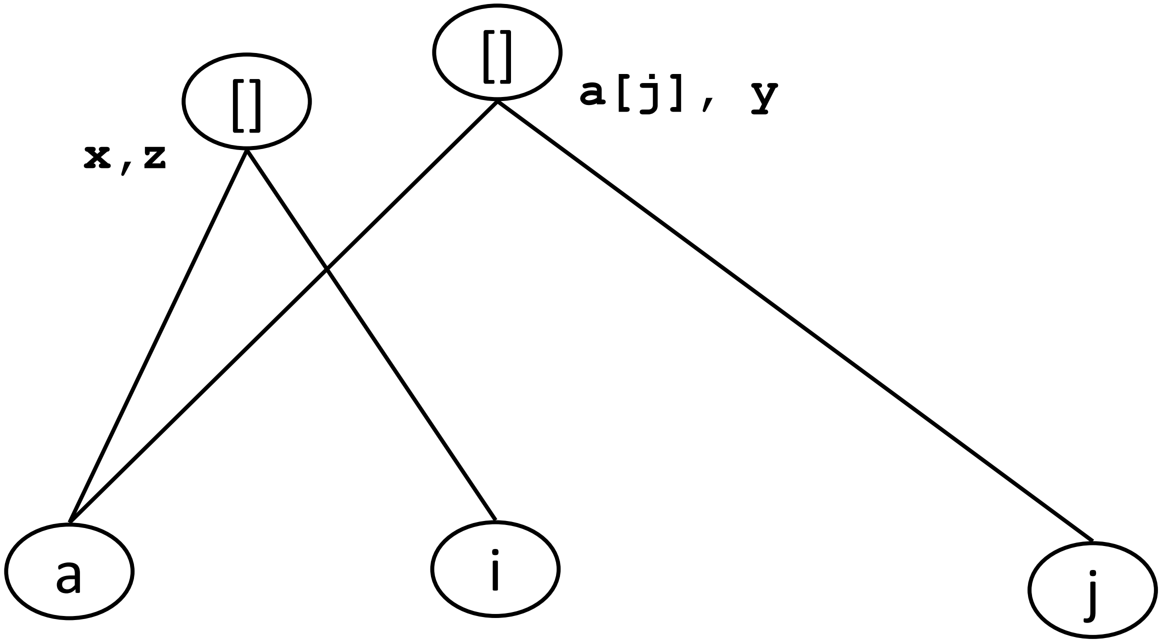
- Assigning to array or pointer “kills” all possible memory lvalues. (I.e., they can’t get any more labels.)

Memory References

```
x = a[i]
a[j] = y
z = a[i]
```

Becomes:

```
x = a[i]
z = x
a[j] = y
```



Build a DAG

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B5: t6     = 4*i
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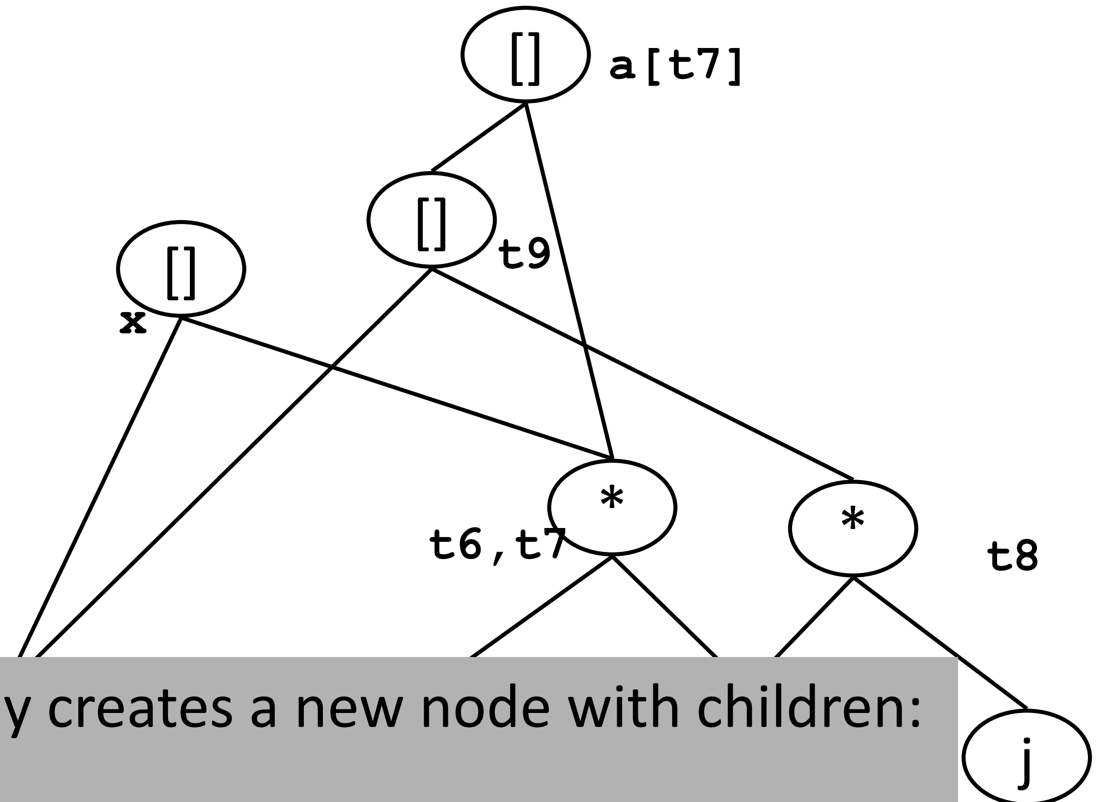

Assignment to an array creates a new node with children:
- index
- old value of array
- value assigned

Build a DAG

```
B5: t6     = 4*i
    x      = a[t6]
    t7     = 4 * i
    t8     = 4 * j
    t9     = a[t8]
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```

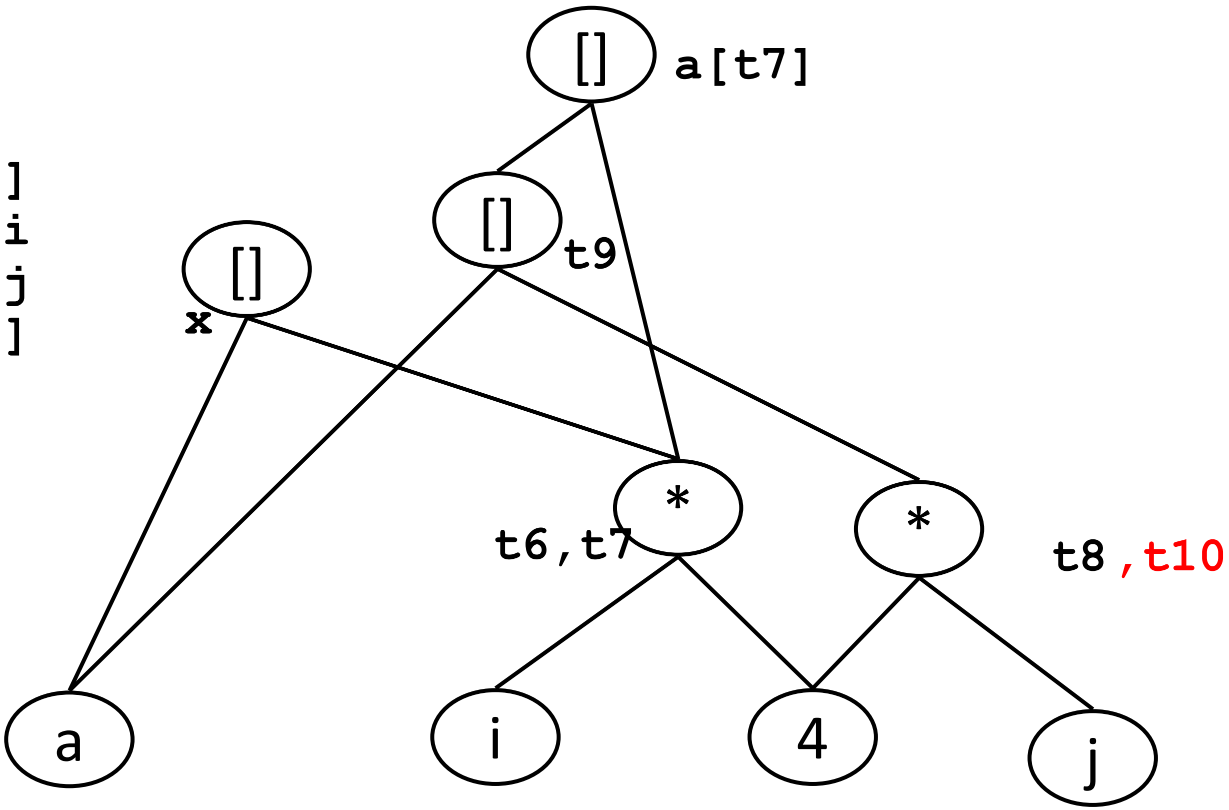

Build a DAG

```
B5: t6     = 4*i
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    jump   B2
```
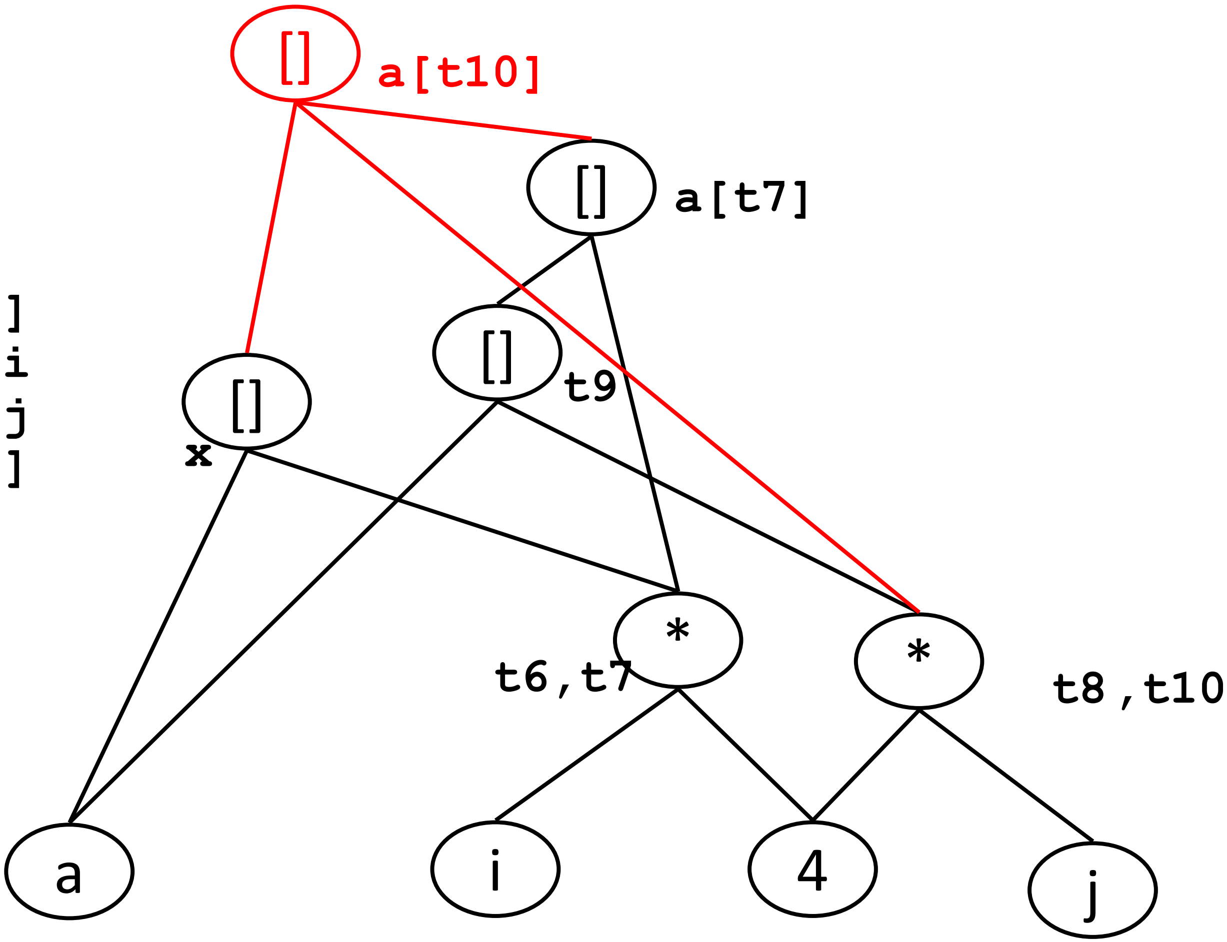



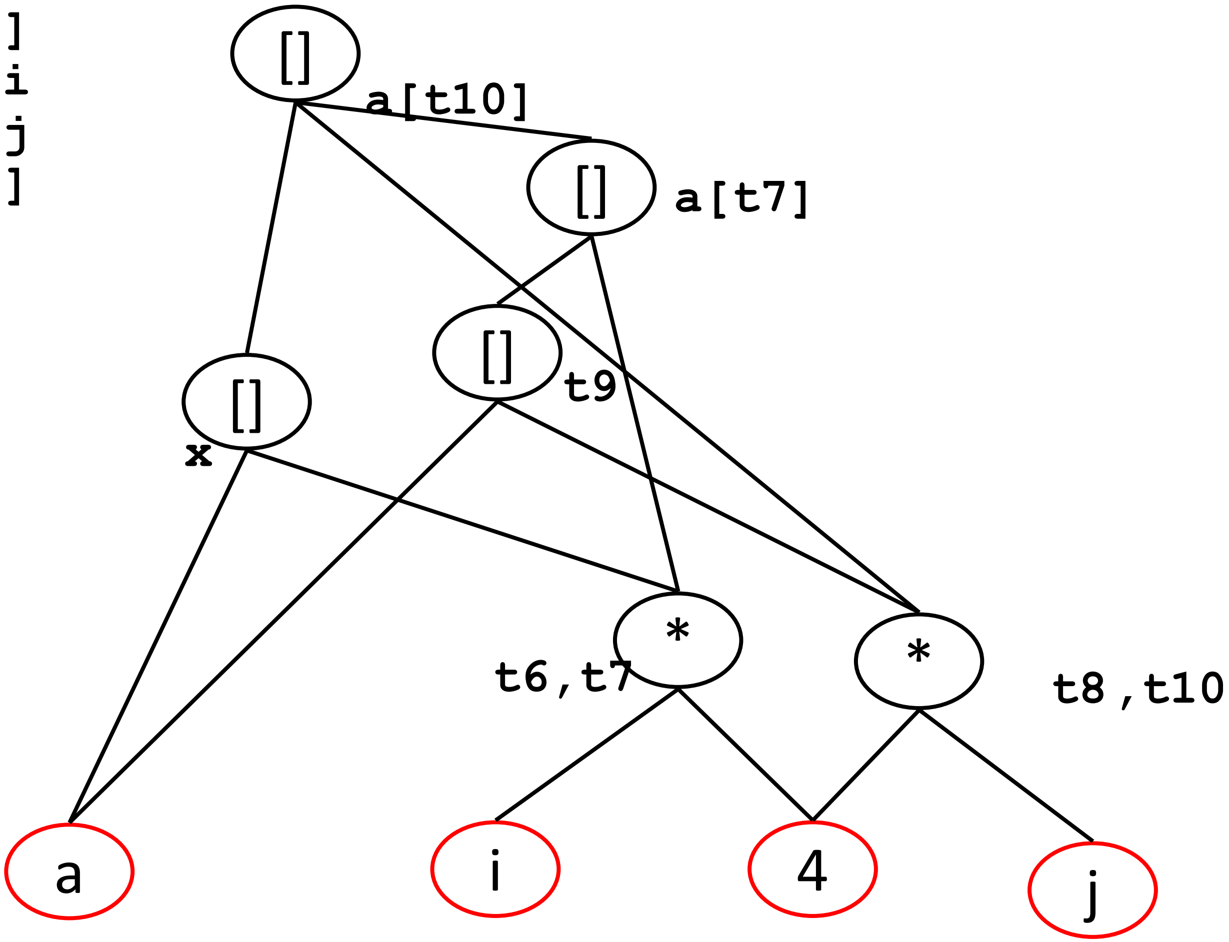
Using the DAG to recreate blocks

- Order of evaluation is any topological sort
- We pick a node. Assign it to ONE of the labels (hopefully one needed later in the program)
- If we end up with identifiers that are needed after this block, insert move statements.
- If a node has no identifiers, make up a new one.
- Caveats:
  - Procedure calls kill nodes
  - A[] = and *p = kill nodes

Recreating the Code

Original

```
B5: t6      = 4*i
    x       = a[t6]
    t7      = 4 * i
    t8      = 4 * j
    t9      = a[t8]
    a[t7]   = t9
    t10     = 4*j
    a[t10] = x
    jump    B2
```



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B5: t6     = 4*i
    x      = a[t6]
    t7     = 4 * i
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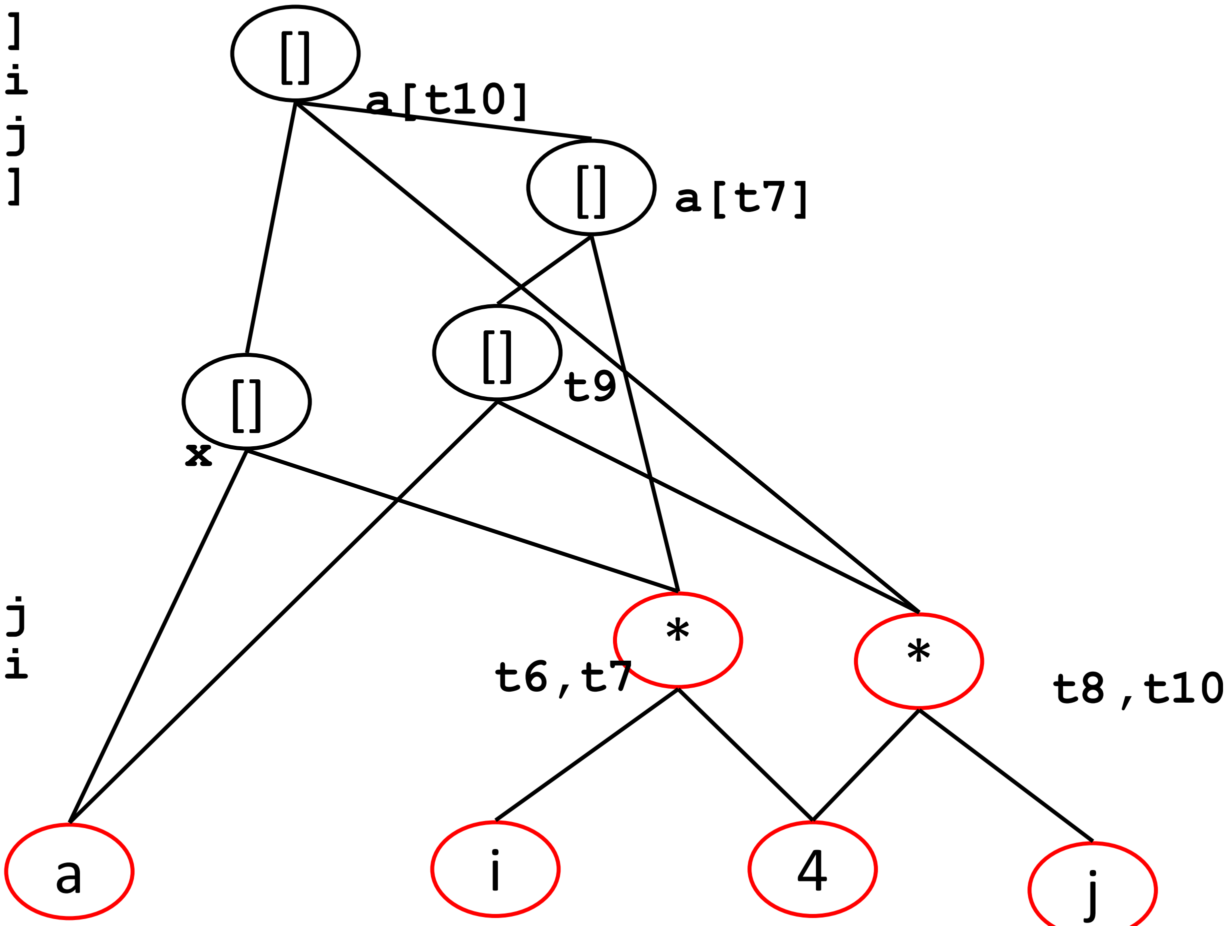
    a[t7]  = t9
    t10    = 4*j
    a[t10] = x
    jump   B2
```

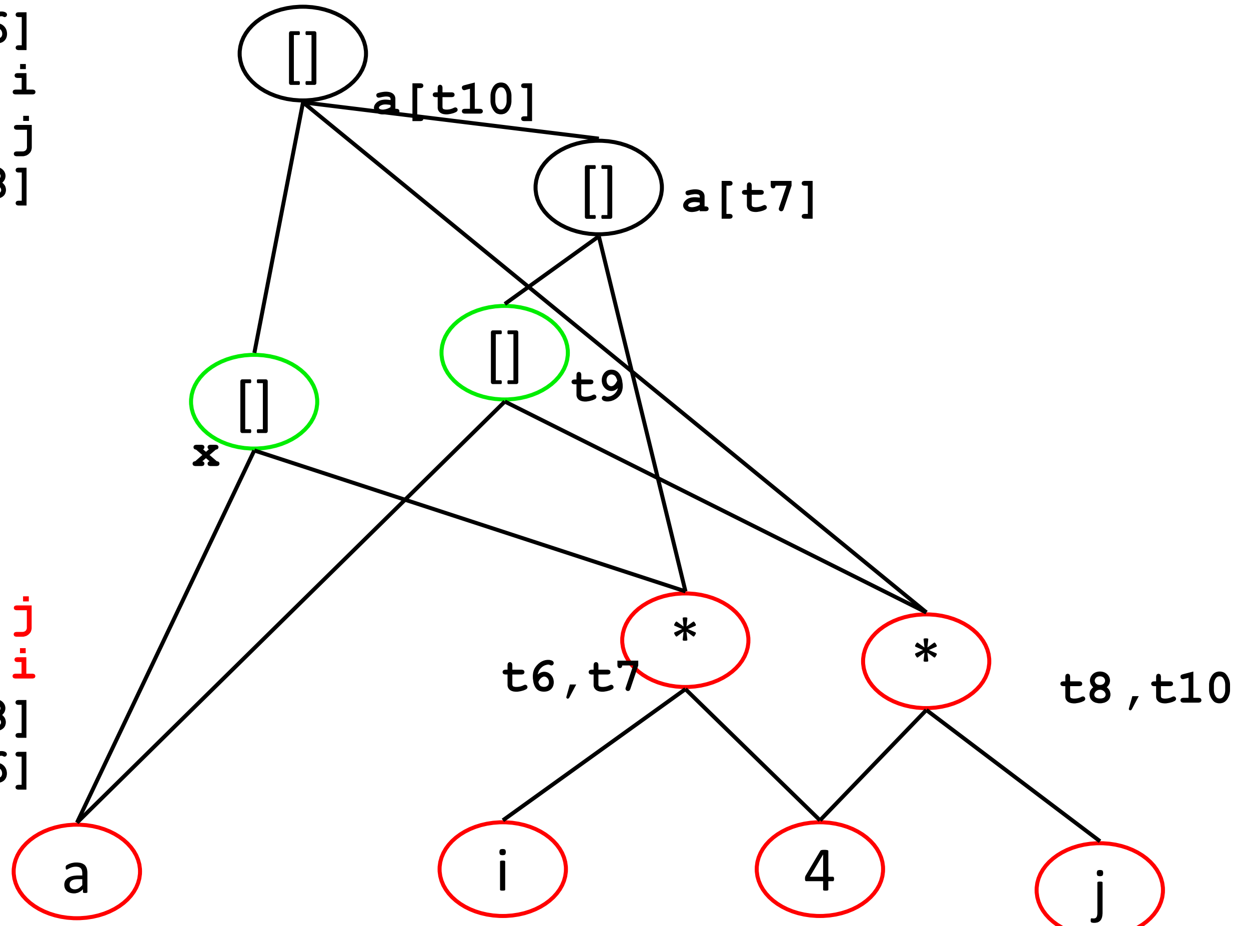



```
B5: t8     = 4 * j
    t6     = 4 * i
```

Recreating the Code

Original

```
B5: t6     = 4*i
    x      = a[t6]
    t7     = 4 * i
    t8     = 4 * j
    t9     = a[t8]
    a[t7]  = t9
    t10    = 4*j
    a[t10] = x
    jump   B2
```

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B5: t8     = 4 * j
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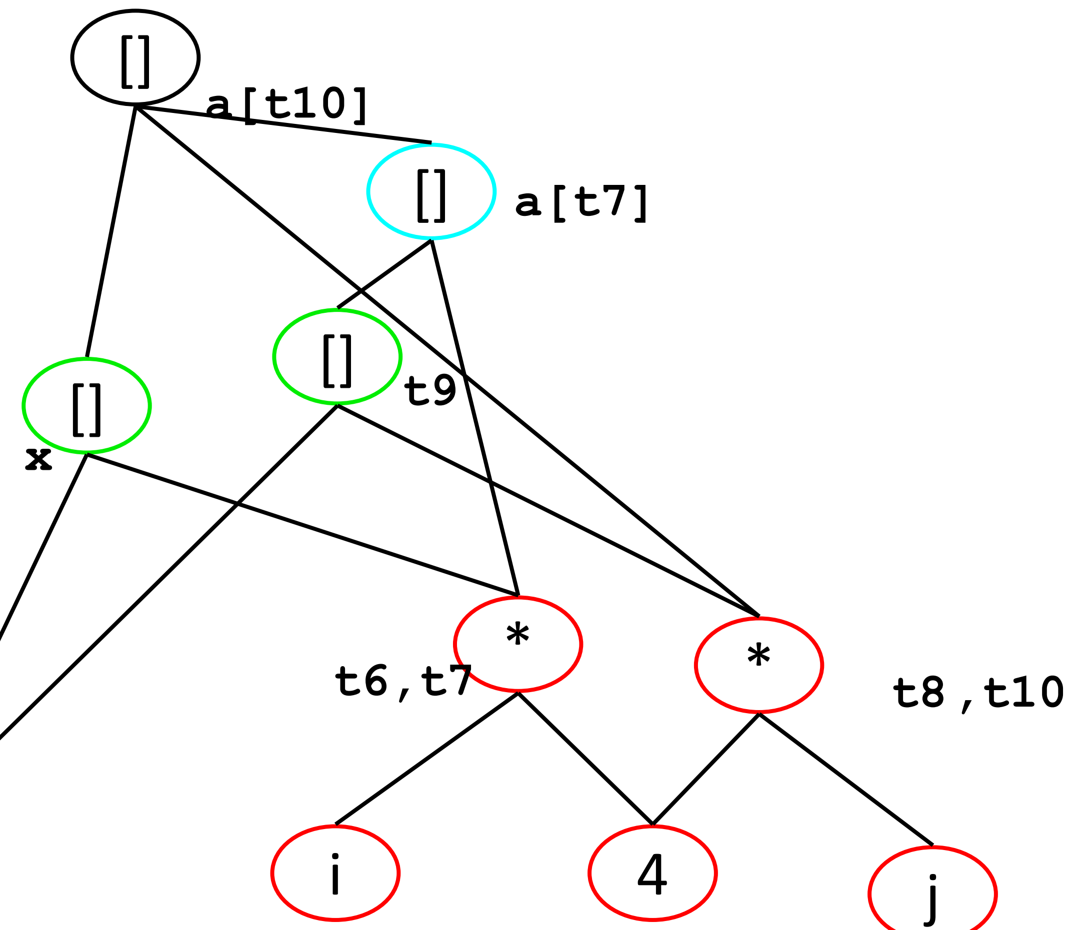


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Recreating the Code

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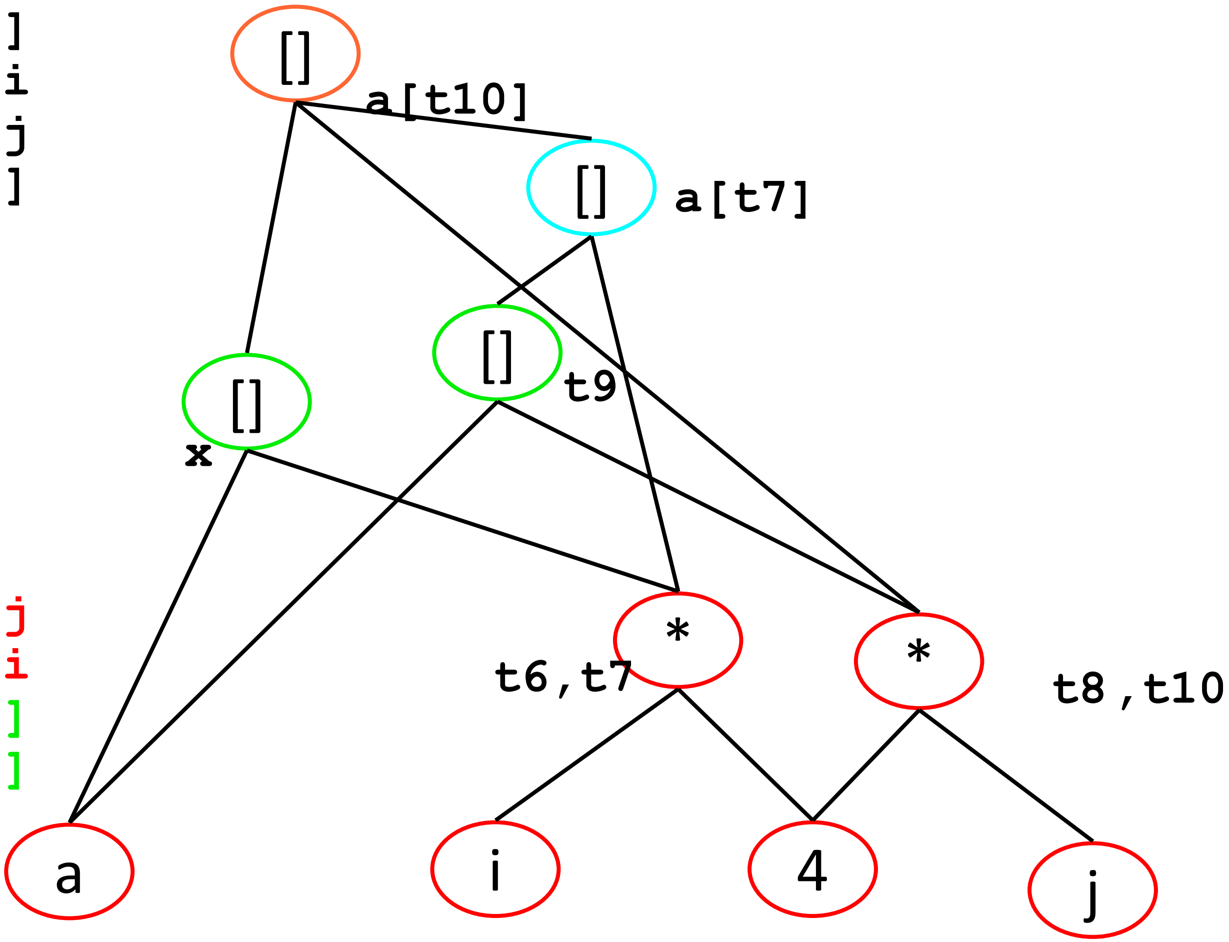
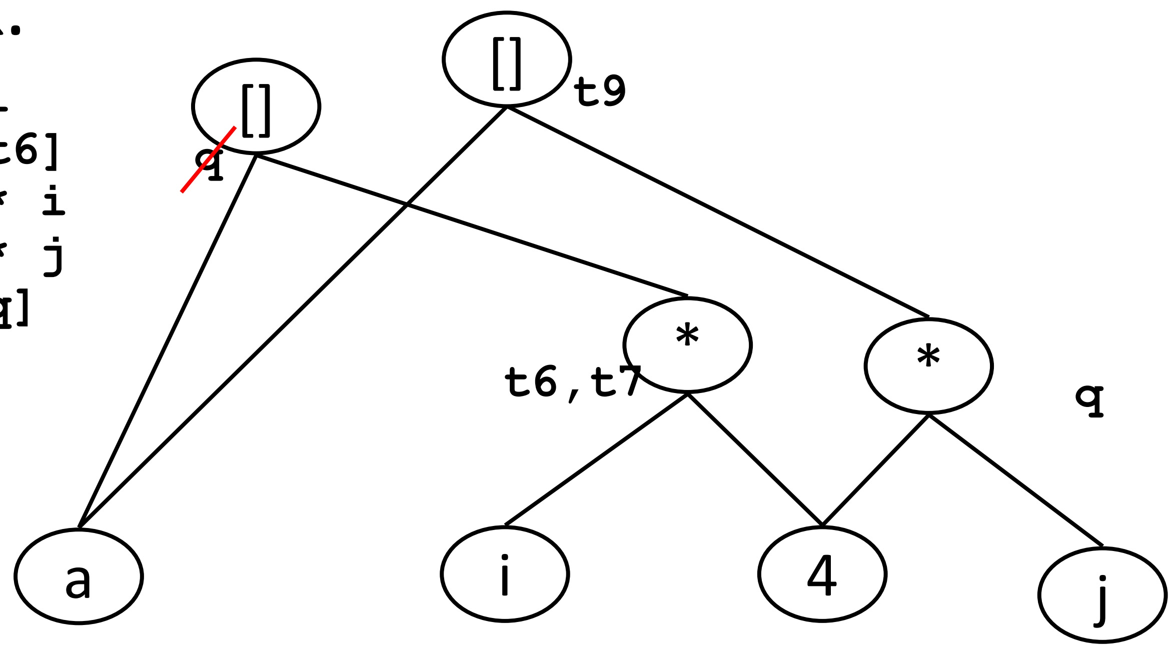


Other uses for DAGs

- Can determine those variables that *can* be live at end of a block.
- Can determine those variables that are live at start of block.

```
B8: t6      = 4*i
    q       = a[t6]
    t7      = 4 * i
    q       = 4 * j
    t9      = a[q]
    …
```



Dead code too?

- Can determine those variables that *can* be live at end of a block.
- Can determine those variables that are live at start of block.

```
B8: t6     = 4*i
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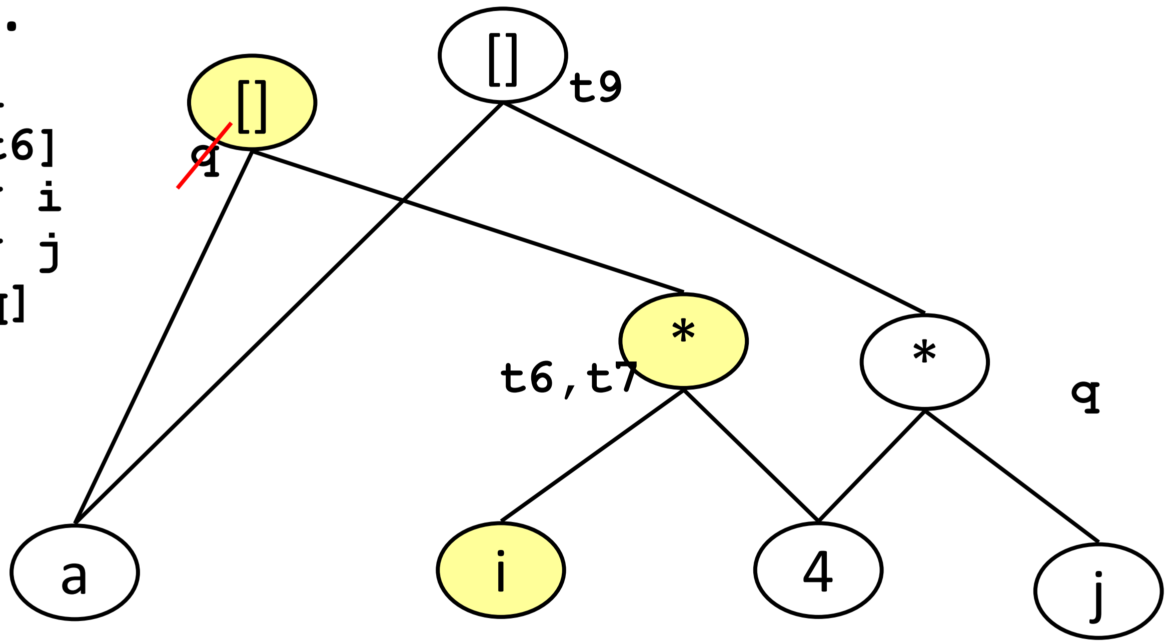
    …
```



Can we do better?

```
B1:     i      = m - 1
        j      = n
        t1     = 4*n
        v      = a[t1]

B2:     i      = i + 1
        t2     = 4 * i
        t3     = a[t2]
        cjump  t3<v   B2, B3


B3:     j      = j - 1
        t4     = 4 * j
        t5     = a[t4]
        cjump  t5>v   B3, B4

B4:     cjump  i >= j B6, B5
```

```
B5:     t6     = 4 * i
        x      = a[t6]

        t8     = 4 * j
        t9     = a[t8]
        a[t6]  = t9

        a[t8]  = x
        jump   B2

B6:     t11    = 4 * i
        x      = a[t11]

        t13    = 4 * n
        t14    = a[t13]
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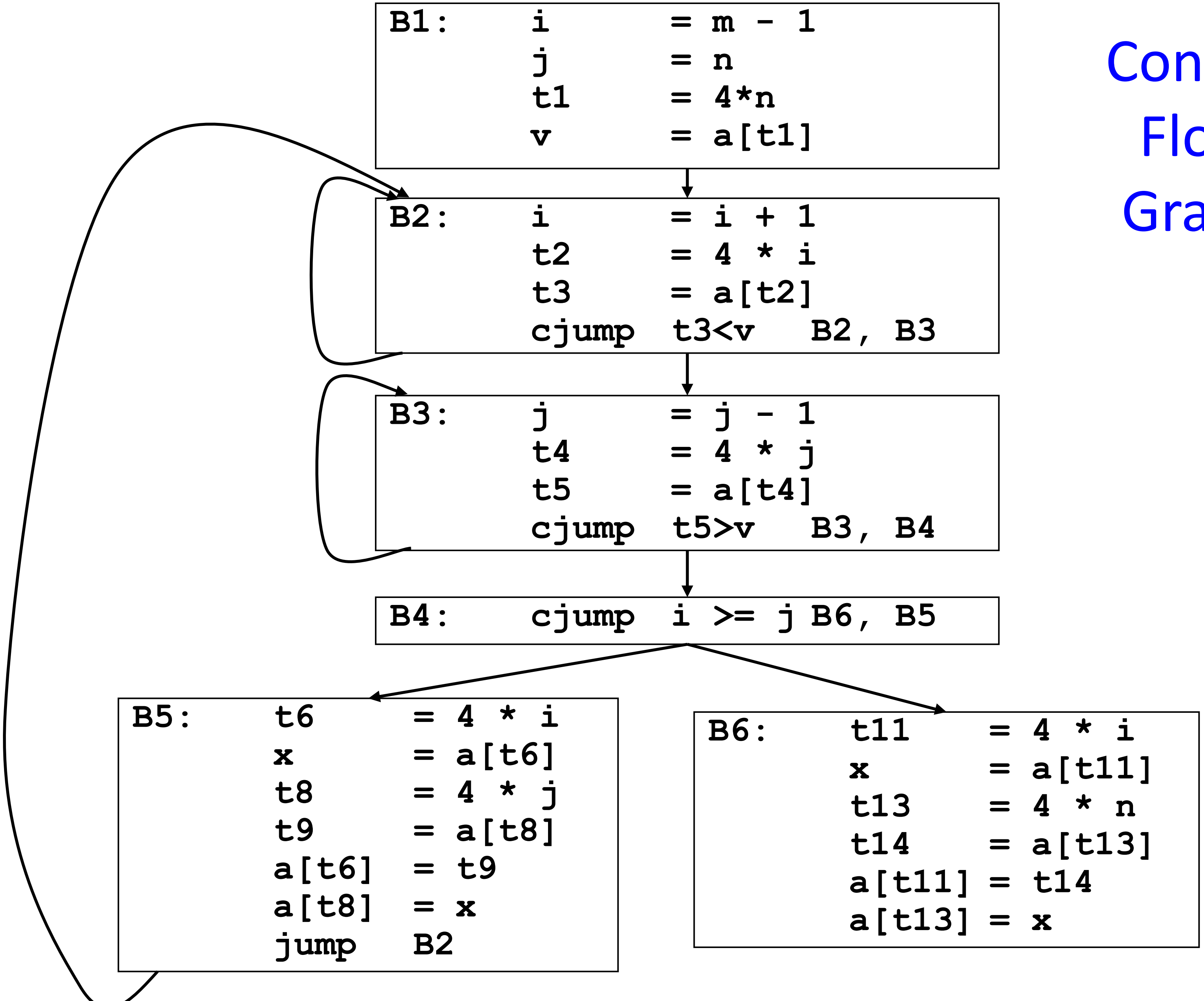
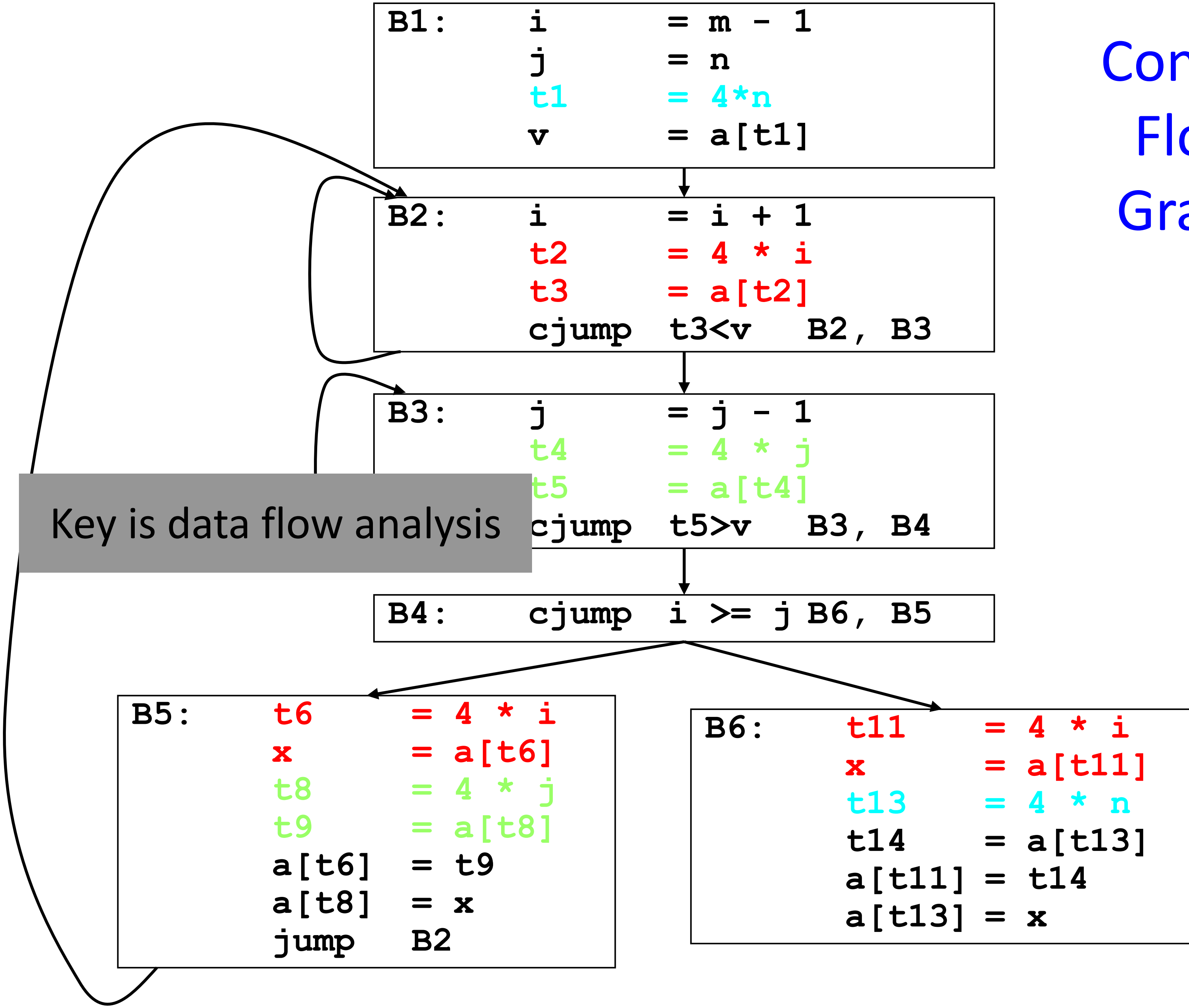
        a[t13] = x
```

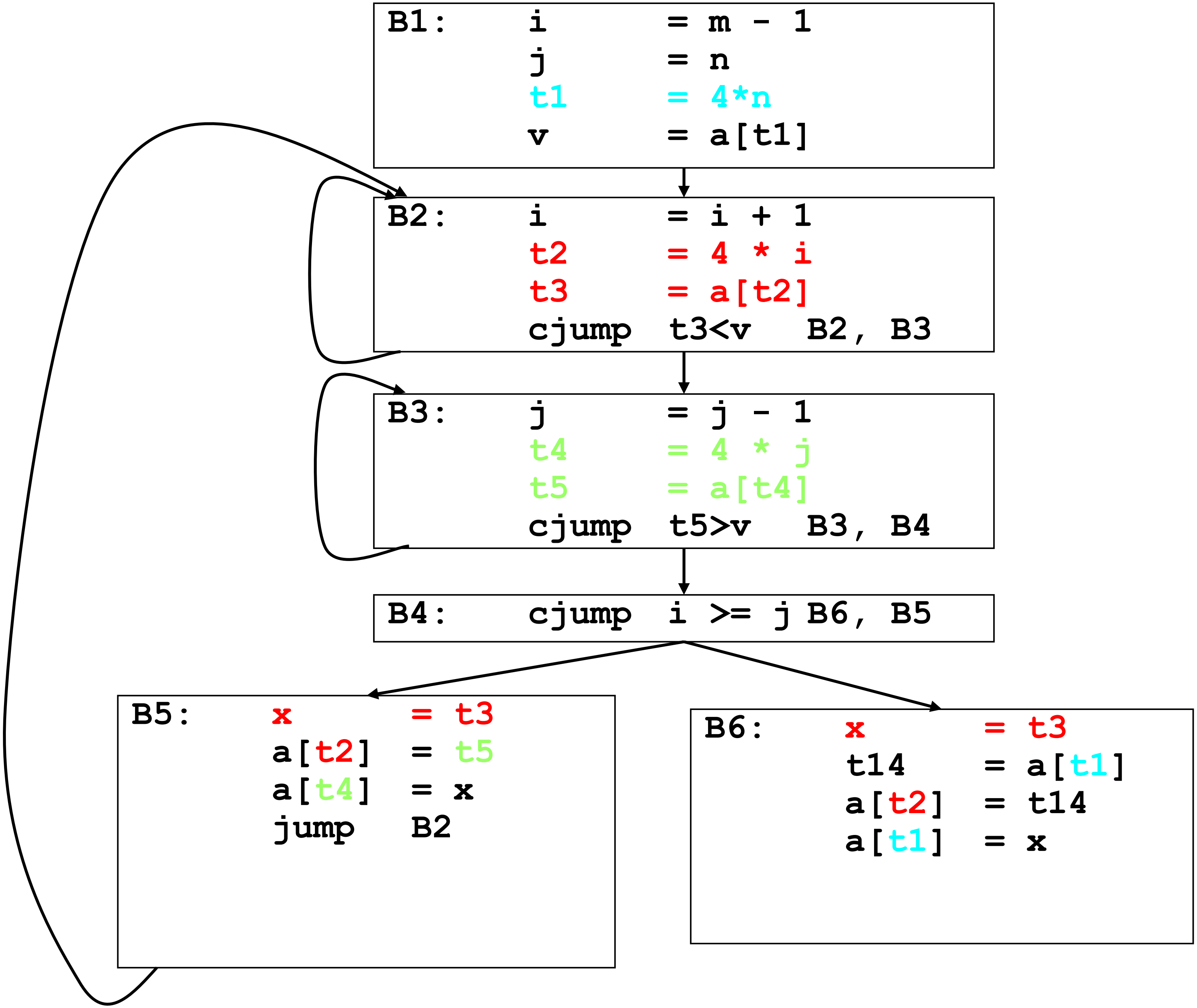
Control Flow Graph



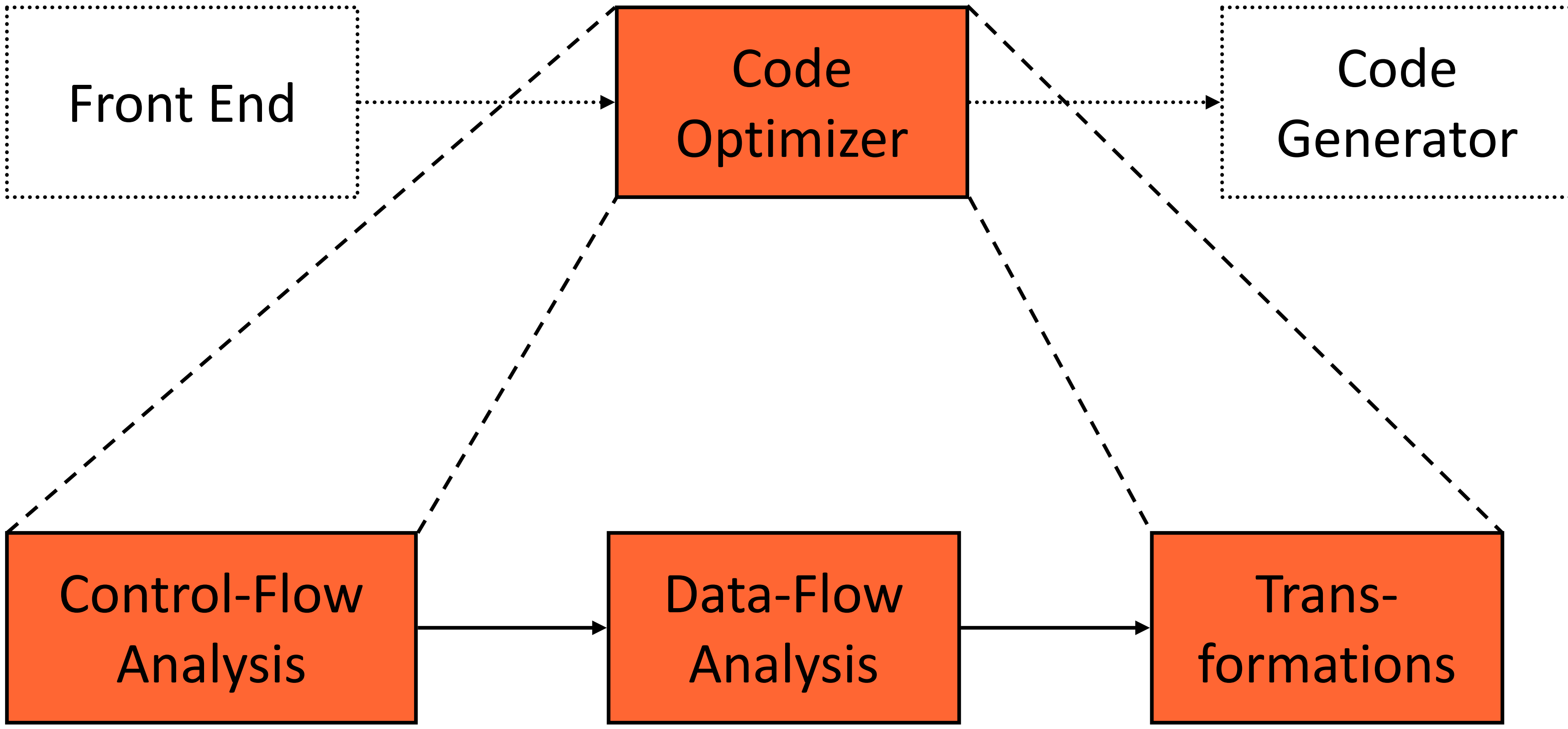
Control Flow Graph



Key is data flow analysis


B1: i = m - 1
j = n
t1 = 4*n
v = a[t1]
B2: i = i + 1
t2 = 4 * i
t3 = a[t2]
cjump t3<v B2, B3
B3: j = j - 1
t4 = 4 * j
t5 = a[t4]
cjump t5>v B3, B4
B4: cjump i >= j B6, B5
B5: x = t3
a[t2] = t5
a[t4] = x
jump B2
B6: x = t3
t14 = a[t1]
a[t2] = t14
a[t1] = x

The Optimizer



Optimizations

- Register Allocation

  Liveness or reaching definitions
- Common subexpression elimination

  Available expressions and reaching expressions
- Constant Propagation
- Copy propagation
- Dead-code elimination
- Loop optimizations
  - Hoisting
  - Induction variable elimination

Dataflow Analysis

- Goal:
  - Answers: Is it legal to perform an optimization?
  - (Not answering: “it is beneficial?”)
- A framework for proving facts about program
- Reasons about lots of little facts
- Little or no interaction between facts
  - Works best on properties about how program computes
- Based on all paths through program
  - including infeasible paths

Today

- Dataflow Analysis
  - reaching definitions
  - liveness
  - available expressions
  - very busy expressions
- Framework

A sample program

```
int fib10(void) {
   int n = 10;
   int older = 0;
   int old = 1;
   int result = 0;
   int i;

   if (n <= 1) return n;
   for (i = 2; i<n; i++) {
        result = old + older;
        older = old;
        old = result;
   }
   return result;
}
```

```
1:     n <- 10
2:     older <- 0
3:     old <- 1
4:     result <- 0
5:     if n <= 1 goto 14
6:     i <- 2
7:     if i > n goto 13

8:     result <- old + older
9:     older <- old
10:    old <- result
11:    i <- i + 1
12:    JUMP 7
13:    return result
14:    return n
```

Simple Constant Propagation

- Can we do SCP?
- How do we recognize it?
- What aren't we doing?
- Metanote:
  - keep opts simple!
  - Use combined power

```
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2:    older <- 0
3:    old <- 1
4:    result <- 0
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6:    i <- 2
7:    if i > n goto 13

8:    result <- old + older
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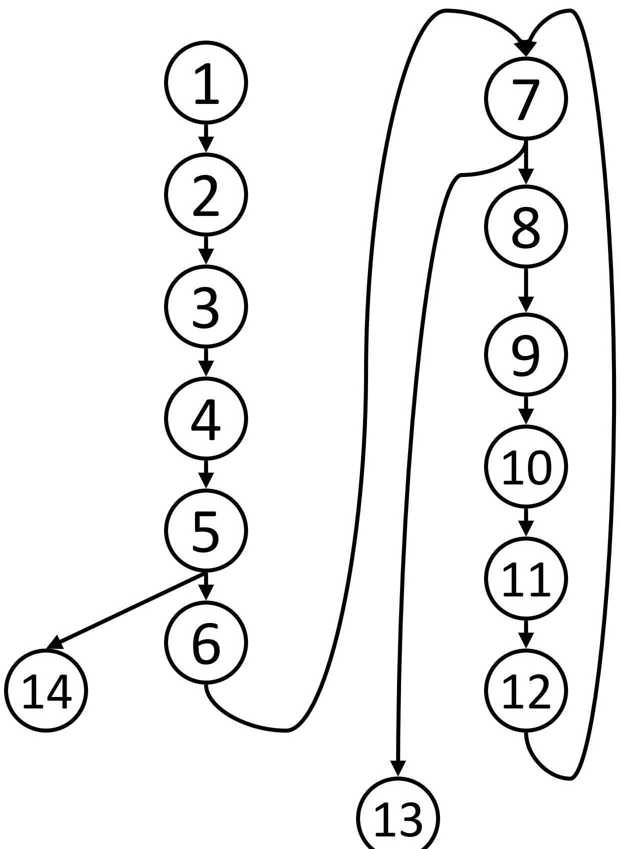
```

Reaching Definitions

A definition of variable v at program point d reaches program point u if there exists a path of control flow edges from d to u that does not contain a definition of v.




```
1:      n <- 10
2:      older <- 0
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4:      result <- 0
5:      if n <= 1 goto 14
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```

Reaching Definitions (ex)

- 1 reaches 5, 7, and 14

14, Really?

Meta-notes:

- (almost) always conservative
- only know what we know
- Keep it simple:
  - What opt(s), if run before this would help
  - What about:

```
1: x <- 0
2: if (false) x<-1
3: … x …
```

- Does 1 reach 3?
- What opt changes this?

```
1:     n <- 10
2:     older <- 0
3:     old <- 1
4:     result <- 0
5:     if n <= 1 goto 14
6:     i <- 2
7:     if i > n goto 13
8:     result <- old + older
9:     older <- old
10:    old <- result
11:    i <- i + 1
12:    JUMP 7
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```

Calculating Reaching Definitions

A definition of variable v at program point d reaches program point u if there exists a path of control flow edges from d to u that does not contain a definition of v.

- Build up RD stmt by stmt
- Stmt s, “d: v <- x op y”, generates d
- Stmt s, “d: v <- x op y”, kills all other defs(v)

Or,

- Gen[s] = { d }
- Kill[s] = defs(v) – { d }

Gen and kill for each stmt

Stmt	Gen	kill
`1: n <- 10`	1	
`2: older <- 0`	2	9
`3: old <- 1`	3	10
`4: result <- 0`	4	8
`5: if n <= 1 goto 14`		
`6: i <- 2`	6	11
`7: if i > n goto 13`		
`8: result <- old + older`	8	4
`9: older <- old`	9	2
`10: old <- result`	10	3
`11: i <- i + 1`	11	6
`12: JUMP 7`		
`13: return result`		
`14: return n`		

we determine the defs that reach a node by using:

- control flow information
- gen and kill info

Computing in[n] and out[n]

- In[n]: the set of defs that reach the beginning of node n
- Out[n]: the set of defs that reach the end of node n

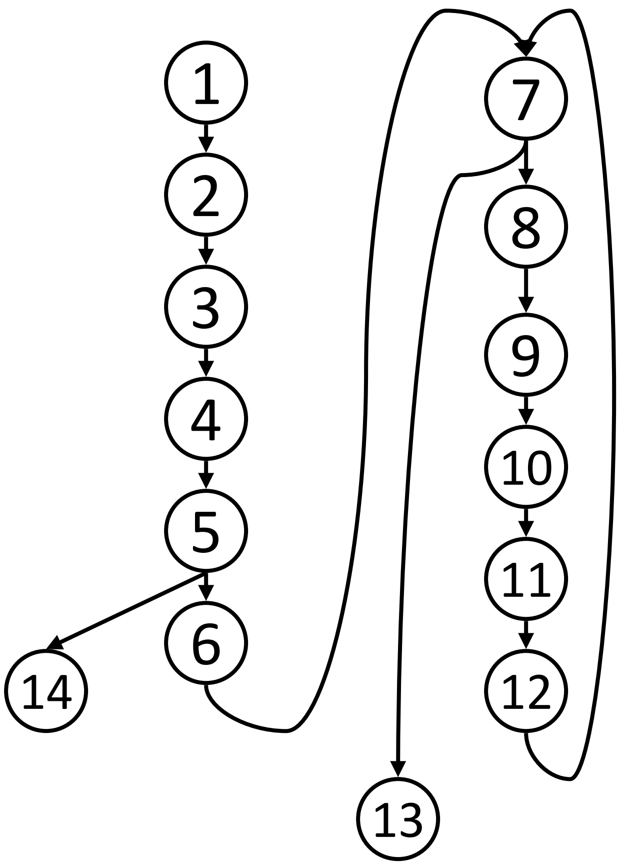
$$in[n] = \bigcup_{p \in pred[n]} out[p]$$

$$out[n] = gen[n] \cup (in[n] - kill[n])$$

- Initialize in[n]=out[n]={} for all n
- Solve iteratively

pred[n]?

- Pred[n] are all nodes that can directly reach n in the control flow graph.
- E.g., pred[7] = { 6, 12 }

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11:     i <- i + 1
12:     JUMP 7
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```
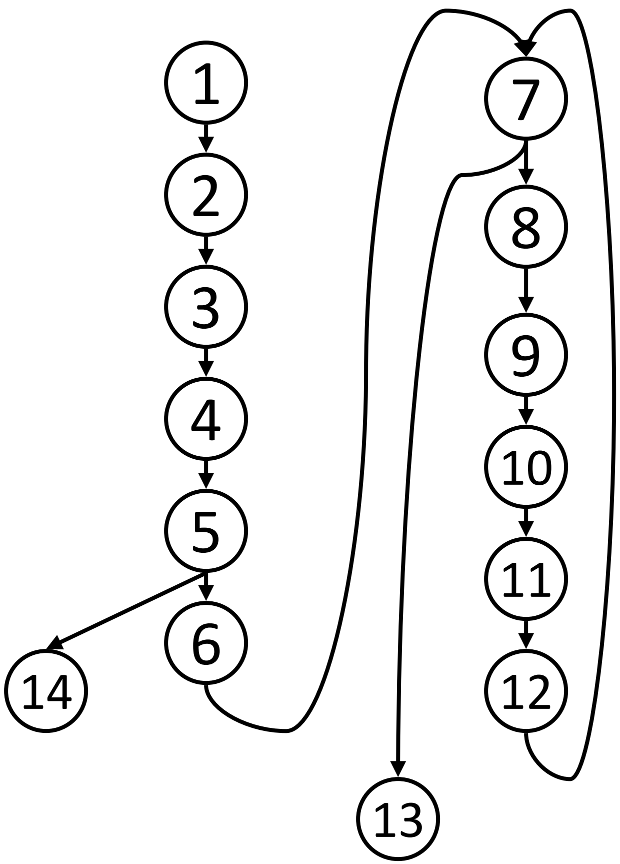

What order to eval nodes?

- Does it matter?
- Lets do: 1,2,3,4,5,14,6,7,13,8,9,10,11,12

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Example:

- Order: 1,2,3,4,5,14,6,7,13,8,9,10,11,12

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Code	Gen	kill	in	out
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12: JUMP 7				
13: return result			1-4,6	1-4,6
14: return n			1-4	1-4

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`11: i <- i + 1`	11	6	1,6,8-10	1,8-11
`12: JUMP 7`			1,8-11	1,8-11
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Example (pass 2)

- Order: 1,2,3,4,5,14,6,7,13,8,9,10,11,12

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3:	`old <- 1`	3	10	1,2	1,2,3
4:	`result <- 0`	4	8	1-3	1-4
5:	`if n <= 1 goto 14`			1-4	1-4
6:	`i <- 2`	6	11	1-4	1-4,6
7:	`if i > n goto 13`			1-4,6,8-11	1-4,6,8-11
8:	`result <- old + older`	8	4	1-4,6,8-11	1-3,6,8-11
9:	`older <- old`	9	2	1-3,6,8	1,3,6,8,9
10:	`old <- result`	10	3	1,3,6,8,9	1,6,8-10
11:	`i <- i + 1`	11	6	1,6,8-10	1,8-11
12:	`JUMP 7`			1,8-11	1,8-11
13:	`return result`			1-4,6,8-11	1-4,6,8-11
14:	`return n`			1-4	1-4

Example (pass 2)

- Order: 1,2,3,4,5,14,6,7,13,8,9,10,11,12

$$in[n] = \bigcup_{p \in pred[n]} out[p] \quad out[n] = gen[n] \cup (in[n] - kill[n])$$

	Gen	kill	in	out
`1: n <- 10`	1			1
`2: older <- 0`	2	9	1	1,2
`3: old <- 1`	3	10	1,2	1,2,3
`4: result <- 0`	4	8	1-3	1-4
`5: if n <= 1 goto 14`			1-4	1-4
`6: i <- 2`	6	11	1-4	1-4,6
`7: if i > n goto 13`			1-4,6,8-11	1-4,6,8-11
`8: result <- old + older`	8	4	1-4,6,8-11	1-3,6,8-11
`9: older <- old`	9	2	1-3,6,8-11	1,3,6,8-11
`10: old <- result`	10	3	1,3,6,8,9	1,6,8-10
`11: i <- i + 1`	11	6	1,6,8-10	1,8-11
`12: JUMP 7`			1,8-11	1,8-11
`13: return result`			1-4,6,8-11	1-4,6,8-11
`14: return n`			1-4	1-4

Example (pass 2)

- Order: 1,2,3,4,5,14,6,7,13,8,9,10,11,12

$$in[n] = \bigcup_{p \in pred[n]} out[p] \quad out[n] = gen[n] \bigcup (in[n] - kill[n])$$

	Gen	kill	in	out
1: n <- 10	1			1
2: older <- 0	2	9	1	1,2
3: old <- 1	3	10	1,2	1,2,3
4: result <- 0	4	8	1-3	1-4
5: if n <= 1 goto 14			1-4	1-4
6: i <- 2	6	11	1-4	1-4,6
7: if i > n goto 13			1-4,6,8-11	1-4,6,8-11
8: result <- old + older	8	4	1-4,6,8-11	1-3,6,8-11
9: older <- old	9	2	1-3,6,8-11	1,3,6,8-11
10: old <- result	10	3	1,3,6,8-11	1,6,8-11
11: i <- i + 1	11	6	1,6,8-10	1,8-11
12: JUMP 7			1,8-11	1,8-11
13: return result			1-4,6,8-11	1-4,6,8-11
14: return n			1-4	1-4

Example (pass 2)

- Order: 1,2,3,4,5,14,6,7,13,8,9,10,11,12

$$in[n] = \bigcup_{p \in pred[n]} out[p] \quad out[n] = gen[n] \cup (in[n] - kill[n])$$

	Gen	kill	in	out
`1: n <- 10`	1			1
`2: older <- 0`	2	9	1	1,2
`3: old <- 1`	3	10	1,2	1,2,3
`4: result <- 0`	4	8	1-3	1-4
`5: if n <= 1 goto 14`			1-4	1-4
`6: i <- 2`	6	11	1-4	1-4,6
`7: if i > n goto 13`			1-4,6,8-11	1-4,6,8-11
`8: result <- old + older`	8	4	1-4,6,8-11	1-3,6,8-11
`9: older <- old`	9	2	1-3,6,8-11	1,3,6,8-11
`10: old <- result`	10	3	1,3,6,8-11	1,6,8-11
`11: i <- i + 1`	11	6	1,6,8-11	1,8-11
`12: JUMP 7`			1,8-11	1,8-11
`13: return result`			1-4,6,8-11	1-4,6,8-11
`14: return n`			1-4	1-4

Example (pass 2)

- Order: 1,2,3,4,5,14,6,7,13,8,9,10,11,12

$$in[n] = \bigcup_{p \in pred[n]} out[p] \quad out[n] = gen[n] \bigcup (in[n] - kill[n])$$

	Gen	kill	in	out
1: n <- 10	1			1
2: older <- 0	2	9	1	1,2
3: old <- 1	3	10	1,2	1,2,3
4: result <- 0	4	8	1-3	1-4
5: if n <= 1 goto 14			1-4	1-4
6: i <- 2	6	11	1-4	1-4,6
7: if i > n goto 13			1-4,6,8-11	1-4,6,8-11
8: result <- old + older	8	4	1-4,6,8-11	1-3,6,8-11
9: older <- old	9	2	1-3,6,8-11	1,3,6,8-11
10: old <- result	10	3	1,3,6,8-11	1,6,8-11
11: i <- i + 1	11	6	1,6,8-11	1,8-11
12: JUMP 7			1,8-11	1,8-11
13: return result			1-4,6,8-11	1-4,6,8-11
14: return n			1-4	1-4

An Improvement: Basic Blocks

- No need to compute this one stmt at a time
- For straight line code:
  - In[s1; s2] = in[s1]
  - Out[s1; s2] = out[s2]
- Combine the gen and kill sets into one per BB.

- Gen[BB]={2,3,4,5}
- Kill[BB]={1,8,11}

	Gen	kill
`1: i <- 1`	1	8,4
`2: j <- 2`	2	
`3: k <- 3 + i`	3	11
`4: i <- j`	4	1,8
`5: m <- i + k`	5	

General Procedure

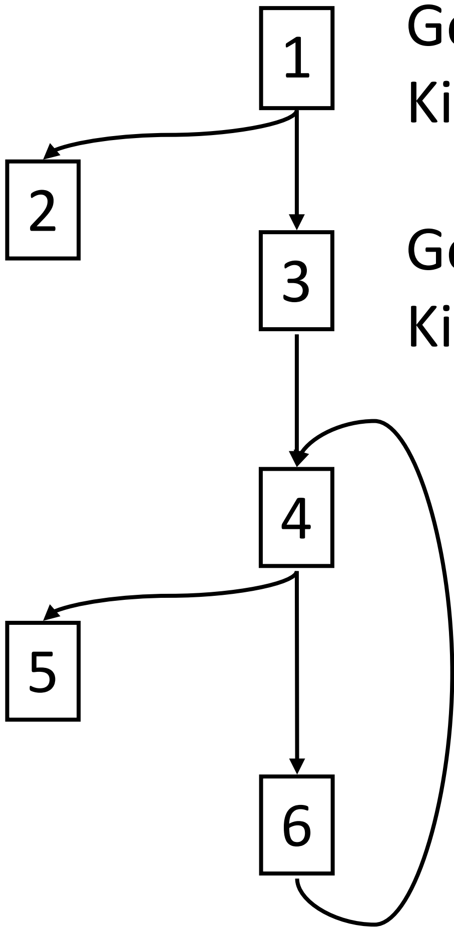
- Create Basic Blocks
- Create Control Flow Graph
- Summarize Basic Block into a single Gen set and a single Kill set
- Iterate over CFG until no changes
- create data flow facts for individual instructions as needed starting with in set of the basic block

BB sets

Block	Statement	Gen	kill	BB Gen	BB Kill
1	`1: n <- 10`	1			
	`2: older <- 0`	2	9		
	`3: old <- 1`	3	10		
	`4: result <- 0`	4	8		
	`5: if n <= 1 goto 14`			1,2,3,4	8,9,10
3	`6: i <- 2`	6	11	6	11
4	`7: if i > n goto 13`				
6	`8: result <- old + older`	8	4		
	`9: older <- old`	9	2		
	`10: old <- result`	10	3		
	`11: i <- i + 1`	11	6		
	`12: JUMP 7`			8-11	2-4,6
5	`13: return result`				
2	`14: return n`				

BB sets

In out



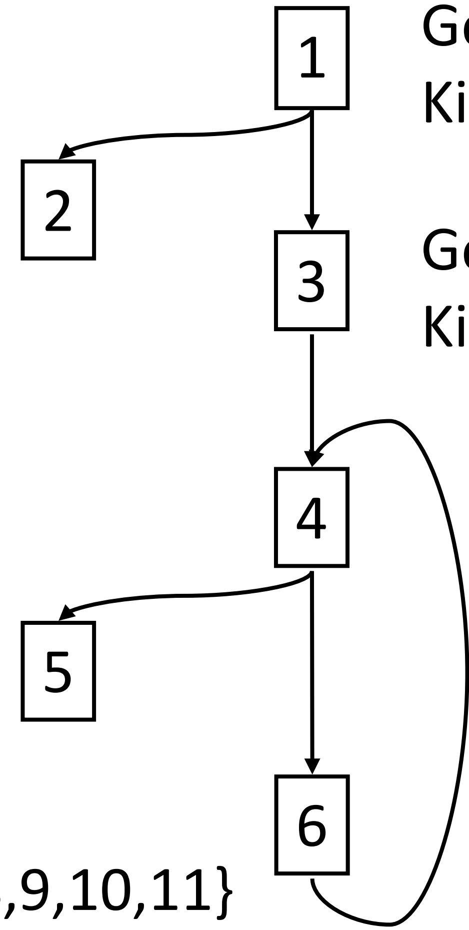
Gen={1,2,3,4}
Kill={8,9,10}

out: 1,2,3,4

Gen={6}
Kill={11}

Gen={8,9,10,11}
Kill={2,3,4,6}

BB sets



Gen={1,2,3,4}
Kill={8,9,10}

Gen={6}
Kill={11}

Gen={8,9,10,11}
Kill={2,3,4,6}

In	out
	1,2,3,4
1,2,3,4	1,2,3,4,6
1-4,6,8-11	1-4,6,8-11
1-4,6,8-11	1,8-11

Forward Dataflow

- Reaching definitions is a forward dataflow problem:
  It propagates information from the predecessors of a node to the node
- Defined by:
  - Basic attributes: (gen and kill)
  - Transfer function: F_{bb} $out[n] = gen[n] \bigcup (in[n] - kill[n])$
  - Meet operator: union $in[n] = \bigcup_{p \in pred[n]} out[p]$
  - Set of values (a lattice, in this case powerset of program points)
  - Initial values for each node b
- Solve for fixed point solution

How to implement?

- Values?
- Gen?
- Kill?
- F_{bb}?
- Order to visit nodes?
- When are we done?
  - In fact, do we know we terminate?

Implementing RD

- Values: bits in a bit vector
- Gen: 1 in each position generated, otherwise 0
- Kill: 0 in each position killed, otherwise 1
- F_{bb}: out[b] = gen[b] | (in[b] & kill[b])
- Init in[b]=out[b]=0

- When are we done?
- What order to visit nodes? Does it matter?

RD Worklist algorithm

Initialize: in[B] = out[b] = $\emptyset$

Initialize: in[entry] = $\emptyset$

Work queue, W = all Blocks in topological order

```
while (|W|  != 0)  {
     remove b from W
     old = out[b]
     in[b] =  {over all pred(p) ∈ b} ∪ out[p]
     out[b] = gen[b] ∪ (in[b] - kill[b])
     if (old != out[b]) W = W ∪ succ(b)
}
```

Storing Rd information

- Use-def chains: for each use of var x in s, a list of definitions of x that reach s

```
1:  n <- 10                    1
2:  older <- 0                 1            1,2
3:  old <- 1                   1,2          1,2,3
4:  result <- 0                1-3          1-4
5:  if n <= 1 goto 14          1-4          1-4
6:  i <- 2                     1-4          1-4,6
7:  if i > n goto 13           1-4,6,8-11   1-4,6,8-11
8:  result <- old + older      1-4,6,8-11   1-3,6,8-11
9:  older <- old               1-3,6,8-11   1,3,6,8-11
10: old <- result              1,3,6,8-11   1,6,8-11
11: i <- i + 1                 1,6,8-11     1,8-11
12: JUMP 7                     1,8-11       1,8-11
13: return result              1-4,6        1-4,6
14: return n                   1-4          1-4
```

Storing Rd information

- Use-def chains: for each use of var x in s, a list of definitions of x that reach s

```
1:  n <- 10                          1
2:  older <- 0                       1               1,2
3:  old <- 1                         1,2             1,2,3
4:  result <- 0                      1-3             1-4
5:  if n <= 1 goto 14                1-4             1-4
6:  i <- 2                           1-4             1-4,6
7:  if i > n goto 13                 1-4,6,8-11      1-4,6,8-11
8:  result <- old + older            1-4,6,8-11      1-3,6,8-11
9:  older <- old                     1-3,6,8-11      1,3,6,8-11
10: old <- result                    1,3,6,8-11      1,6,8-11
11: i <- i + 1                       1,6,8-11        1,8-11
12: JUMP 7                           1,8-11          1,8-11
13: return result                    1-4,6           1-4,6
14: return n                         1-4             1-4
```

Storing Rd information

- Use-def chains: for each use of var x in s, a list of definitions of x that reach s

1: n <- 10	1	
2: older <- 0	1	1,2
3: old <- 1	1,2	1,2,3
4: result <- 0	1-3	1-4
5: if n <= 1 goto 14	1-4	1-4
6: i <- 2	1-4	1-4,6
7: if i > n goto 13	1-4,6,8-11	1-4,6,8-11
8: result <- old + older	1-4,6,8-11	1-3,6,8-11
9: older <- old	1-3,6,8-11	1,3,6,8-11
10: old <- result	1,3,6,8-11	1,6,8-11
11: i <- i + 1	1,6,8-11	1,8-11
12: JUMP 7	1,8-11	1,8-11
13: return result	1-4,6	1-4,6
14: return n	1-4	1-4

Storing Rd information

- Use-def chains: for each use of var x in s, a list of definitions of x that reach s

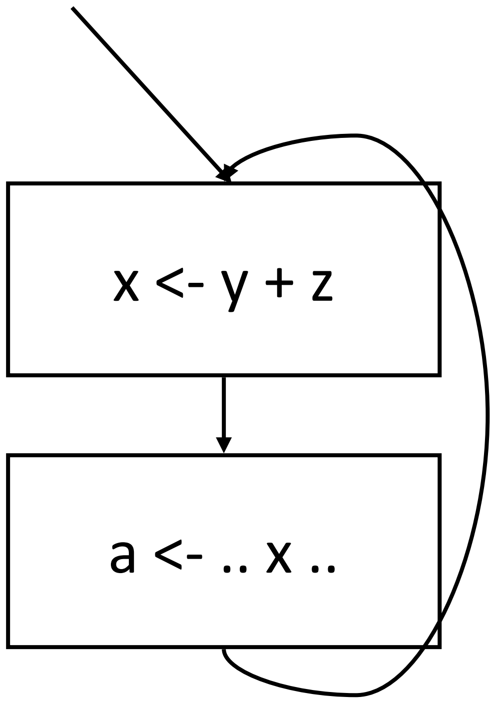
1: n <- 10	1	
2: older <- 0	1	1,2
3: old <- 1	1,2	1,2,3
4: result <- 0	1-3	1-4
5: if n <= 1 goto 14	1-4	1-4
6: i <- 2	1-4	1-4,6
7: if i > n goto 13	1-4,6,8-11	1-4,6,8-11
8: result <- old + older	1-4,6,8-11	1-3,6,8-11
9: older <- old	1-3,6,8-11	1,3,6,8-11
10: old <- result	1,3,6,8-11	1,6,8-11
11: i <- i + 1	1,6,8-11	1,8-11
12: JUMP 7	1,8-11	1,8-11
13: return result	1-4,6	1-4,6
14: return n	1-4	1-4

Constant Folding + DCE

~~1: n <- 10~~	~~1~~	
2: older <- 0	1	1,2
3: old <- 1	1,2	1,2,3
4: result <- 0	1-3	1-4
~~5: if 10 <= 1 goto 14~~	~~1-4~~	~~1-4~~
6: i <- 2	1-4	1-4,6
7: if i > 10 goto 13	1-4,6,8-11	1-4,6,8-11
8: result <- old + older	1-4,6,8-11	1-3,6,8-11
9: older <- old	1-3,6,8-11	1,3,6,8-11
10: old <- result	1,3,6,8-11	1,6,8-11
11: i <- i + 1	1,6,8-11	1,8-11
12: JUMP 7	1,8-11	1,8-11
13: return result	1-4,6	1-4,6
~~14: return 10~~	~~1-4~~	~~1-4~~

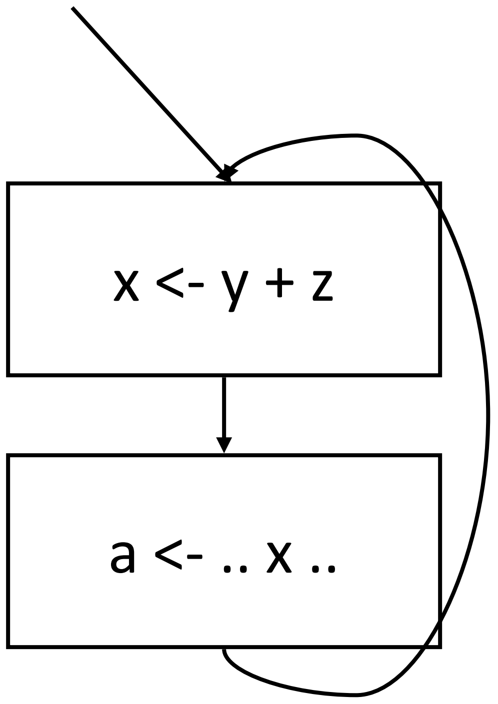
Loop Invariant Code Motion

- When can expression be moved out of a loop?



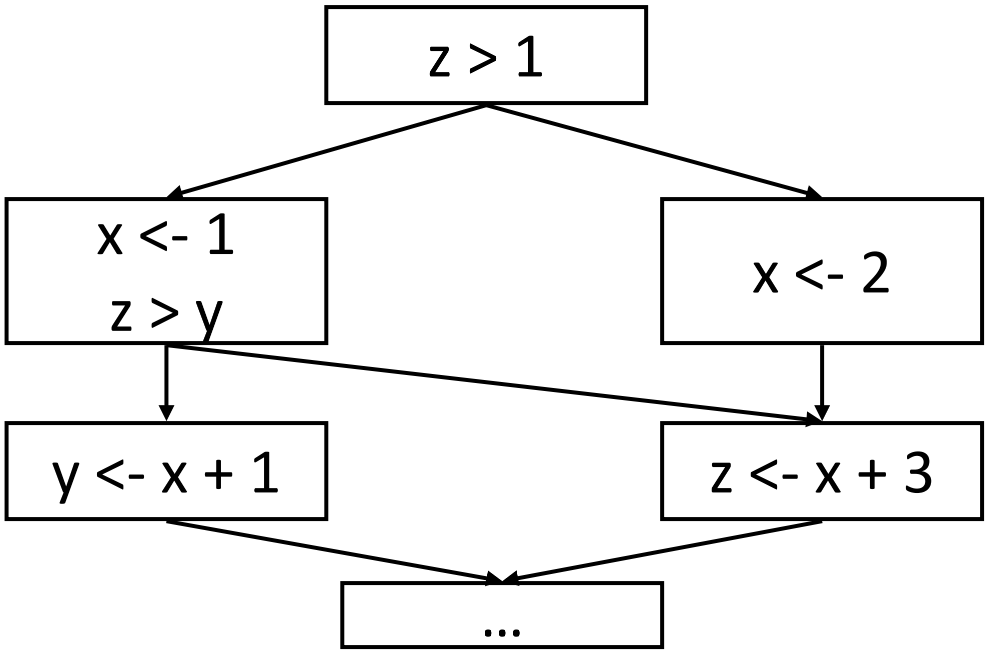
Loop Invariant Code Motion

- When can expression be moved out of a loop?
- When all reaching definitions of operands are outside of loop, expression is loop invariant
- Use ud-chains to detect



Def-use chains are valuable too

- Def-use chain: for each definition of var x, a list of all uses of that definition
- Computed from liveness analysis, a backward dataflow problem
- Def-use is NOT symmetric to use-def



Liveness (def-use chains)

- A variable x is live-out of a stmt s if x can be used along some path starting a s, otherwise x is dead.

Liveness as a dataflow problem

- This is a backwards analysis
  - A variable is live out if used by a successor
  - Gen: For a use: indicate it is live coming into s
  - Kill: Defining a variable v in s makes it dead before s (unless s uses v to define v)
  - Lattice is just live (top) and dead (bottom)
- Values are variables
- In[n] = variables live before n

  $$= (out[n] - kill[n]) \cup gen[n]$$

- Out[n] = variables live after n

  $$= \bigcup_{s \in succ(n)} In[s]$$

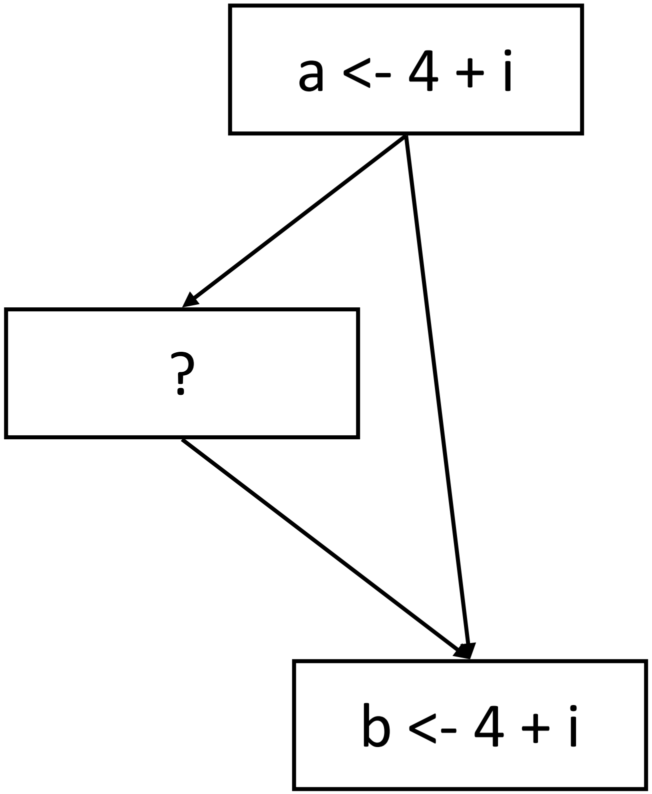
Dead Code Elimination

- Code is dead if it has no effect on the outcome of the program.
- When is code dead?

Dead Code Elimination

- Code is dead if it has no effect on the outcome of the program.
- When is code dead?
  - When the definition is dead, and
  - When the instruction has no side effects
- So:
  - run liveness
  - Construct def-use chains
  - Any instruction which has no users and has no side effects can be eliminated

When can we do CSE?



Available Expressions

- X+Y is “available” at statement S if
  - x+y is computed along every path from the start to S AND
  - neither x nor y is modified after the last evaluation of x+y

```
a <- b+c

b <- a-d

c <- b+c

d <- a-d
```

Available Expressions

- X+Y is “available” at statement S if
  - x+y is computed along every path from the start to S AND
  - neither x nor y is modified after the last evaluation of x+y

a <- b+c

b <- a-d

c <- b+c ⟵ b+c Not available, since b redefined

d <- a-d ⟵ a-d is available

Computing Available Expressions

- Forward or backward?
- Values?
- Lattice?
- gen[b] =
- kill[b] =
- in[b] =
- out[b] =
- initialization?

Computing Available Expressions

- Forward
- Values: all expressions
- Lattice: available, not-avail
- gen[b] = if b evals expr e and doesn't
  define variables used in e
- kill[b] = if b assigns to x,
  then all exprs using x are killed.
- out[b] = (in[b] – kill[b]) $\cup$ gen[b]
- in[b] = what to do at a join point?
- initialization?

Computing Available Expressions

- Forward
- Values: all expressions
- Lattice: available, not-avail
- gen[b] = if b evals expr e and doesn't
  define variables used in e
- kill[b] = if b assigns to x, exprs(x) are killed
  $out[b] = (in[b] - kill[b]) \cup gen[b]$
- in[b] = An expr is avail only if avail on ALL edges, so:
  $in[b] = \cap$ over all $p \in pred(b)$, out[p]
- Initialization
  - All nodes, but entry are set to ALL avail
  - Entry is set to NONE avail

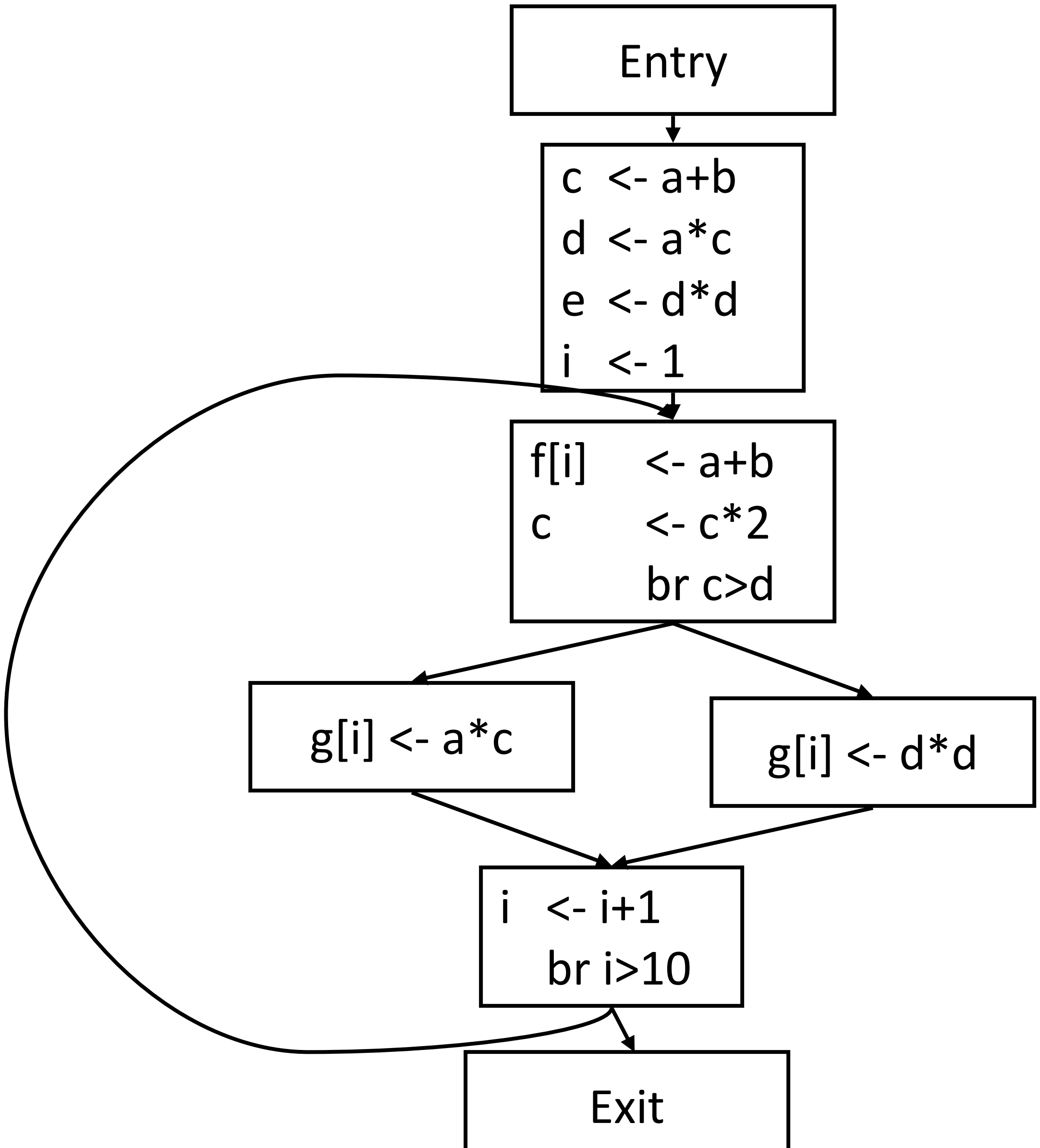
Constructing Gen & Kill

Stmt s	Gen	Kill
t <- x op y	{x op y}-kill[s]	{exprs containing t}
t <- M[a]	{M[a]}-kill[s]	
M[a] <- b		
f(a, …)		{M[x] for all x}
t <- f(a,…)		

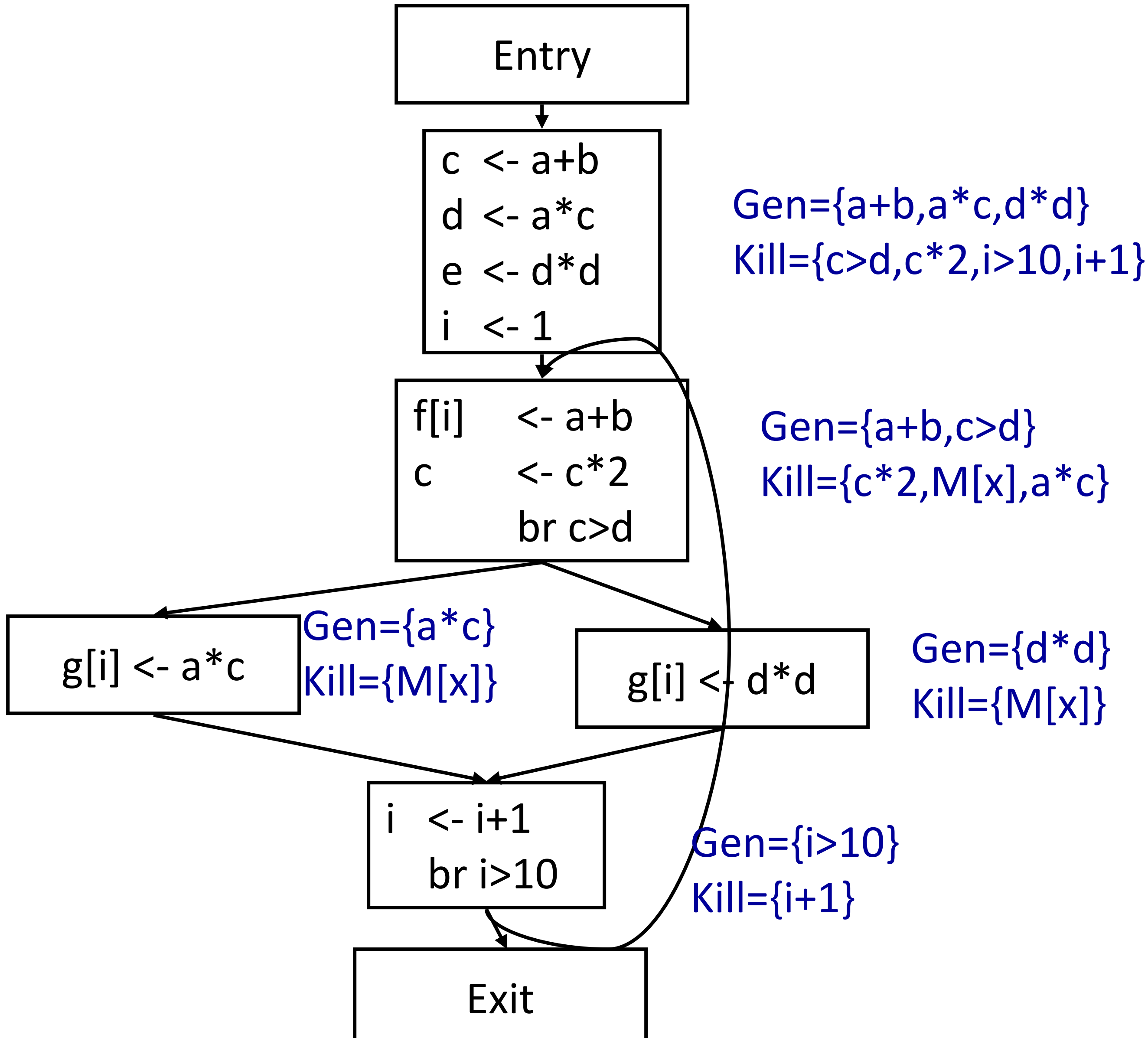
Constructing Gen & Kill

Stmt s	Gen	Kill
t <- x op y	{x op y}-kill[s]	{exprs containing t}
t <- M[a]	{M[a]}-kill[s]	{exprs containing t}
M[a] <- b	{}	{for all x, M[x]}
f(a, …)	{}	{for all x, M[x]}
t <- f(a,…)	{}	{exprs containing t for all x, M[x]}

Example

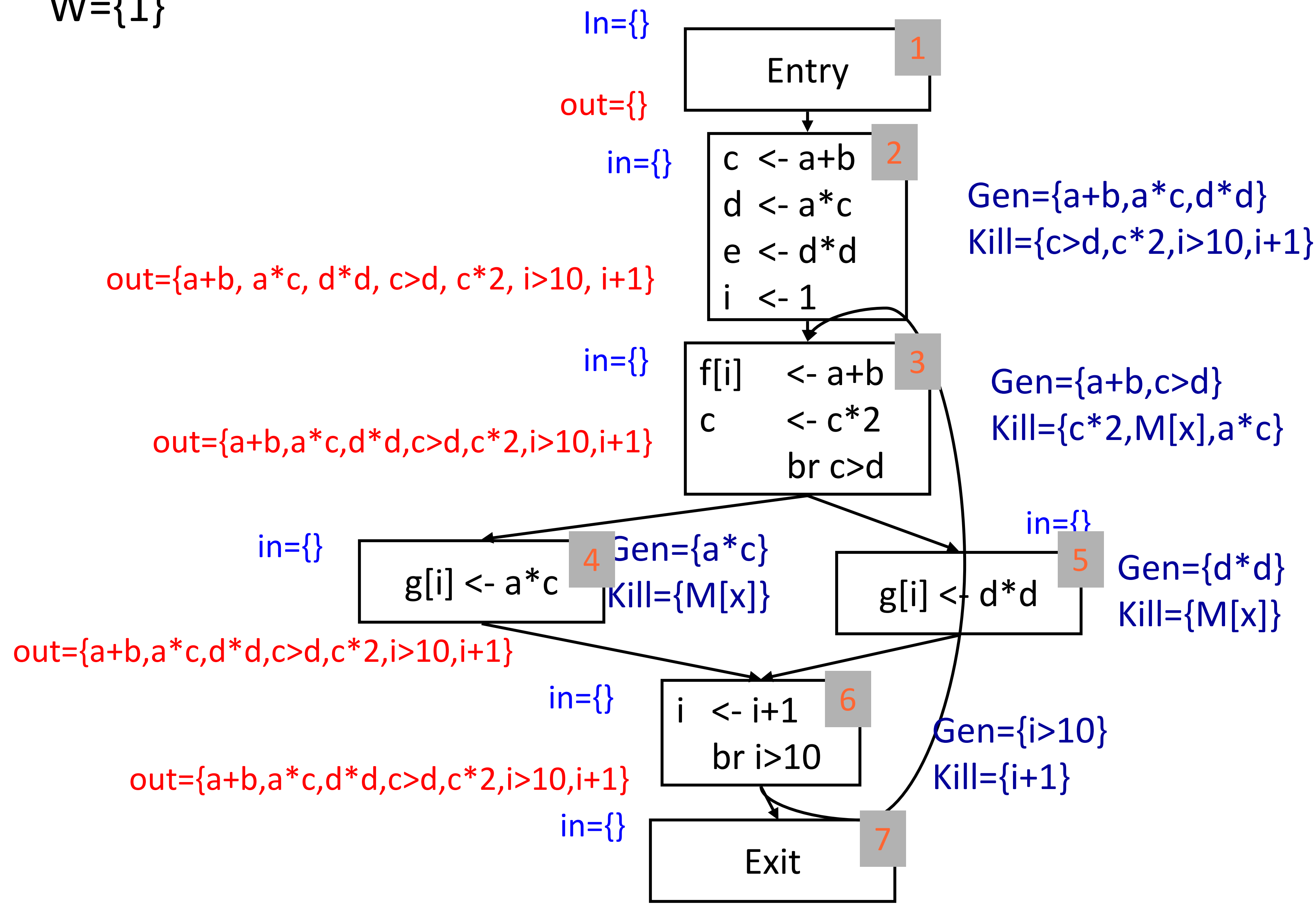


Example


Entry
c <- a+b
d <- a*c
e <- d*d
i <- 1
Gen={a+b,a*c,d*d}
Kill={c>d,c*2,i>10,i+1}
f[i] <- a+b
c <- c*2
br c>d
Gen={a+b,c>d}
Kill={c*2,M[x],a*c}
g[i] <- a*c
Gen={a*c}
Kill={M[x]}
g[i] <- d*d
Gen={d*d}
Kill={M[x]}
i <- i+1
br i>10
Gen={i>10}
Kill={i+1}
Exit

Example

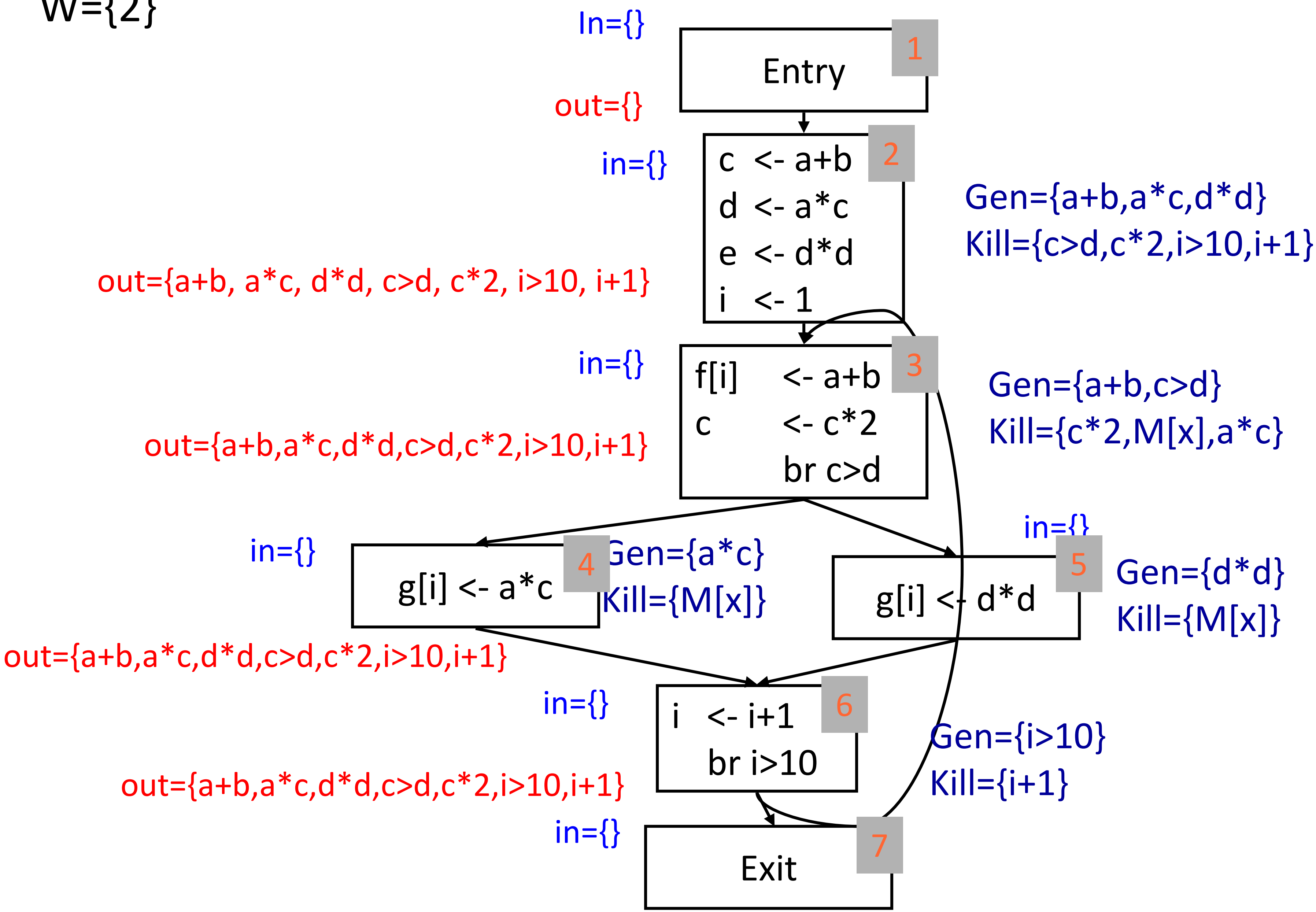
W={1}



In={}
Entry
1
out={}
in={}
c <- a+b
d <- a*c
e <- d*d
i <- 1
2
Gen={a+b,a*c,d*d}
Kill={c>d,c*2,i>10,i+1}
out={a+b, a*c, d*d, c>d, c*2, i>10, i+1}
in={}
f[i] <- a+b
c <- c*2
br c>d
3
Gen={a+b,c>d}
Kill={c*2,M[x],a*c}
out={a+b,a*c,d*d,c>d,c*2,i>10,i+1}
in={}
g[i] <- a*c
4
Gen={a*c}
Kill={M[x]}
in={}
g[i] <- d*d
5
Gen={d*d}
Kill={M[x]}
out={a+b,a*c,d*d,c>d,c*2,i>10,i+1}
in={}
i <- i+1
br i>10
6
Gen={i>10}
Kill={i+1}
out={a+b,a*c,d*d,c>d,c*2,i>10,i+1}
in={}
Exit
7

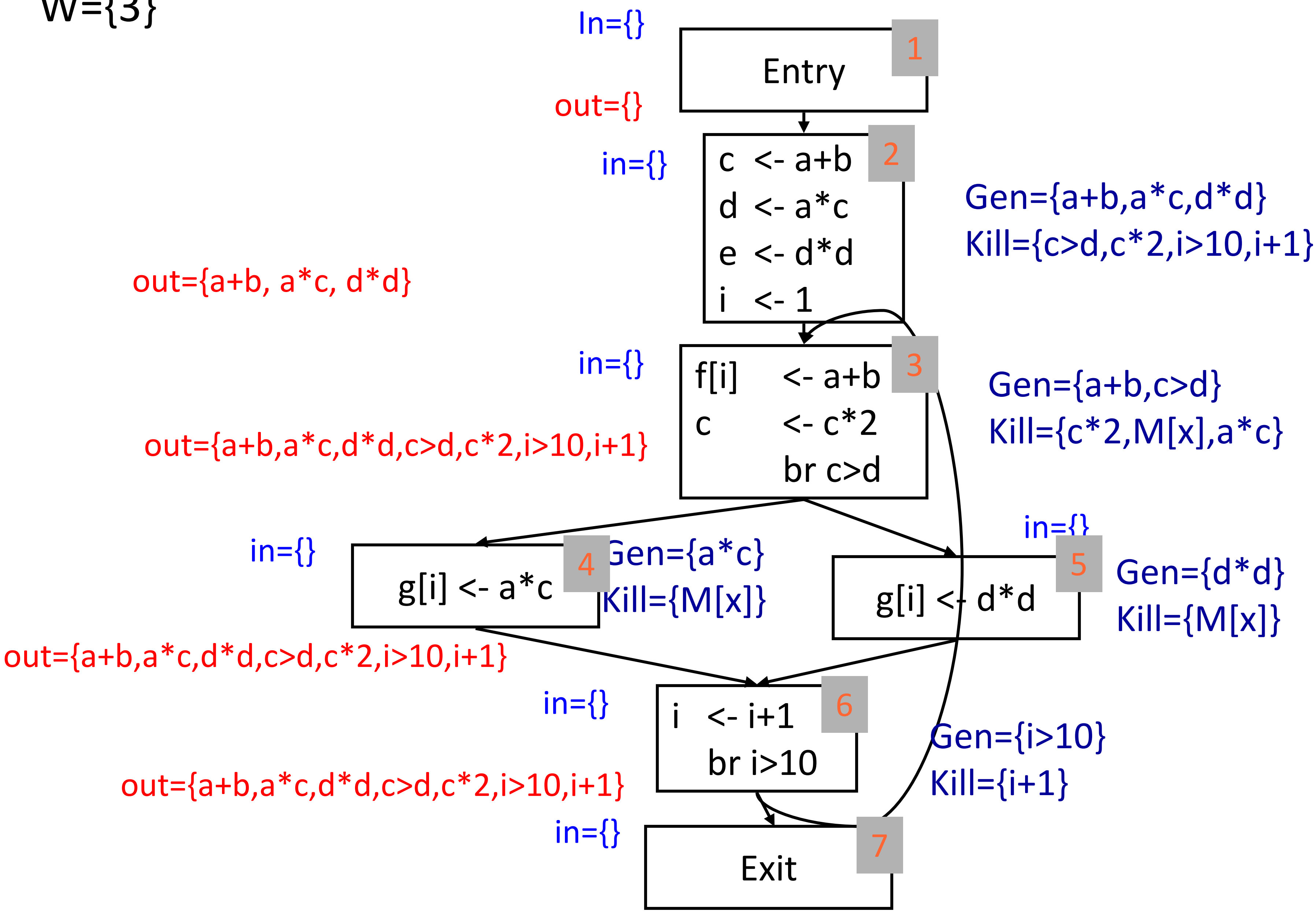
Example

W={2}


In={}
Entry
1
out={}
in={}
c <- a+b
d <- a*c
e <- d*d
i <- 1
2
Gen={a+b,a*c,d*d}
Kill={c>d,c*2,i>10,i+1}
out={a+b, a*c, d*d, c>d, c*2, i>10, i+1}
in={}
f[i] <- a+b
c <- c*2
br c>d
3
Gen={a+b,c>d}
Kill={c*2,M[x],a*c}
out={a+b,a*c,d*d,c>d,c*2,i>10,i+1}
in={}
g[i] <- a*c
4
Gen={a*c}
Kill={M[x]}
in={}
g[i] <- d*d
5
Gen={d*d}
Kill={M[x]}
out={a+b,a*c,d*d,c>d,c*2,i>10,i+1}
in={}
i <- i+1
br i>10
6
Gen={i>10}
Kill={i+1}
out={a+b,a*c,d*d,c>d,c*2,i>10,i+1}
in={}
Exit
7

Example

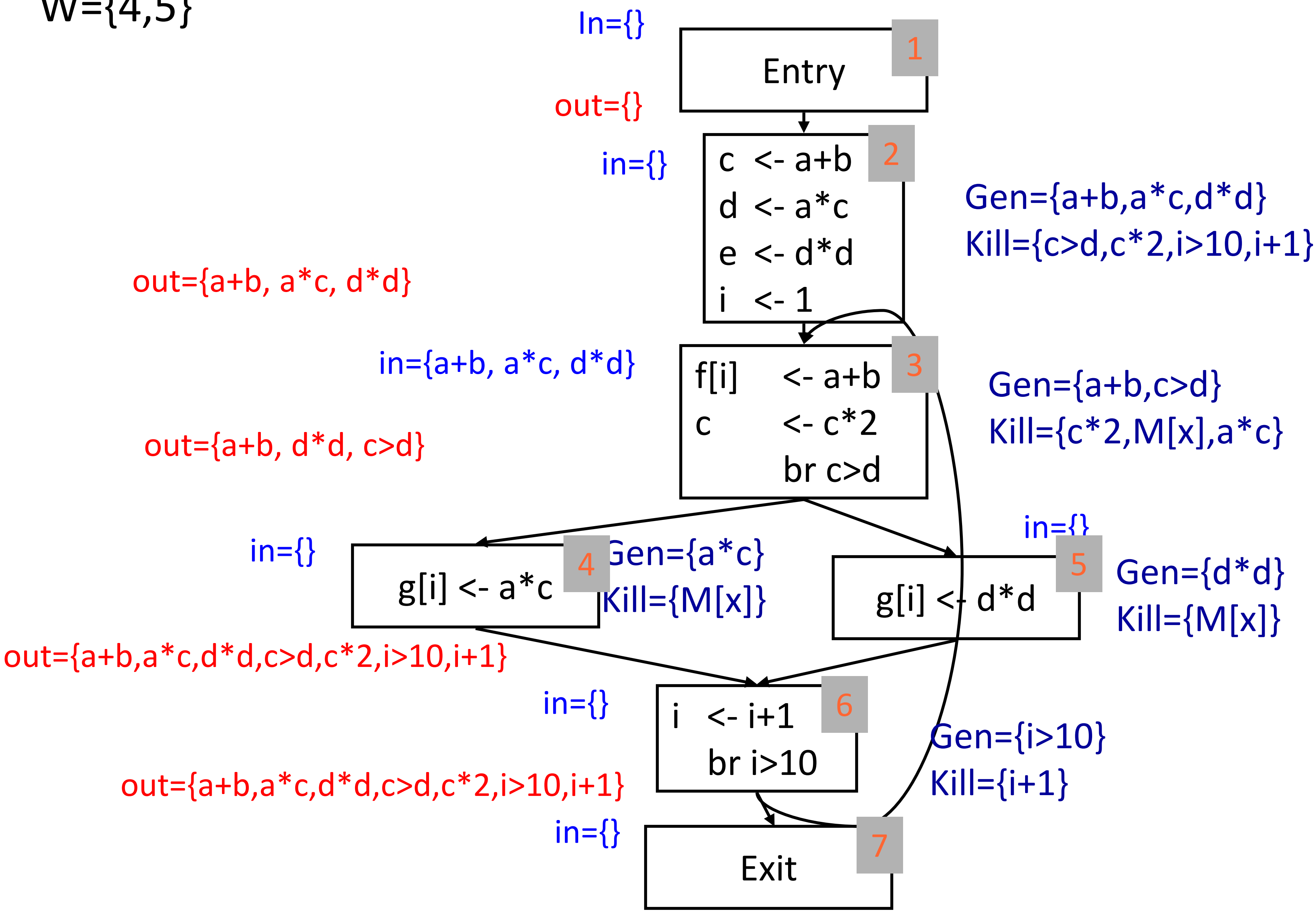
W={3}



In={}
Entry
1
out={}
in={}
c <- a+b
d <- a*c
e <- d*d
i <- 1
2
Gen={a+b,a*c,d*d}
Kill={c>d,c*2,i>10,i+1}
out={a+b, a*c, d*d}
in={}
f[i] <- a+b
c <- c*2
br c>d
3
Gen={a+b,c>d}
Kill={c*2,M[x],a*c}
out={a+b,a*c,d*d,c>d,c*2,i>10,i+1}
in={}
g[i] <- a*c
4
Gen={a*c}
Kill={M[x]}
in={}
g[i] <- d*d
5
Gen={d*d}
Kill={M[x]}
out={a+b,a*c,d*d,c>d,c*2,i>10,i+1}
in={}
i <- i+1
br i>10
6
Gen={i>10}
Kill={i+1}
out={a+b,a*c,d*d,c>d,c*2,i>10,i+1}
in={}
Exit
7

Example

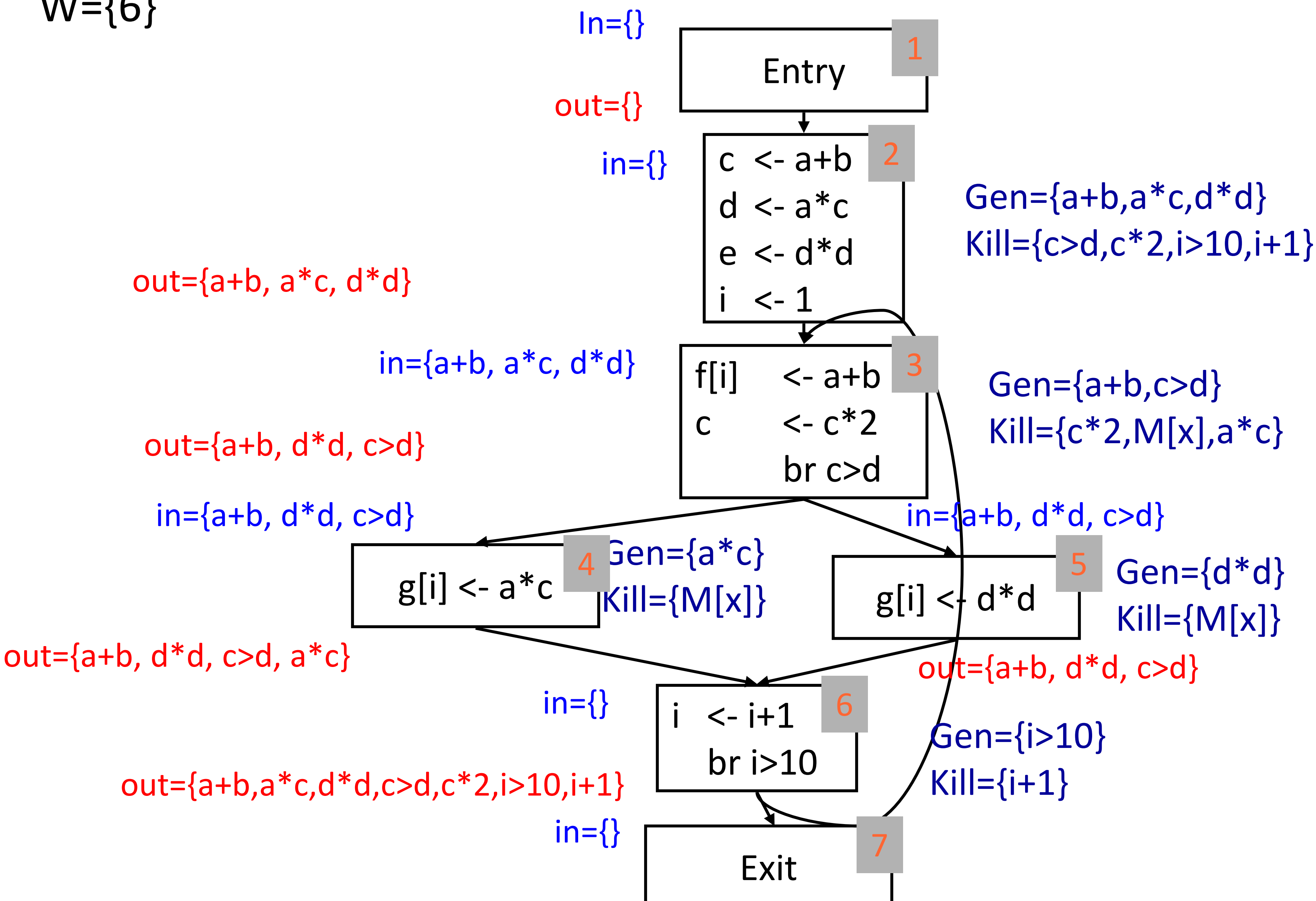
W={4,5}



In={}
Entry
1
out={}
in={}
c <- a+b
d <- a*c
e <- d*d
i <- 1
2
Gen={a+b,a*c,d*d}
Kill={c>d,c*2,i>10,i+1}
out={a+b, a*c, d*d}
in={a+b, a*c, d*d}
f[i] <- a+b
c <- c*2
br c>d
3
Gen={a+b,c>d}
Kill={c*2,M[x],a*c}
out={a+b, d*d, c>d}
in={}
g[i] <- a*c
4
Gen={a*c}
Kill={M[x]}
in={}
g[i] <- d*d
5
Gen={d*d}
Kill={M[x]}
out={a+b,a*c,d*d,c>d,c*2,i>10,i+1}
in={}
i <- i+1
br i>10
6
Gen={i>10}
Kill={i+1}
out={a+b,a*c,d*d,c>d,c*2,i>10,i+1}
in={}
Exit
7

Example

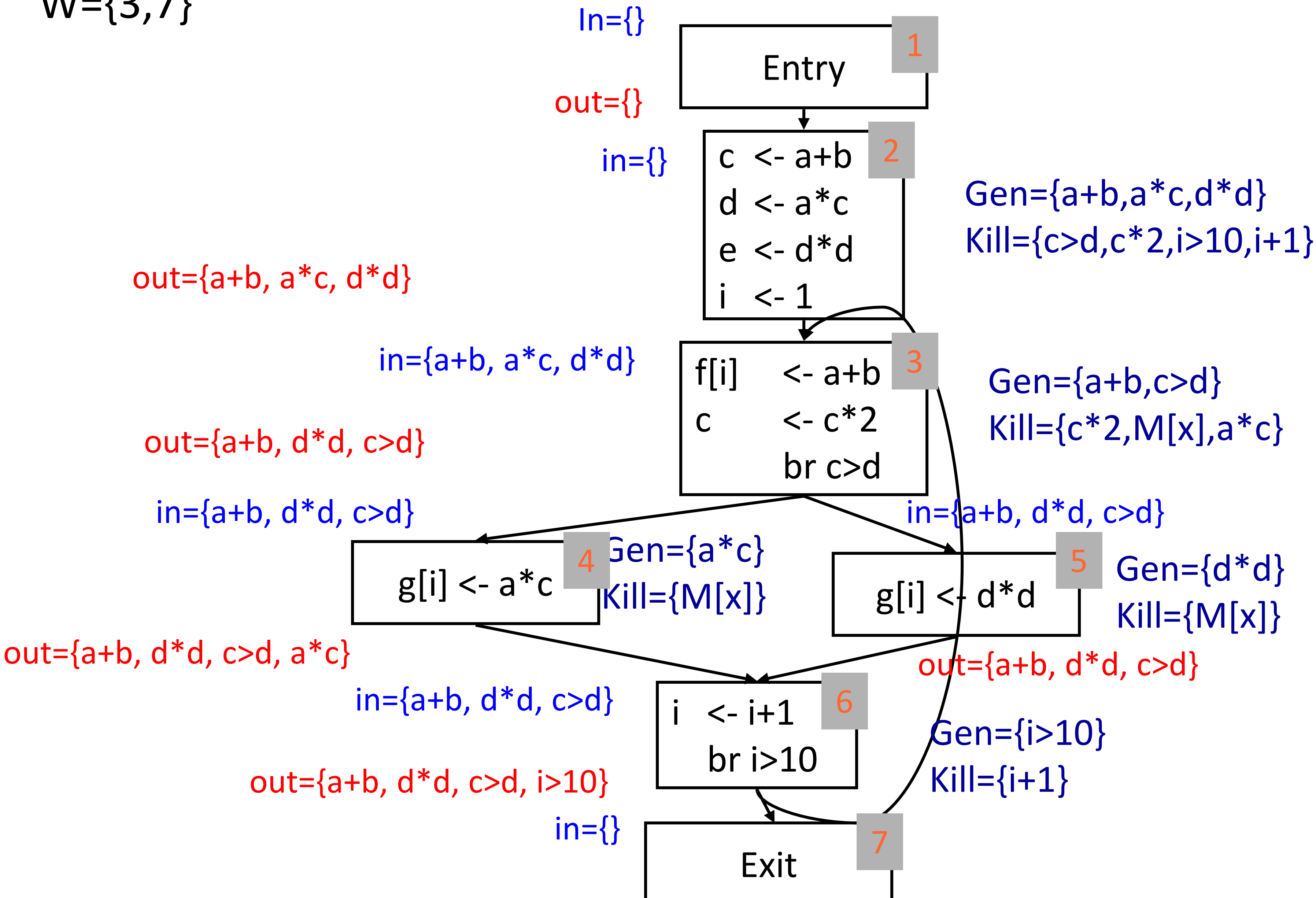
W={6}



In={}
Entry
1
out={}
in={}
c <- a+b
d <- a*c
e <- d*d
i <- 1
2
Gen={a+b,a*c,d*d}
Kill={c>d,c*2,i>10,i+1}
out={a+b, a*c, d*d}
in={a+b, a*c, d*d}
f[i] <- a+b
c <- c*2
br c>d
3
Gen={a+b,c>d}
Kill={c*2,M[x],a*c}
out={a+b, d*d, c>d}
in={a+b, d*d, c>d}
in={a+b, d*d, c>d}
g[i] <- a*c
4
Gen={a*c}
Kill={M[x]}
g[i] <- d*d
5
Gen={d*d}
Kill={M[x]}
out={a+b, d*d, c>d, a*c}
out={a+b, d*d, c>d}
in={}
i <- i+1
br i>10
6
Gen={i>10}
Kill={i+1}
out={a+b,a*c,d*d,c>d,c*2,i>10,i+1}
in={}
Exit
7

Example

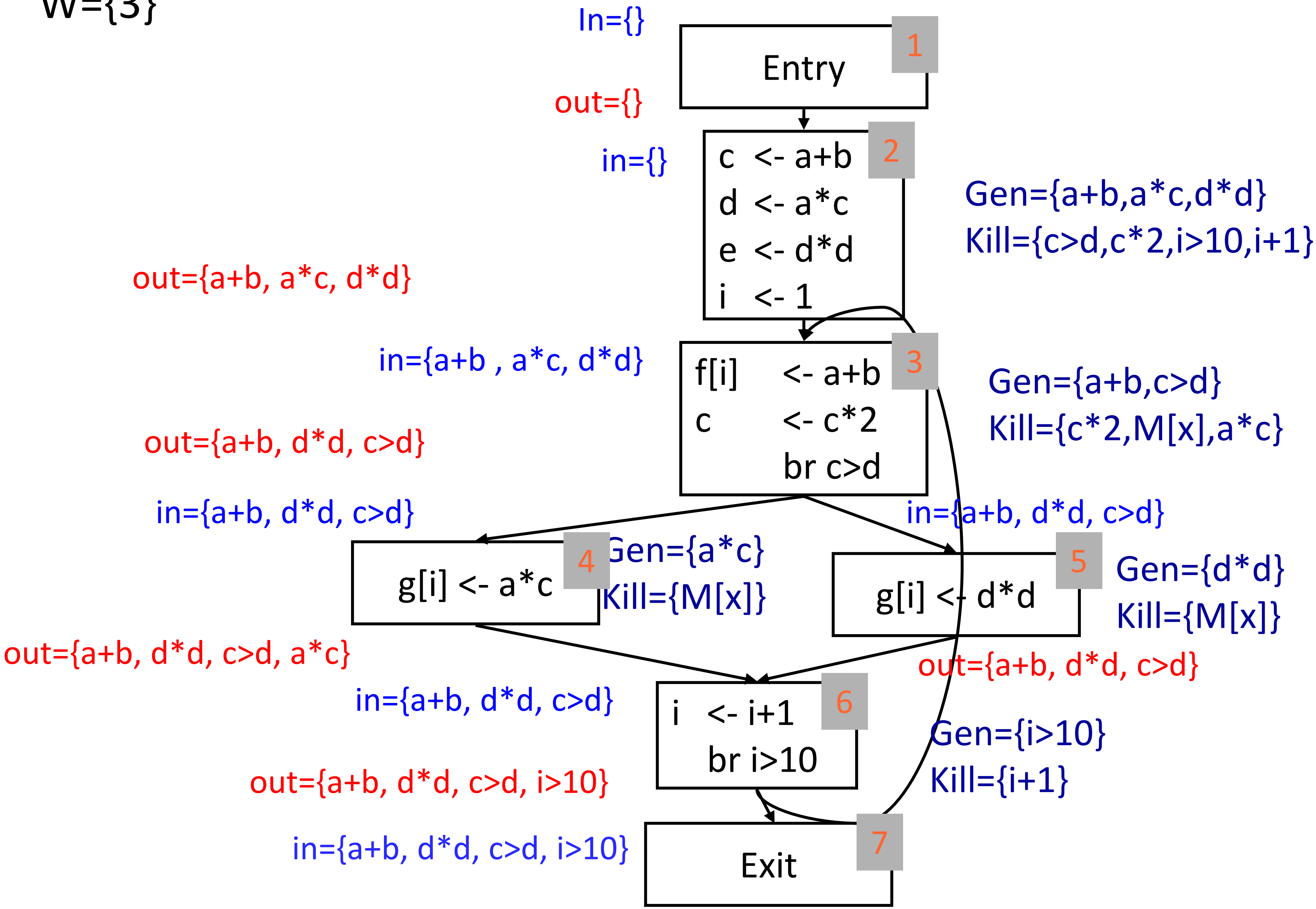
W={3,7}



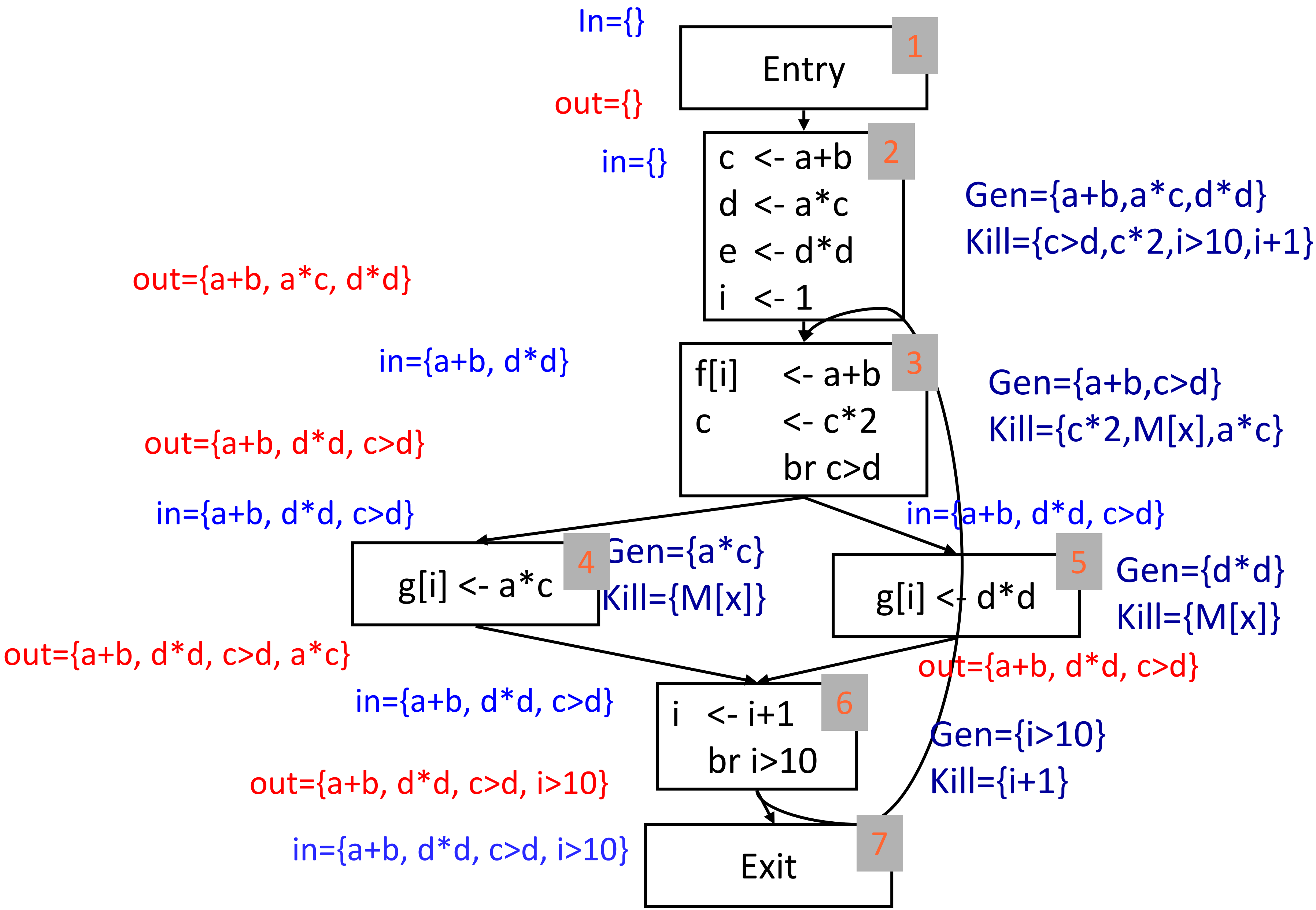
In={}
Entry
1
out={}
in={}
c <- a+b
d <- a*c
e <- d*d
i <- 1
2
Gen={a+b,a*c,d*d}
Kill={c>d,c*2,i>10,i+1}
out={a+b, a*c, d*d}
in={a+b, a*c, d*d}
f[i] <- a+b
c <- c*2
br c>d
3
Gen={a+b,c>d}
Kill={c*2,M[x],a*c}
out={a+b, d*d, c>d}
in={a+b, d*d, c>d}
in={a+b, d*d, c>d}
g[i] <- a*c
4
Gen={a*c}
Kill={M[x]}
g[i] <- d*d
5
Gen={d*d}
Kill={M[x]}
out={a+b, d*d, c>d, a*c}
out={a+b, d*d, c>d}
in={a+b, d*d, c>d}
i <- i+1
br i>10
6
Gen={i>10}
Kill={i+1}
out={a+b, d*d, c>d, i>10}
in={}
Exit
7

Example

W={3}


In={}
1
Entry
out={}
in={}
2
c <- a+b
d <- a*c
e <- d*d
i <- 1
Gen={a+b,a*c,d*d}
Kill={c>d,c*2,i>10,i+1}
out={a+b, a*c, d*d}
in={a+b , a*c, d*d}
3
f[i] <- a+b
c <- c*2
br c>d
Gen={a+b,c>d}
Kill={c*2,M[x],a*c}
out={a+b, d*d, c>d}
in={a+b, d*d, c>d}
in={a+b, d*d, c>d}
4
g[i] <- a*c
Gen={a*c}
Kill={M[x]}
5
g[i] <- d*d
Gen={d*d}
Kill={M[x]}
out={a+b, d*d, c>d, a*c}
out={a+b, d*d, c>d}
in={a+b, d*d, c>d}
6
i <- i+1
br i>10
Gen={i>10}
Kill={i+1}
out={a+b, d*d, c>d, i>10}
in={a+b, d*d, c>d, i>10}
7
Exit

Example


In={}
Entry
1
out={}
in={}
c <- a+b
d <- a*c
e <- d*d
i <- 1
2
Gen={a+b,a*c,d*d}
Kill={c>d,c*2,i>10,i+1}
out={a+b, a*c, d*d}
in={a+b, d*d}
f[i] <- a+b
c <- c*2
br c>d
3
Gen={a+b,c>d}
Kill={c*2,M[x],a*c}
out={a+b, d*d, c>d}
in={a+b, d*d, c>d}
in={a+b, d*d, c>d}
g[i] <- a*c
4
Gen={a*c}
Kill={M[x]}
g[i] <- d*d
5
Gen={d*d}
Kill={M[x]}
out={a+b, d*d, c>d, a*c}
out={a+b, d*d, c>d}
in={a+b, d*d, c>d}
i <- i+1
br i>10
6
Gen={i>10}
Kill={i+1}
out={a+b, d*d, c>d, i>10}
in={a+b, d*d, c>d, i>10}
Exit
7

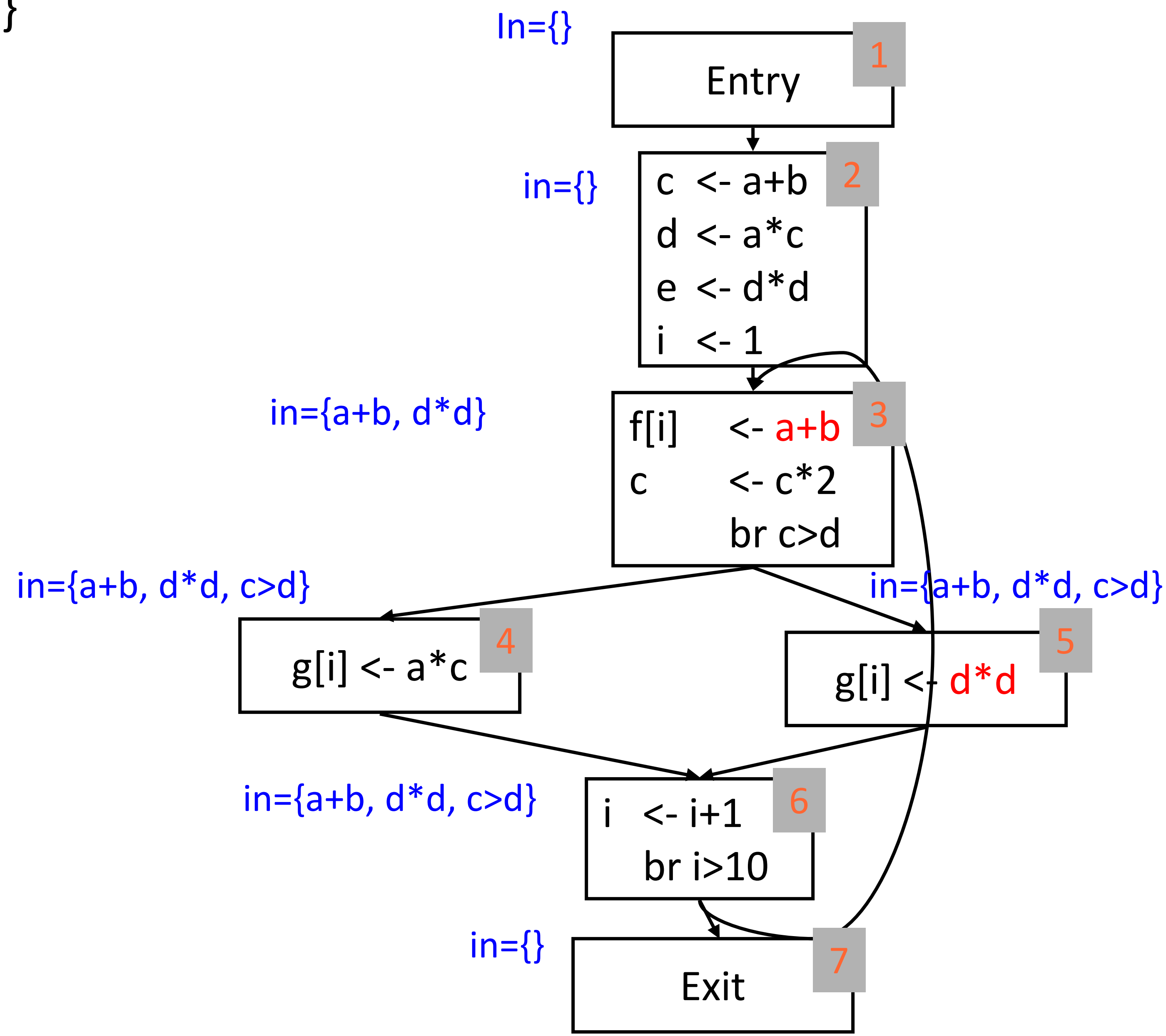
CSE

- Calculate Available expressions
- For every stmt in program

```
If expression, x op y, is available {
    Compute reaching expressions for "x op y"
        at this stmt
    foreach stmt in RE of the form t <- x op y
            rewrite at:   t' <- x op y
                          t <- t'
    }
    replace "x op y" in stmt with t'
}
```

Find x op y available

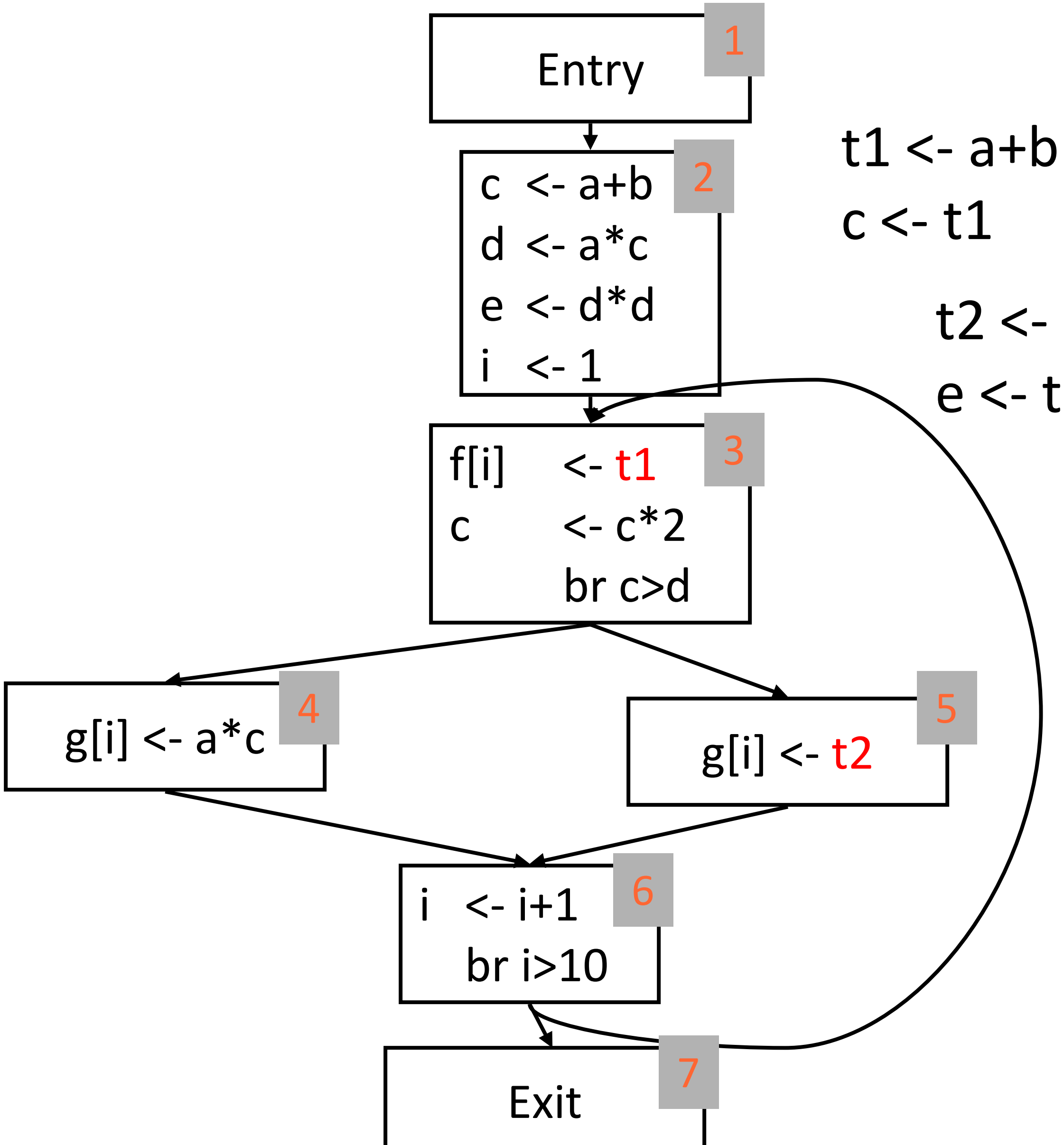
W={3}



Calculating Reaching Expressions

- Could be dataflow problem, but not needed enough, so …
- To find RE for "x op y" at stmt S
  - traverse cfg backward from S until
    - reach t <- x + y (& put into RE)
    - reach definition of x or y

Example



t1 <- a+b

c <- t1

t2 <- d*d

e <- t2

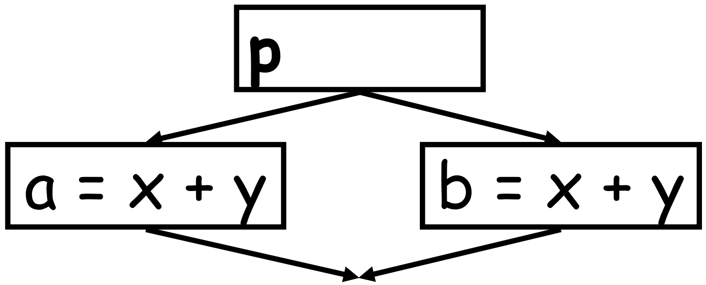
Dataflow Summary

	Union (may)	intersection (must)
Forward	Reaching defs	Available exprs
Backward	Live variables	very busy exprs

Later in course we look at bidirectional dataflow

Very Busy Expressions

- A Backward, Must data flow analysis
- An expression e is *very busy at point p* if On every path from p, e is evaluated before the value of e is changed
- Optimization
  - Can hoist very busy expression computation



Forward Must Data Flow Algorithm

Out(s) = Gen(s) for all statements s

W = {all statements}

Repeat

    Take s from W

    In(s) = $\bigcap_{s' \in pred(s)}$ Out(s')

    Temp = Gen(s) $\cup$ (In(s) – Kill(s))

    If (temp != Out(s)) {

        Out(s) = temp

        W = W $\cup$ succ(s)

    }

Until W = $\varnothing$

Forward May Data Flow Algorithm

Out(s) = Gen(s) for all statements s

W = {all statements}

Repeat

Take s from W

In(s) = $\cup_{s' \in pred(s)}$ Out(s')

Temp = Gen(s) $\cup$ (In(s) – Kill(s))

If (temp != Out(s)) {

Out(s) = temp

W = W $\cup$ succ(s)

}

Until W = $\emptyset$

Backward May Data Flow Algorithm

In(s) = Gen(s) for all statements s

W = {all statements} (worklist)

Repeat

    Take s from W

    $\text{Out}(s) = \cup_{s' \in \text{succ}(s)} \text{In}(s')$

    $\text{Temp} = \text{Gen}(s) \cup (\text{Out}(s) - \text{Kill}(s))$

    If (temp != In(s)) {

        In(s) = temp

        $W = W \cup \text{pred}(s)$

    }

Until $W = \varnothing$

Backward Must Data Flow Algorithm

In(s) = Gen(s) for all statements s

W = {all statements} (worklist)

Repeat

    Take s from W

    Out(s) = $\bigcap_{s' \in succ(s)}$ In(s')

    Temp = Gen(s) $\cup$ (Out(s) – Kill(s))

    If (temp != In(s)) {

        In(s) = temp

        W = W $\cup$ pred(s)

    }

Until W = $\emptyset$