

# Color



# Course announcements

- Homework 4 has been posted.
  - Due Friday March 23<sup>rd</sup> (one-week homework!)
  - Any questions about the homework?
  - How many of you have looked at/started/finished homework 4?
- Talk this week: Katie Bouman, “Imaging the Invisible”.
  - Wednesday, March 21st 10:00 AM GHC6115.

# Overview of today's lecture

- Color and human color perception.
- Retinal color space.
- Color matching.
- Linear color spaces.
- Chromaticity.
- Non-linear color spaces.
- Example computer vision application using color.

# Slide credits

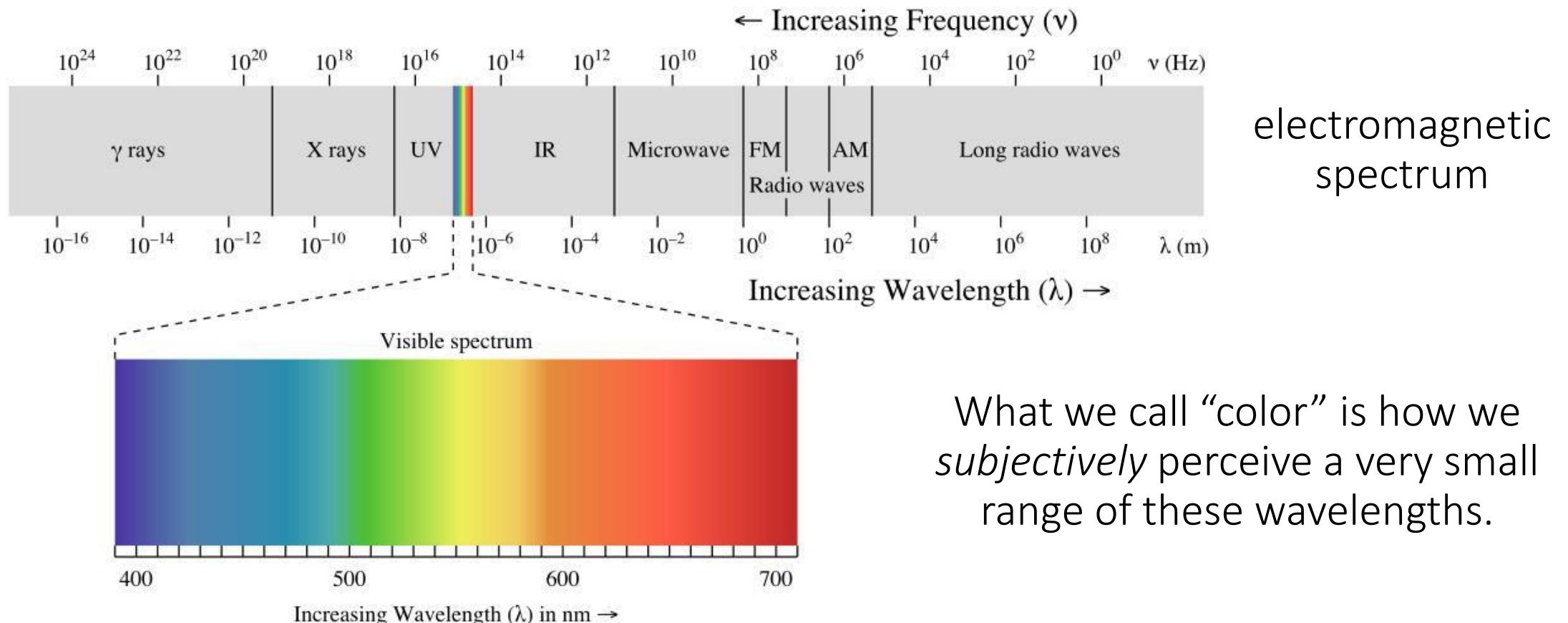
Many of these slides were inspired or adapted from:

- Todd Zickler (Harvard).
- Fredo Durand (MIT).

Color and human color perception

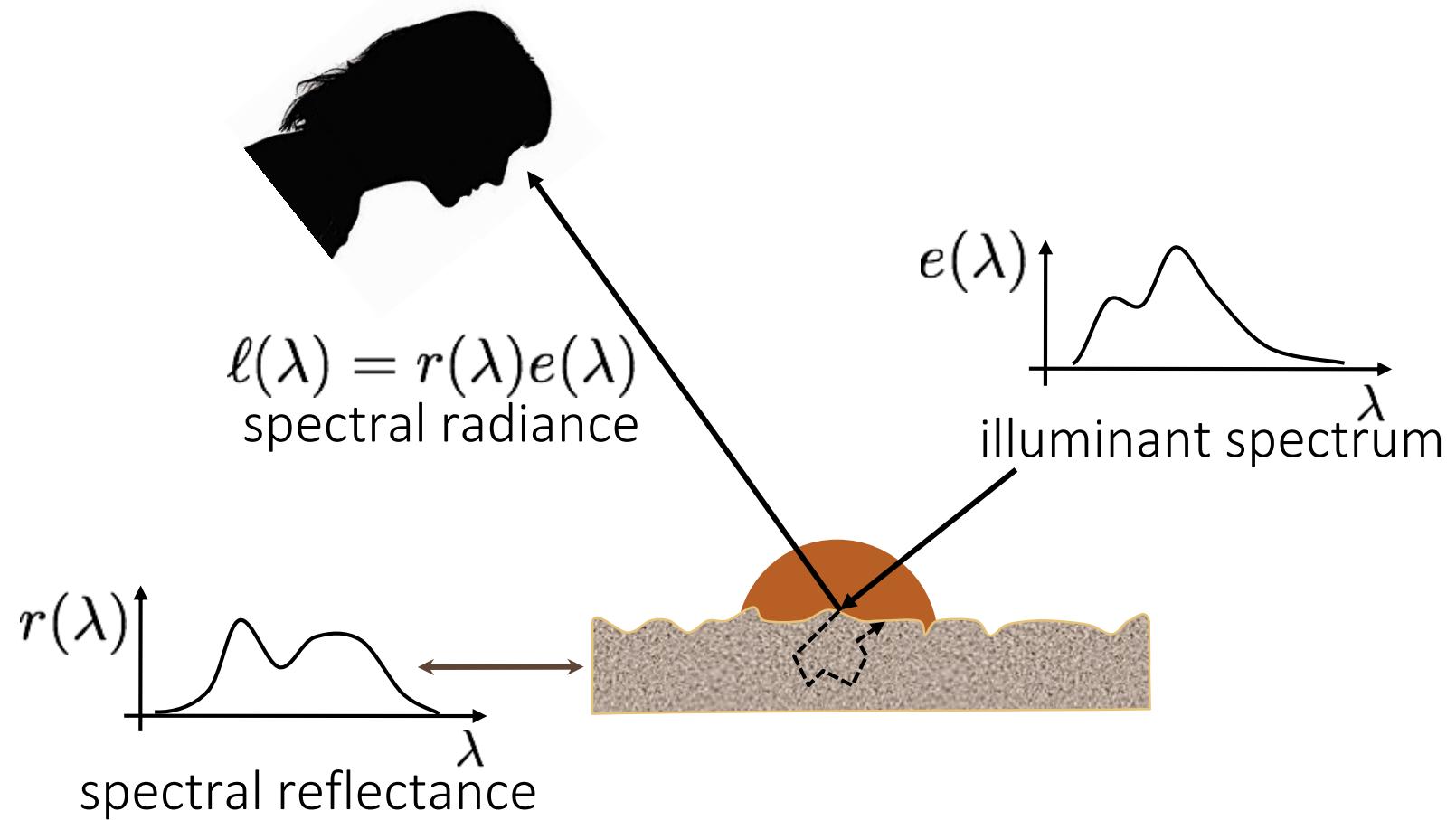
# Color is an artifact of human perception

- “Color” is not an *objective* physical property of light (electromagnetic radiation).
- Instead, light is characterized by its wavelength.

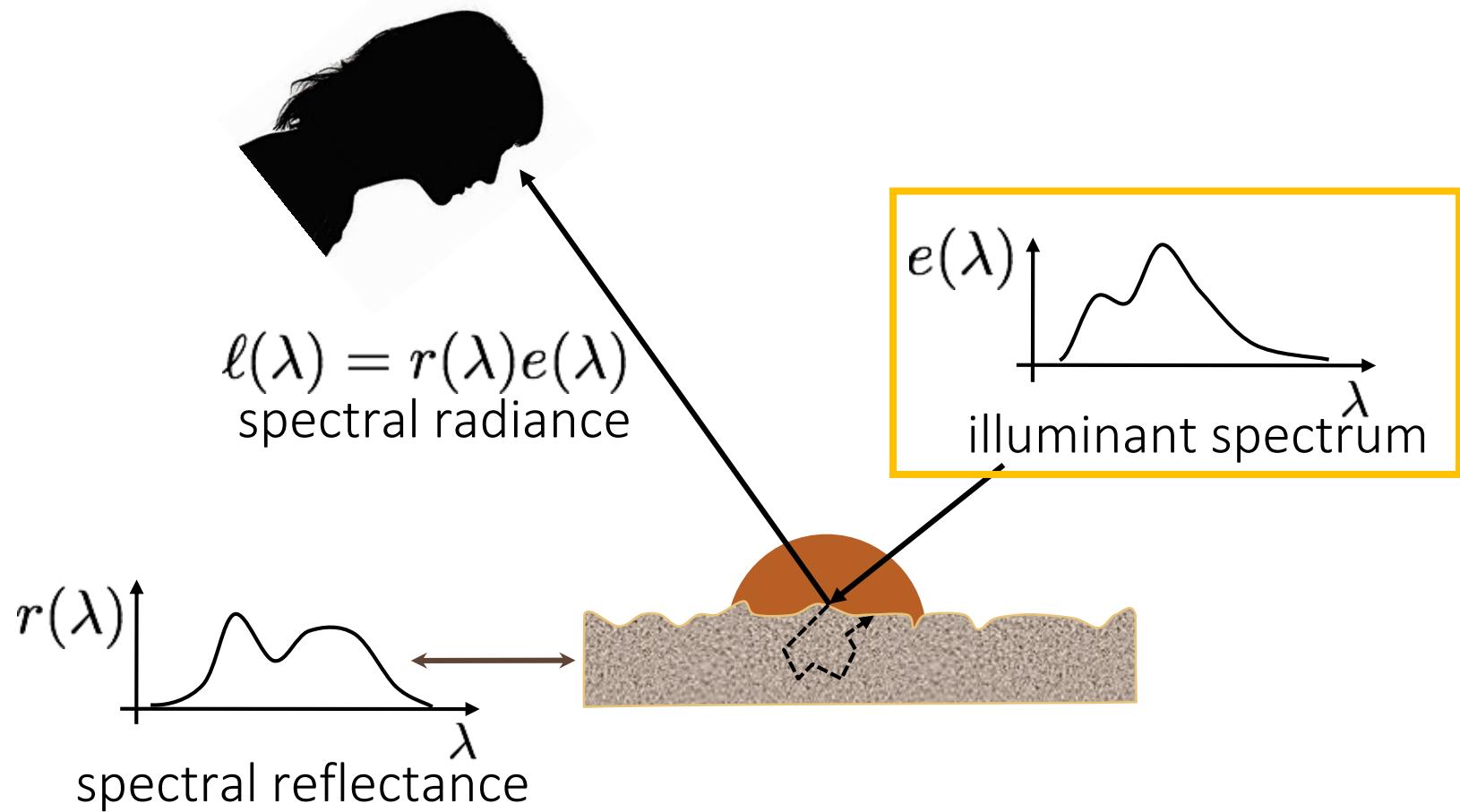


What we call “color” is how we *subjectively* perceive a very small range of these wavelengths.

# Light-material interaction



# Light-material interaction

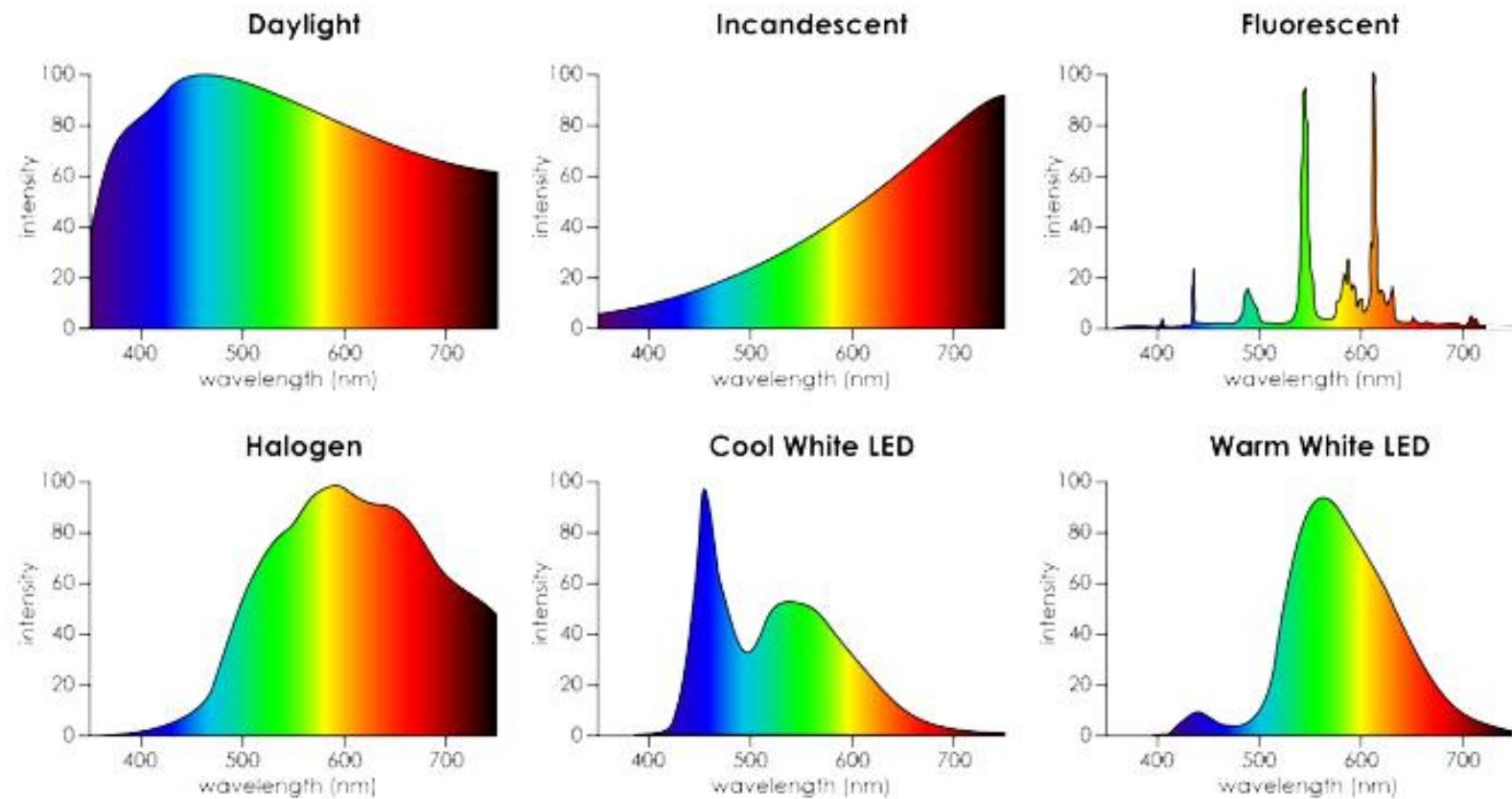


# Illuminant Spectral Power Distribution (SPD)

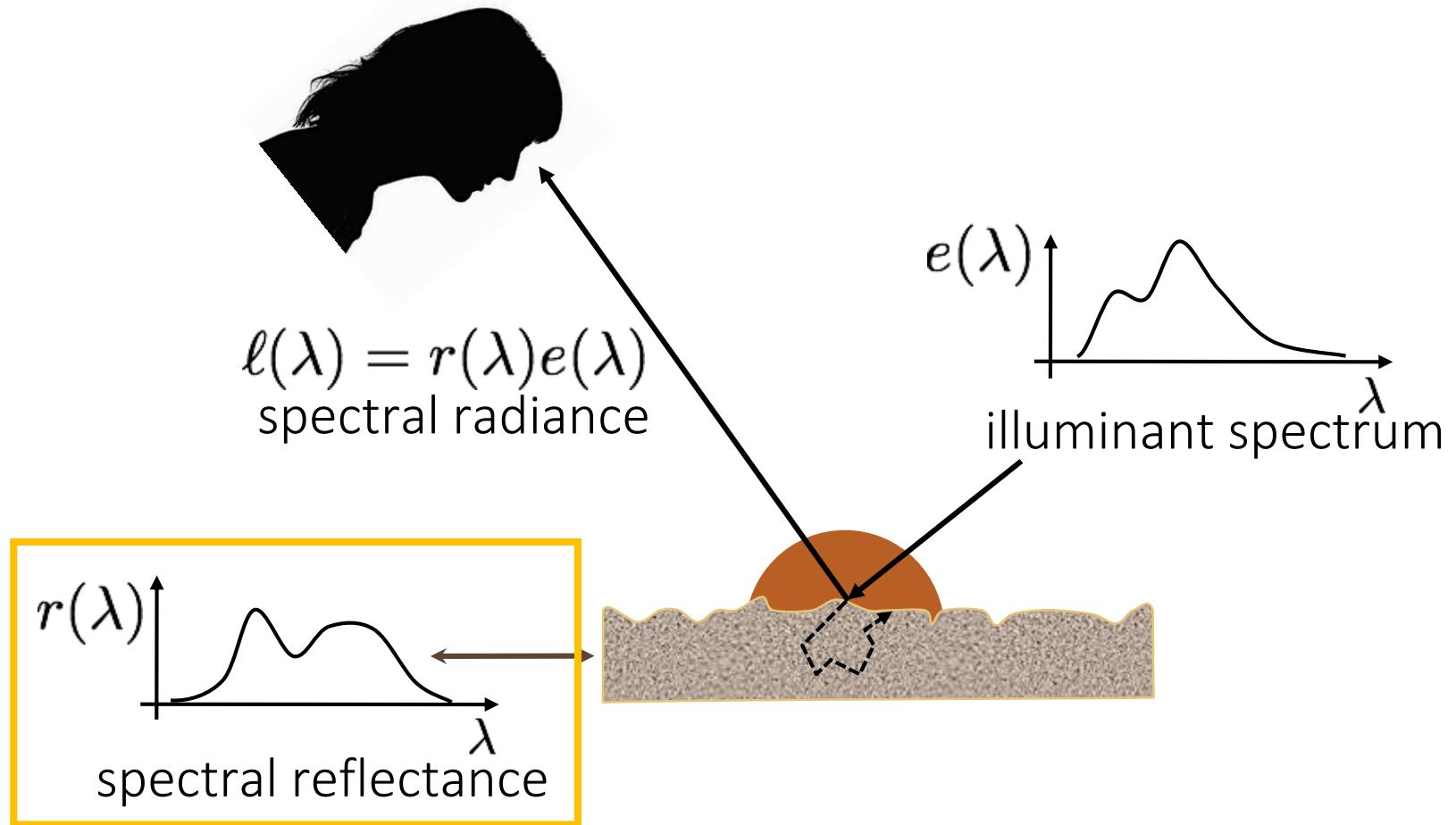
- Most types of light “contain” more than one wavelengths.
- We can describe light based on the distribution of power over different wavelengths.



We call our sensation of all of these distributions “white”.

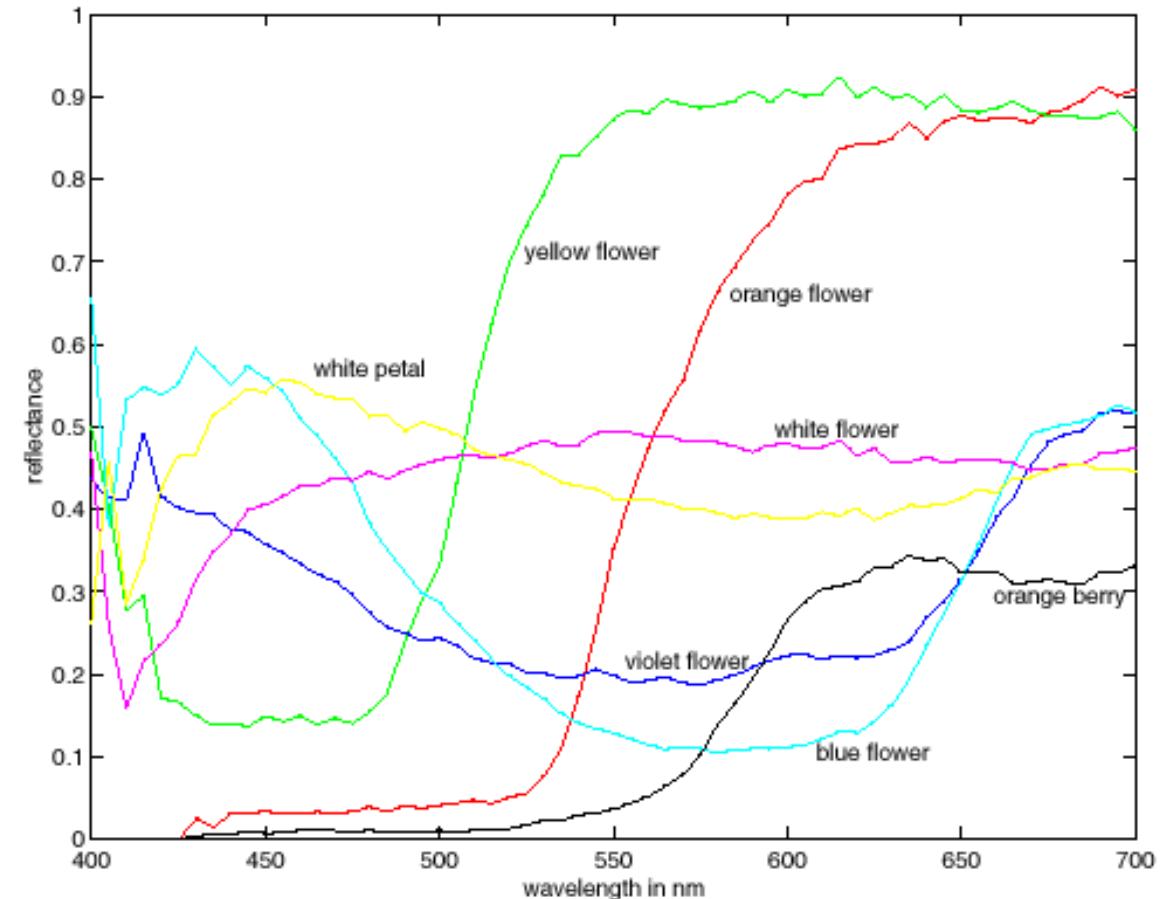


# Light-material interaction

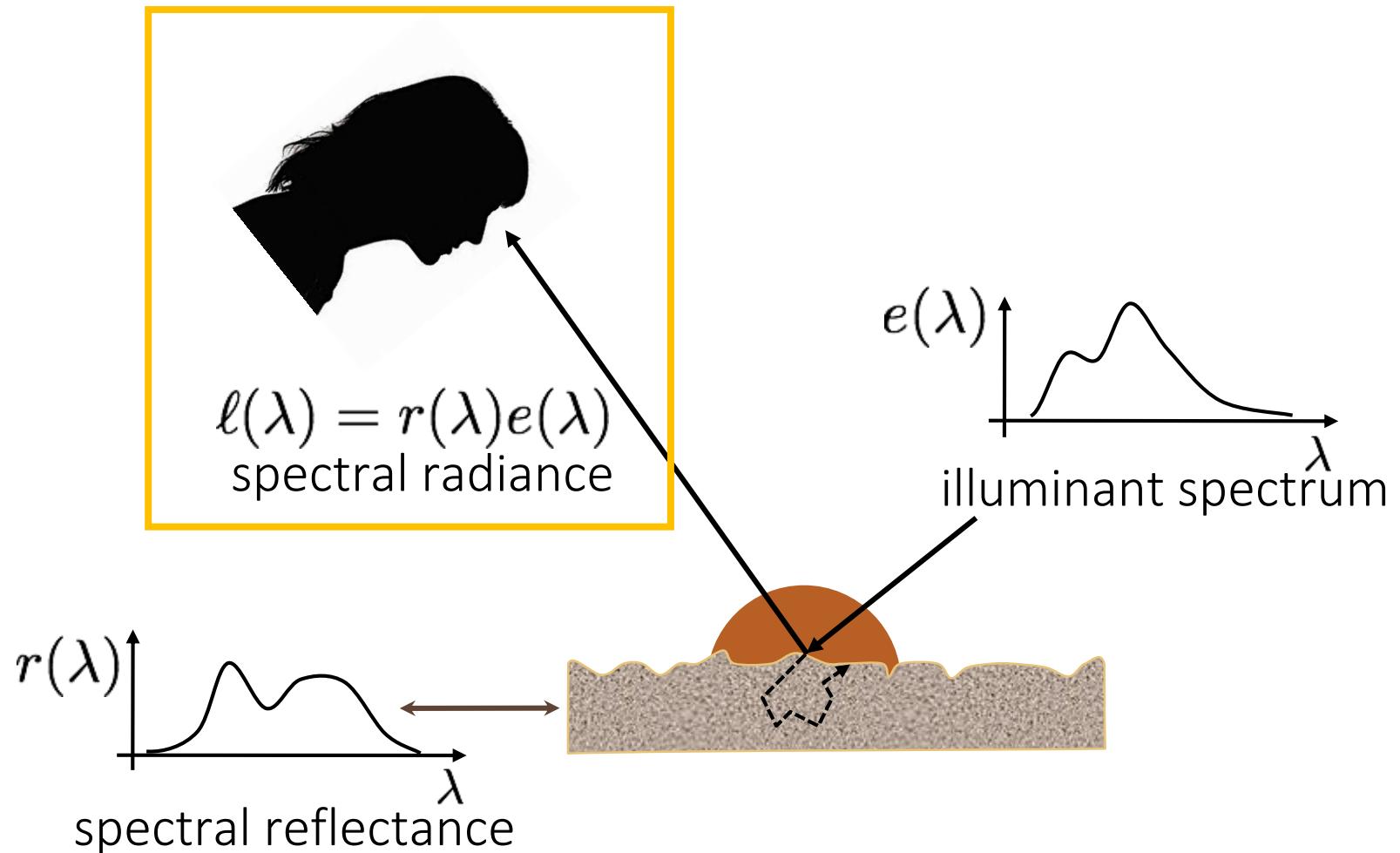


# Spectral reflectance

- Most materials absorb and reflect light differently at different wavelengths.
- We can describe this as a ratio of reflected vs incident light over different wavelengths.



# Light-material interaction

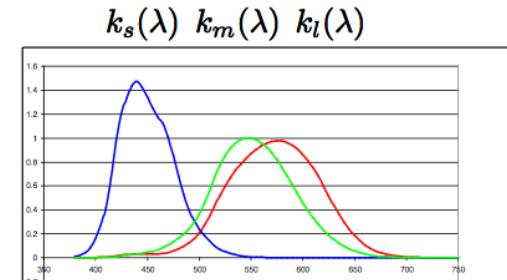


# Human color vision

retinal color

$$\mathbf{c}(\ell(\lambda)) = (c_s, c_m, c_l)$$

$$c_s = \int k_s(\lambda) \ell(\lambda) d\lambda$$

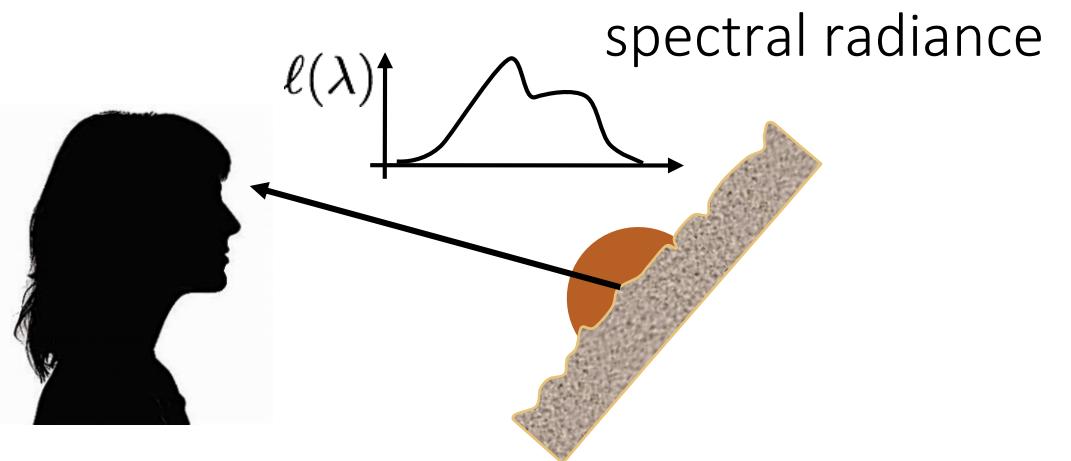


LMS sensitivity functions

perceived color

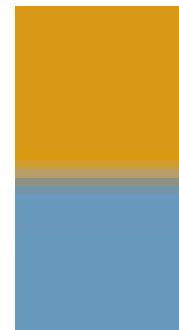
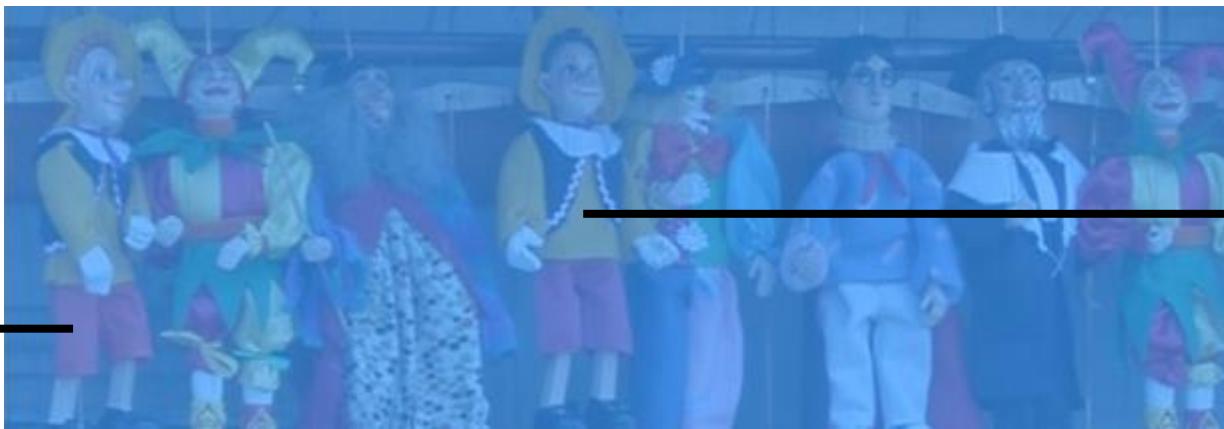
object color

color names



# Retinal vs perceived color

Retinal vs  
perceived color.



# Retinal vs perceived color

- Our visual system tries to “adapt” to illuminant.
- We may interpret the same retinal color very differently.



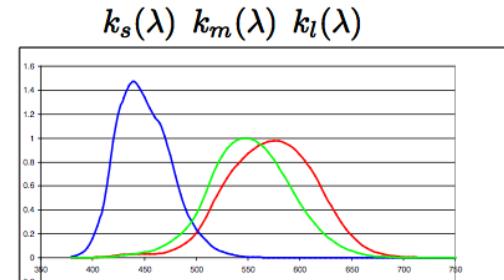
# Human color vision

We will exclusively discuss retinal color in this course

retinal color

$$\mathbf{c}(\ell(\lambda)) = (c_s, c_m, c_l)$$

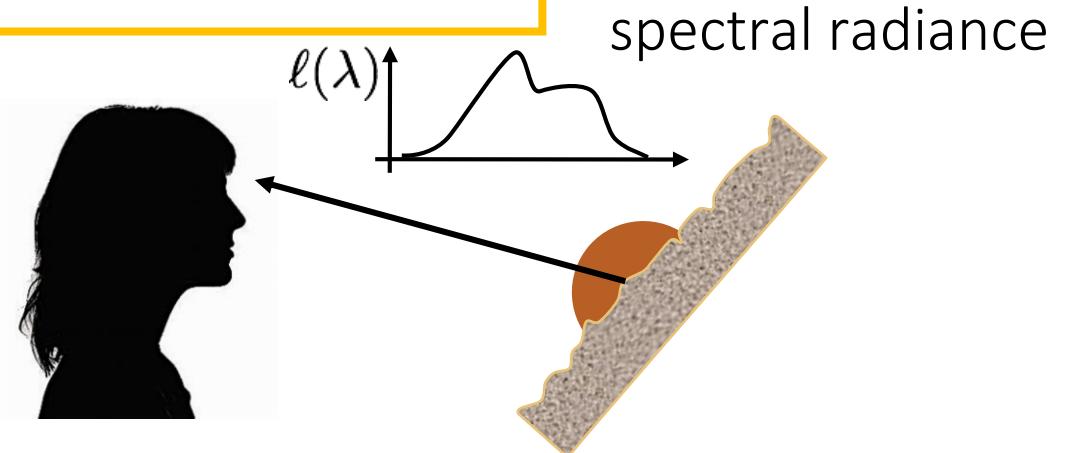
$$c_s = \int k_s(\lambda) \ell(\lambda) d\lambda$$



perceived color

object color

color names



Retinal color space

# Spectral Sensitivity Function (SSF)

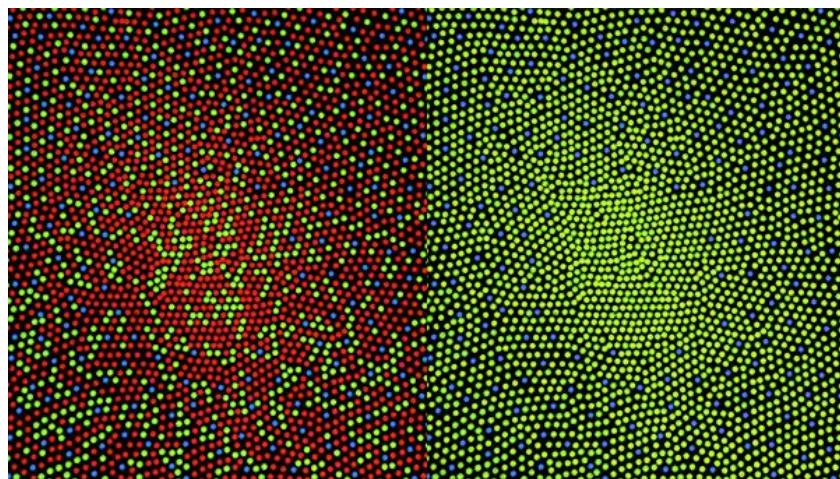
- Any light sensor (digital or not) has different sensitivity to different wavelengths.
- This is described by the sensor's *spectral sensitivity function*  $f(\lambda)$ .
- When measuring light of a some SPD  $\Phi(\lambda)$ , the sensor produces a *scalar* response:

$$\text{sensor response} \longrightarrow R = \int_{\lambda} \text{light SPD} \quad \text{sensor SSF} \quad \Phi(\lambda) f(\lambda) d\lambda$$

Weighted combination of light's SPD: light contributes more at wavelengths where the sensor has higher sensitivity.

# Spectral Sensitivity Function of Human Eye

- The human eye is a collection of light sensors called cone cells.
- There are three types of cells with different spectral sensitivity functions.
- Human color perception is three-dimensional (*tristimulus color*).

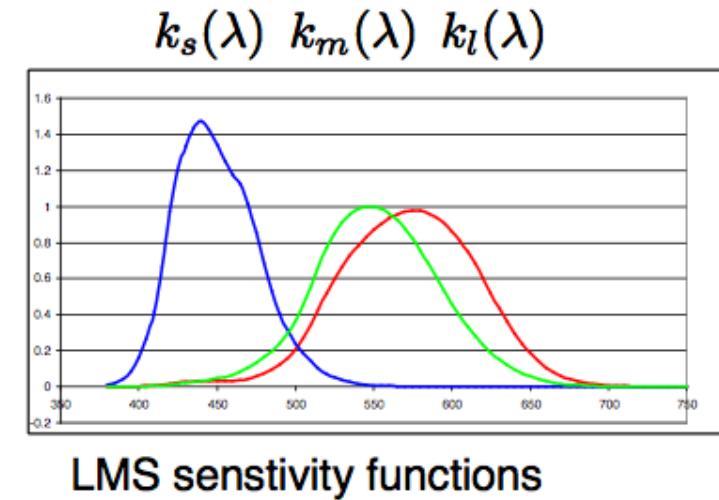


cone distribution  
for normal vision  
(64% L, 32% M)

“short”  $S = \int_{\lambda} \Phi(\lambda)S(\lambda)d\lambda$

“medium”  $M = \int_{\lambda} \Phi(\lambda)M(\lambda)d\lambda$

“long”  $L = \int_{\lambda} \Phi(\lambda)L(\lambda)d\lambda$

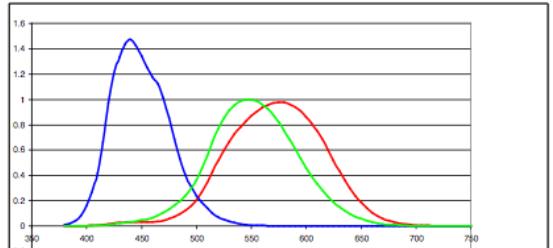


# The retinal color space

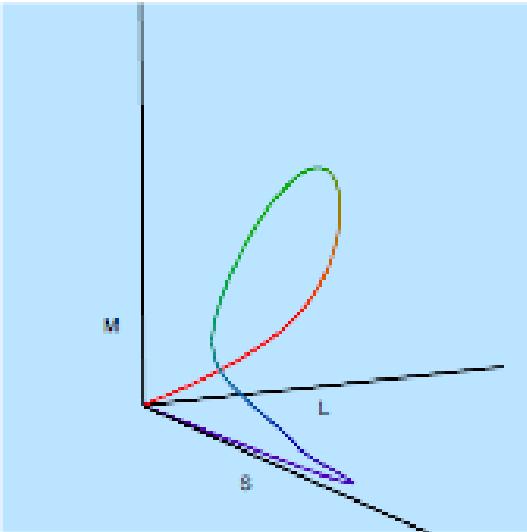
$$\mathbf{c}(\ell_{\lambda_i}) = (c_s, c_m, c_l)$$



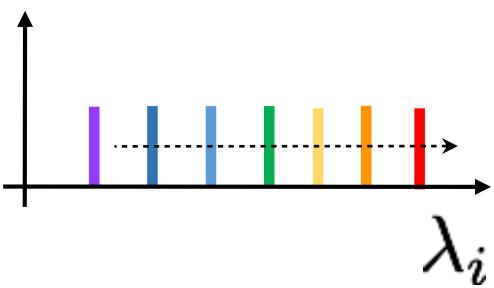
$$k_s(\lambda) \ k_m(\lambda) \ k_l(\lambda)$$



LMS sensitivity functions



$$\ell_{\lambda_i}$$



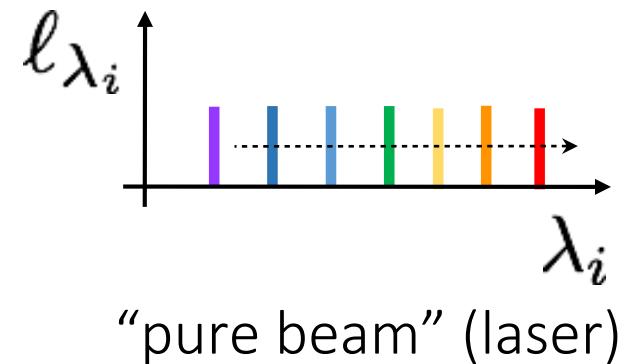
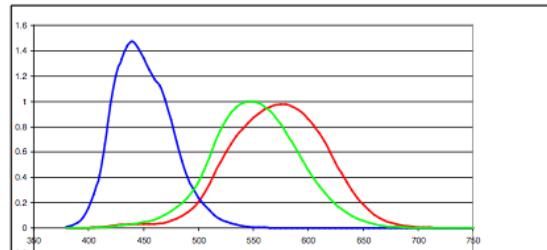
“pure beam” (laser)

# The retinal color space

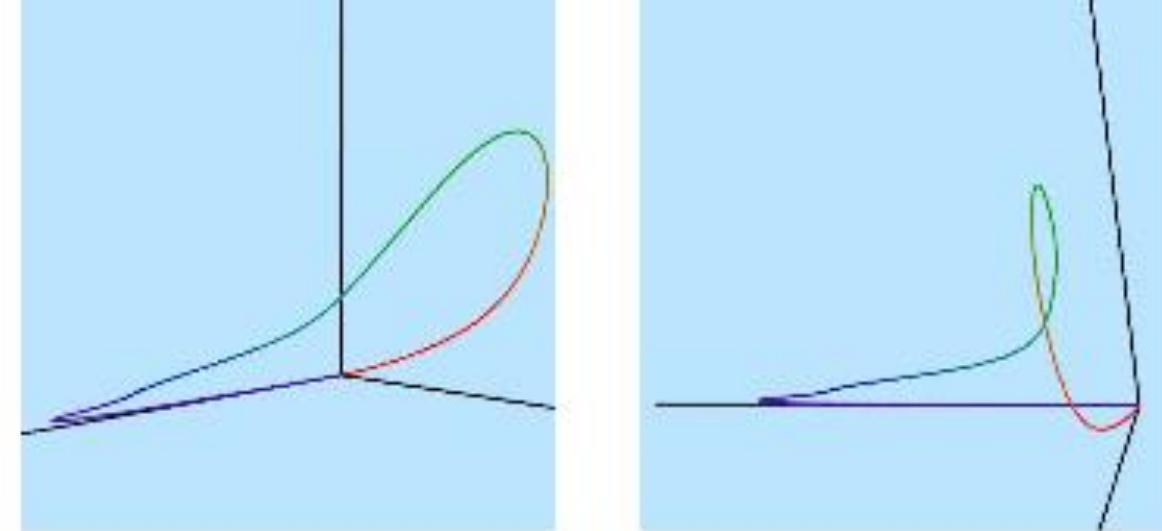
$$\mathbf{c}(\ell_{\lambda_i}) = (c_s, c_m, c_l)$$



$$k_s(\lambda) \ k_m(\lambda) \ k_l(\lambda)$$



“pure beam” (laser)

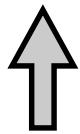


- “lasso curve”
- contained in positive octant
- parameterized by wavelength
- starts and ends at origin
- never comes close to M axis

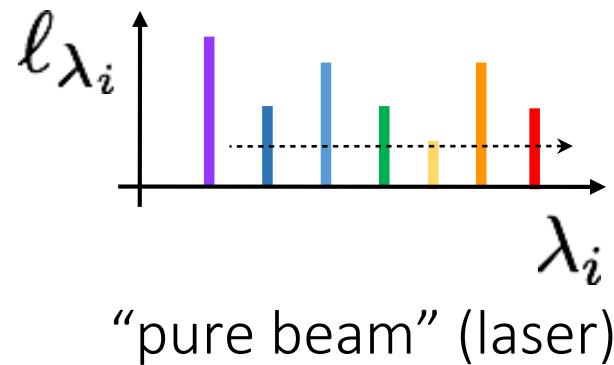
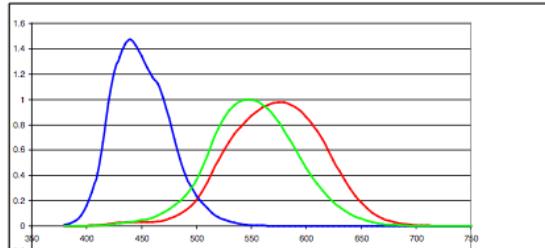
← why?  
← why?

# The retinal color space

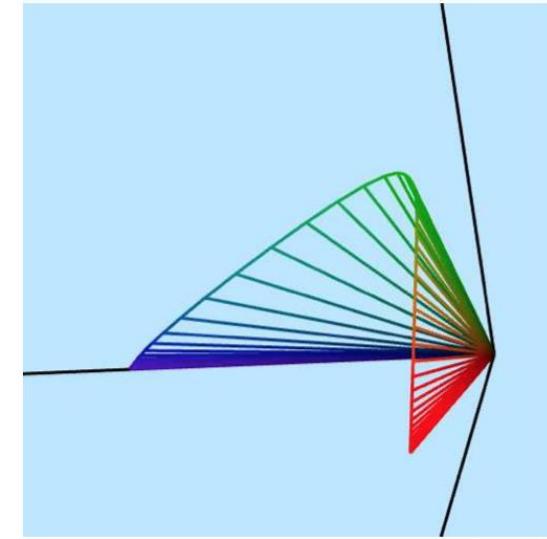
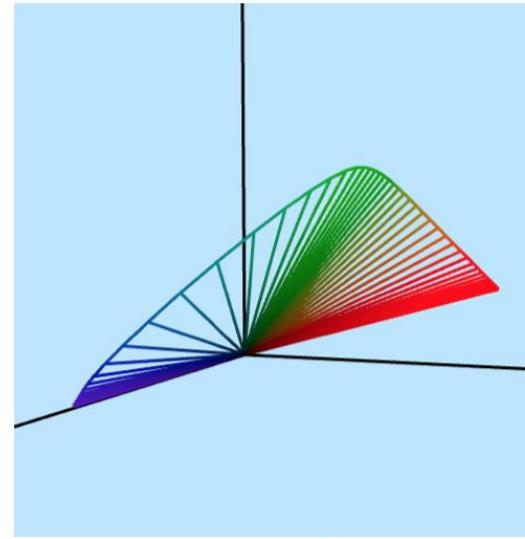
$$\mathbf{c}(\ell_{\lambda_i}) = (c_s, c_m, c_l)$$



$$k_s(\lambda) \ k_m(\lambda) \ k_l(\lambda)$$



“pure beam” (laser)



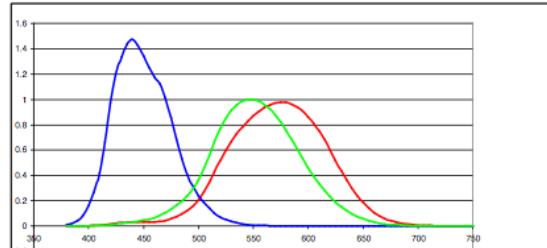
if we also consider variations in the *strength* of the laser this “lasso” turns into (convex!) radial cone with a “horse-shoe shaped” radial cross-section

# The retinal color space

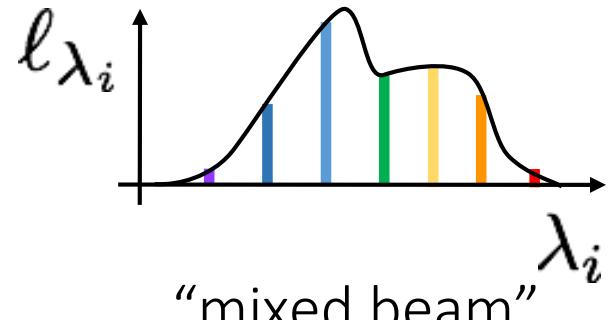
$$\mathbf{c}(\ell_{\lambda_i}) = (c_s, c_m, c_l)$$



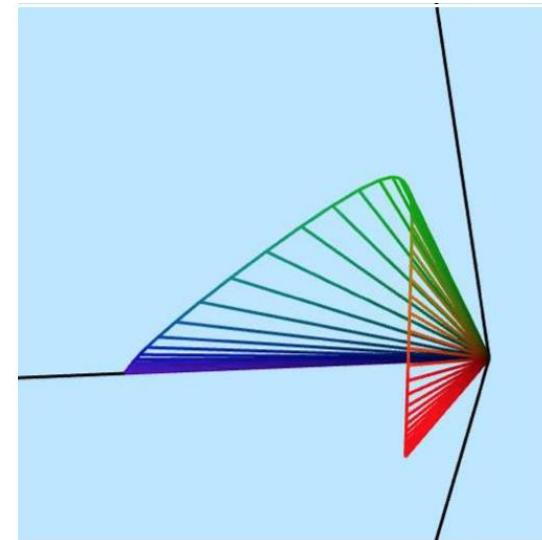
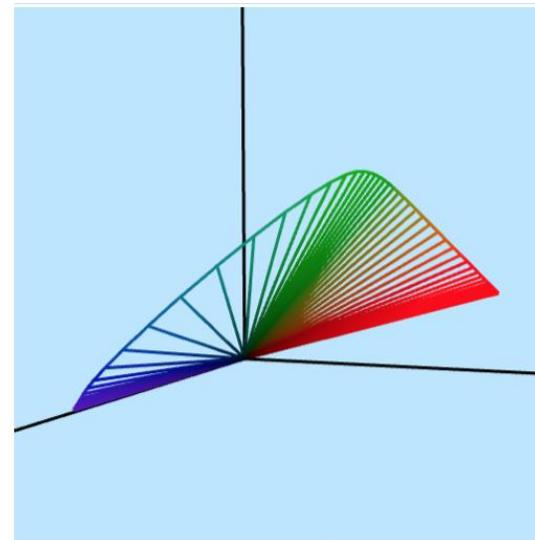
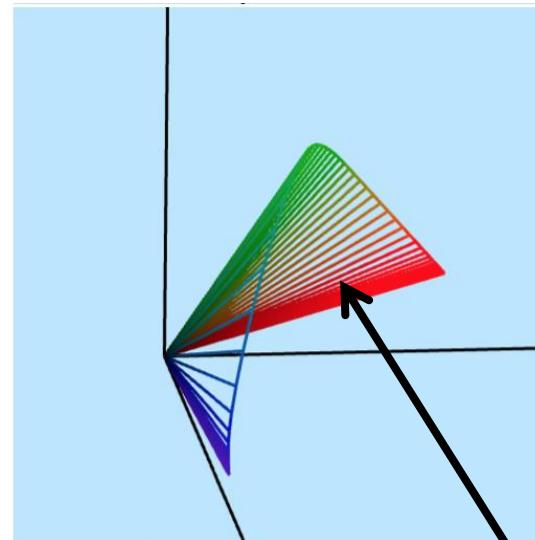
$$k_s(\lambda) \ k_m(\lambda) \ k_l(\lambda)$$



LMS sensitivity functions



= positive combination of pure colors



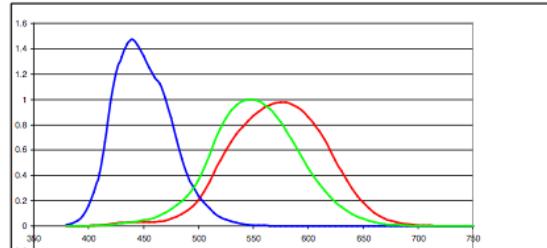
colors of mixed beams are  
inside of convex cone

# The retinal color space

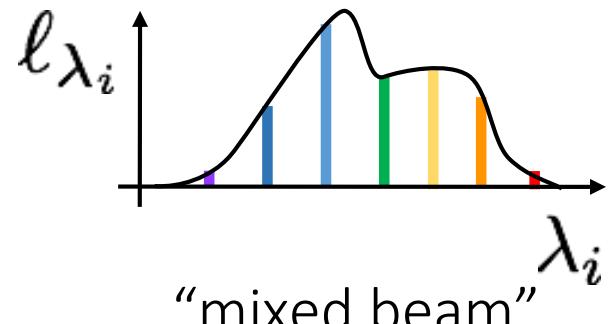
$$\mathbf{c}(\ell_{\lambda_i}) = (c_s, c_m, c_l)$$



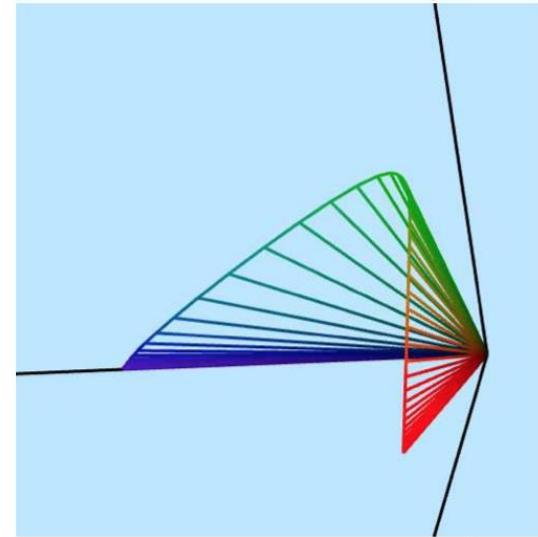
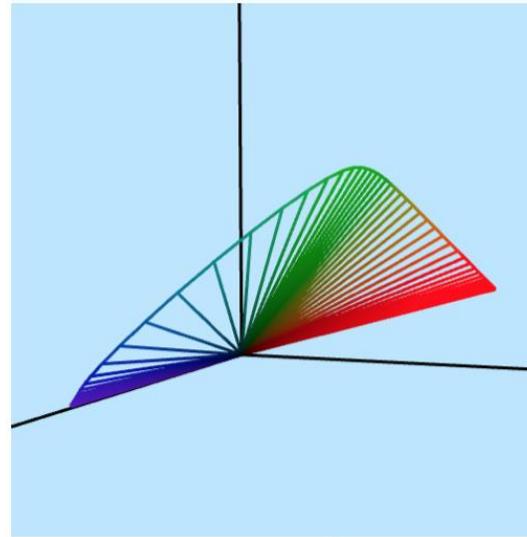
$$k_s(\lambda) \ k_m(\lambda) \ k_l(\lambda)$$



LMS sensitivity functions

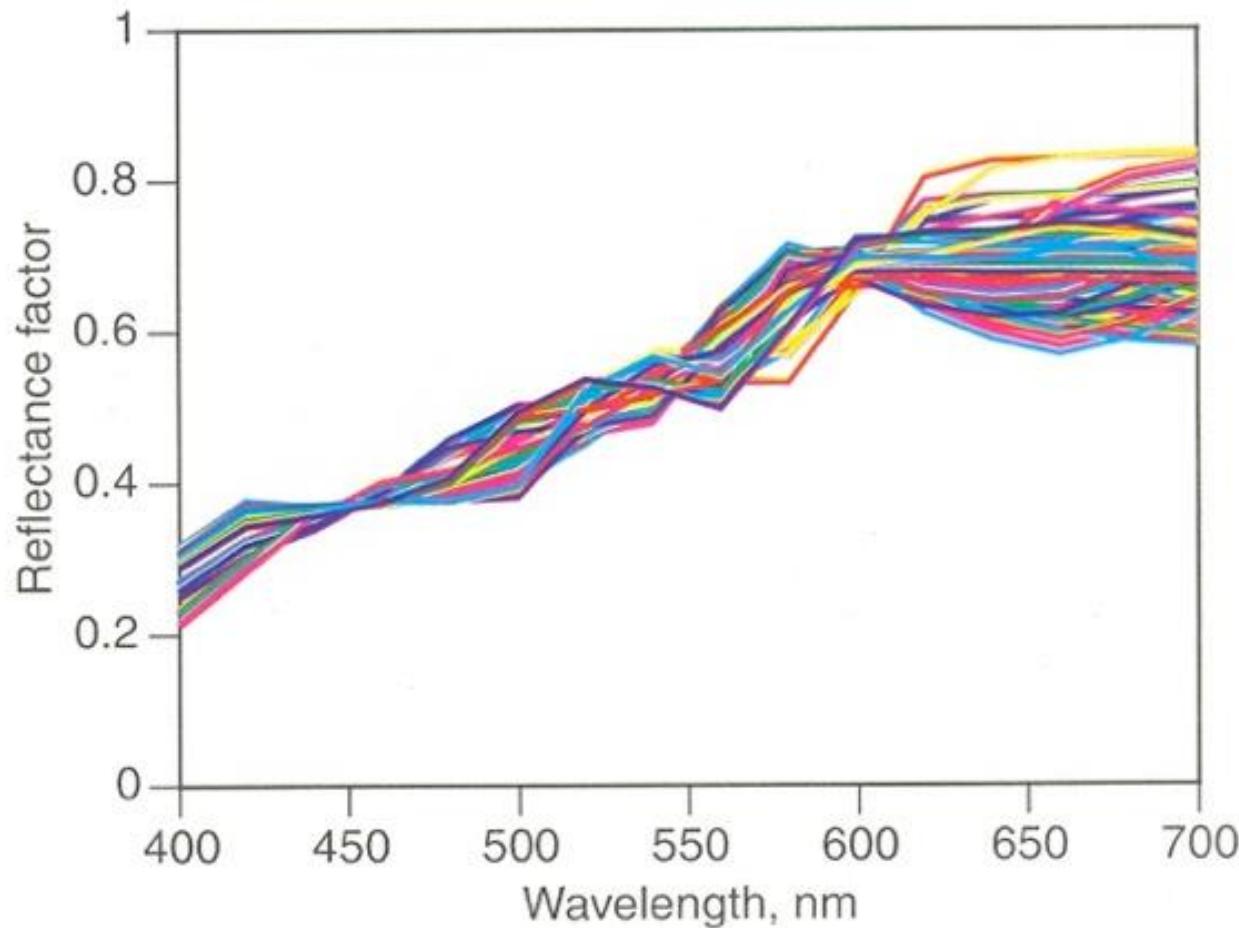


= positive combination of pure colors



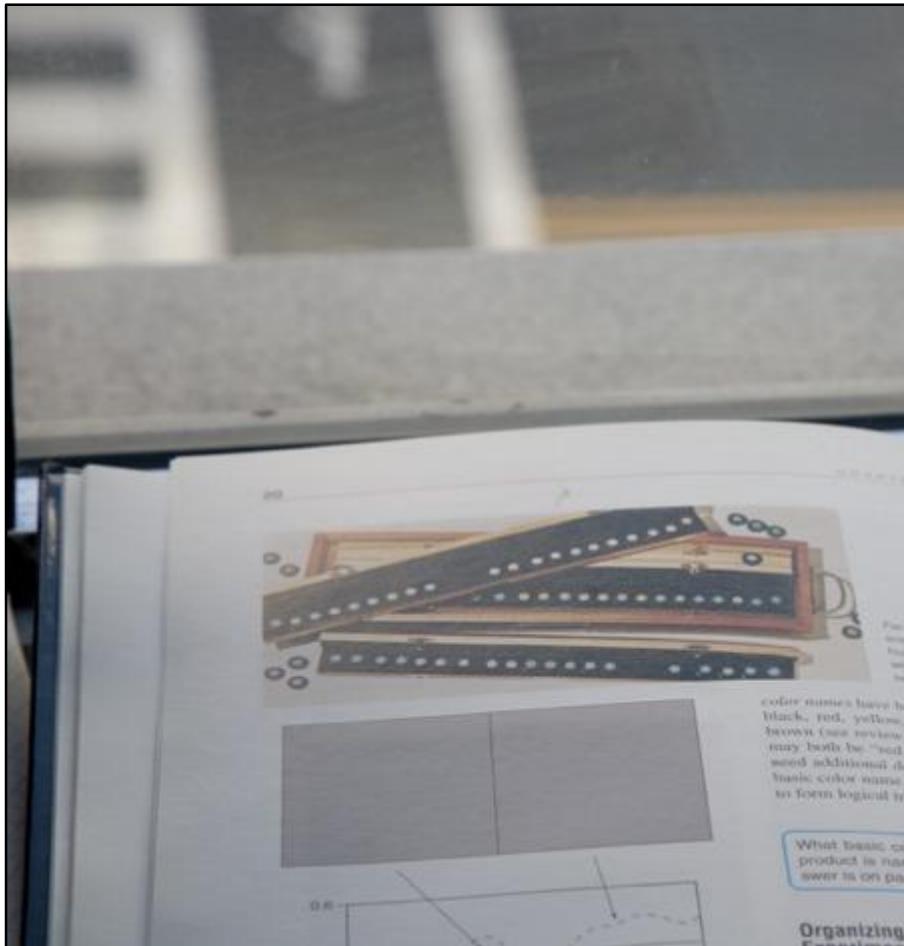
- distinct mixed beams can produce the same retinal color
- These beams are called *metamers*

# There is an infinity of metamers

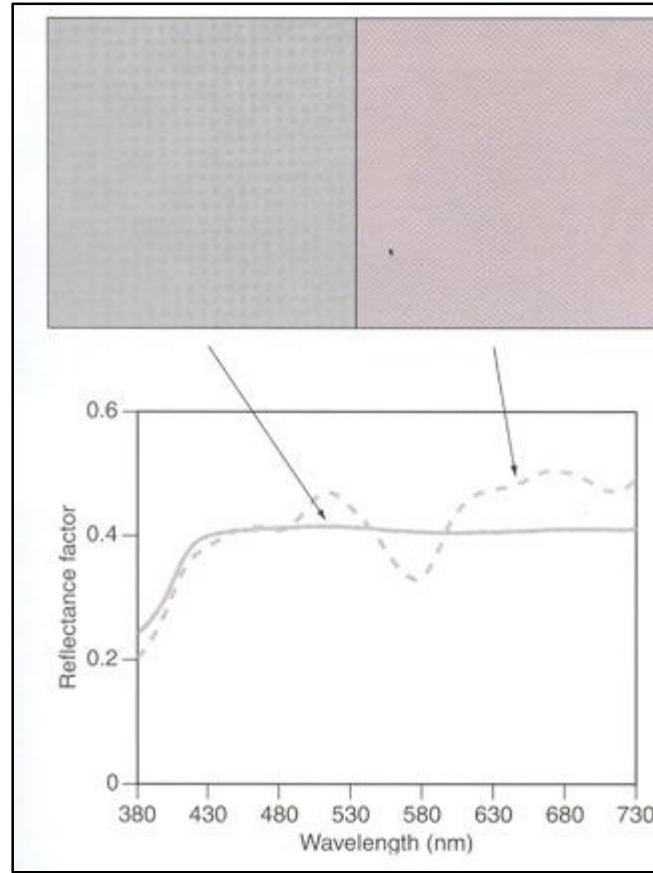


Ensemble of spectral reflectance curves corresponding to three chromatic-pigment recipes all matching a tan material when viewed by an average observer under daylight illumination. [Based on Berns (1988b).]

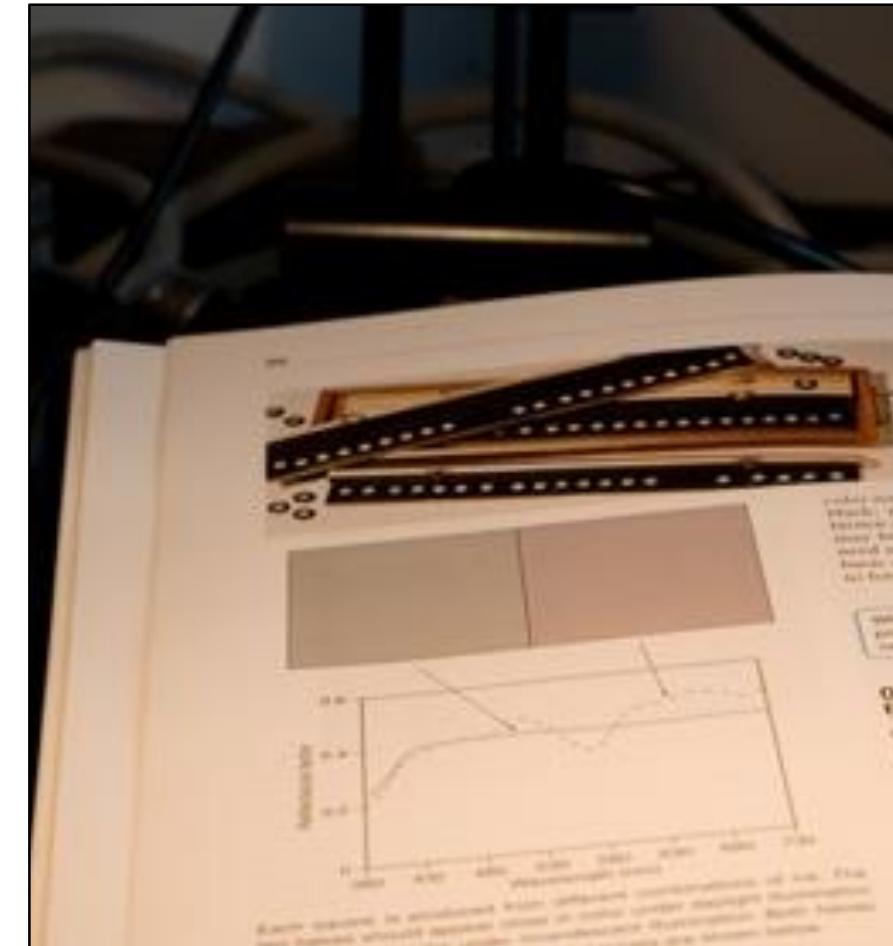
# Example: illuminant metamerism



day light



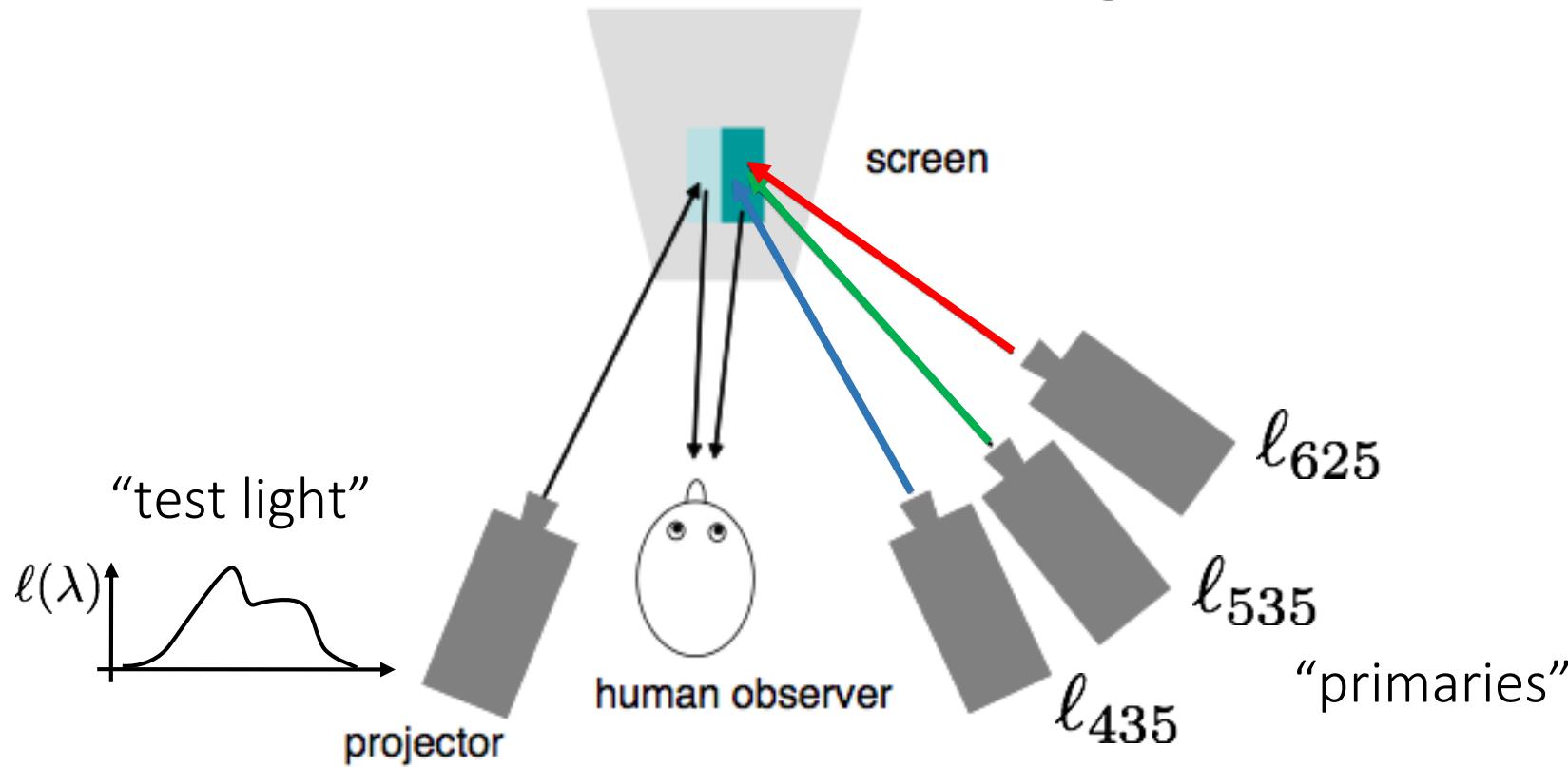
scanned copy



hallogen light

Color matching

# CIE color matching

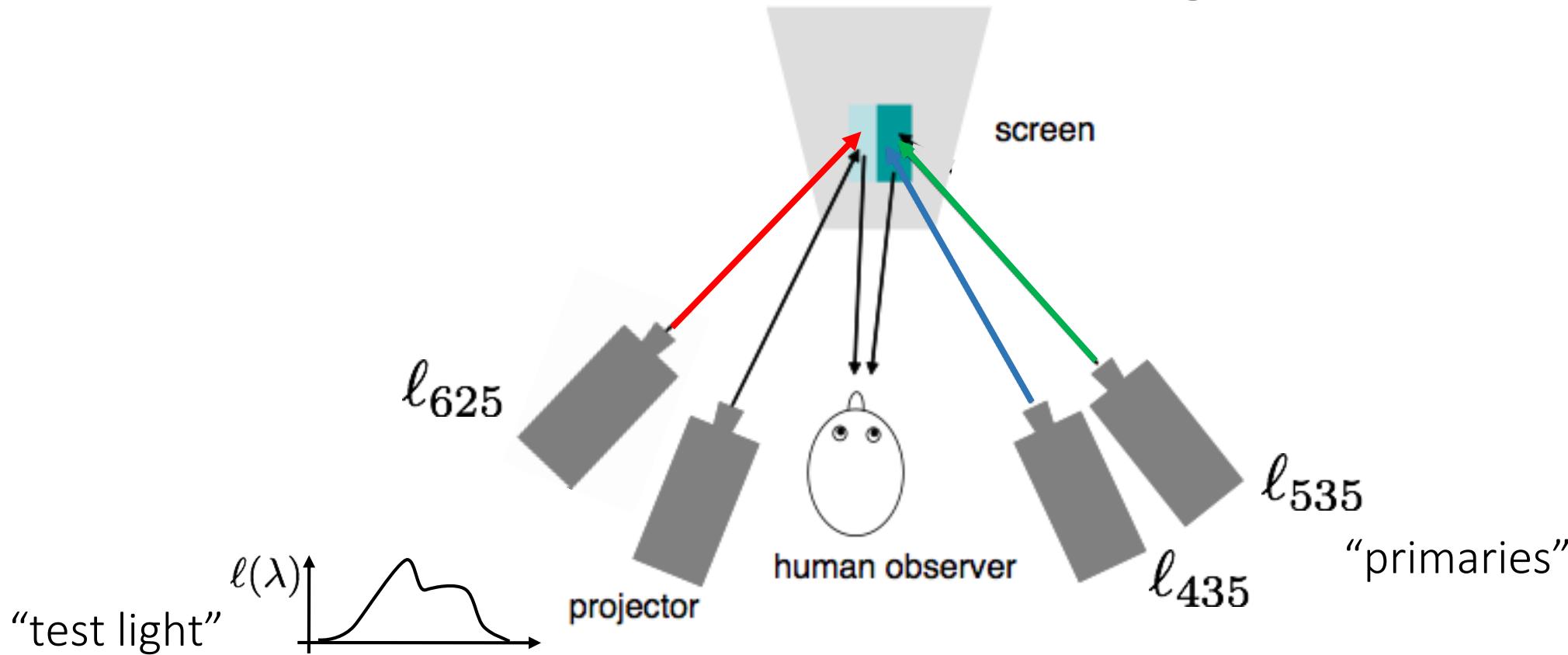


Adjust the strengths of the primaries until they re-produce the test color. Then:

$$\mathbf{c}(\ell(\lambda)) = \alpha \mathbf{c}(\ell_{435}) + \beta \mathbf{c}(\ell_{535}) + \gamma \mathbf{c}(\ell_{625})$$

↑ equality symbol means “has the same  
retinal color as” or “is metameric to”

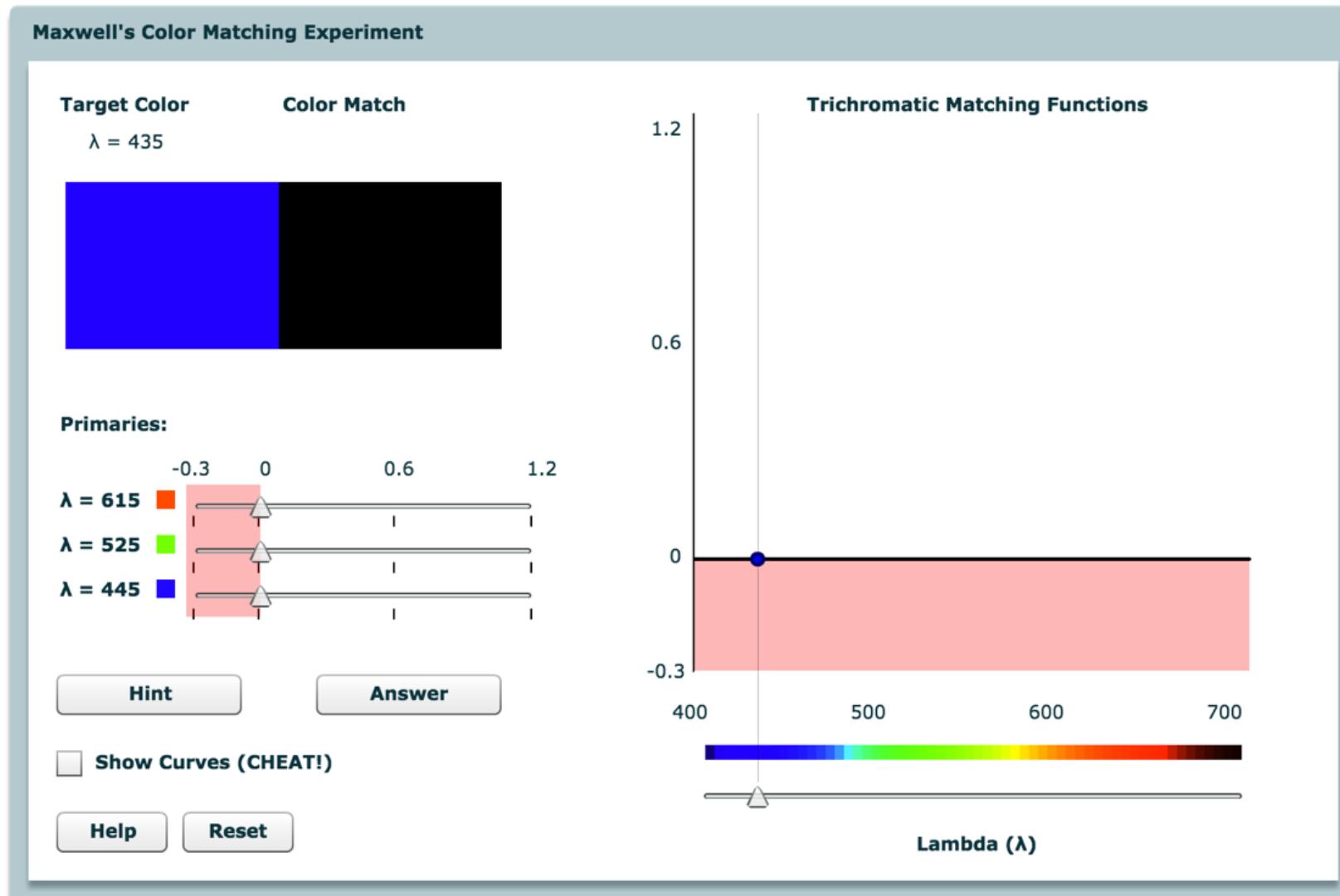
# CIE color matching



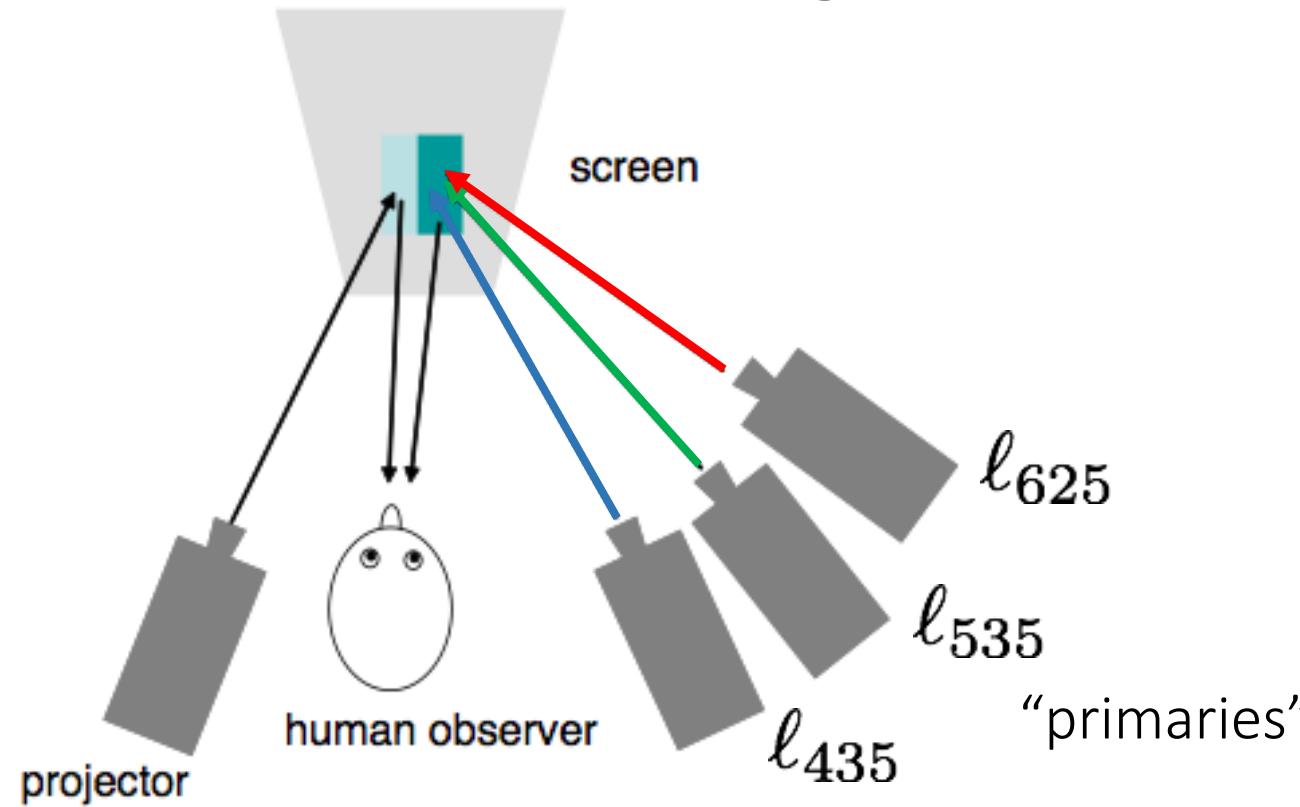
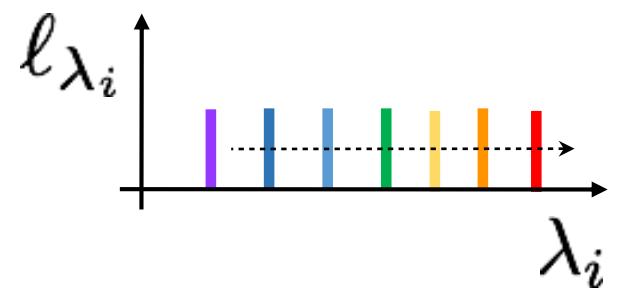
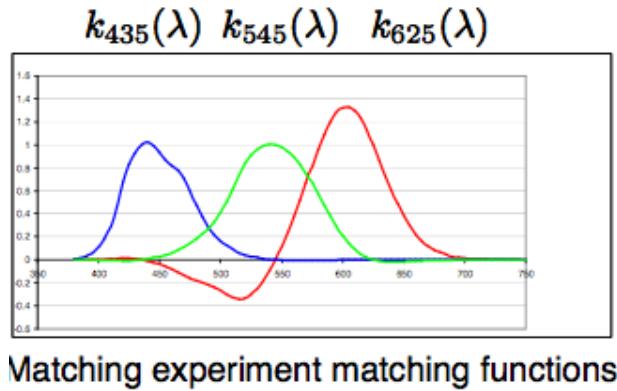
To match some test colors, you need to add some primary beam on the left (same as "subtracting light" from the right)

$$\begin{aligned} \mathbf{c}(\ell(\lambda)) + \gamma \mathbf{c}(\ell_{625}) &= \alpha \mathbf{c}(\ell_{435}) + \beta \mathbf{c}(\ell_{535}) \\ \rightarrow \mathbf{c}(\ell(\lambda)) &= \alpha \mathbf{c}(\ell_{435}) + \beta \mathbf{c}(\ell_{535}) - \gamma \mathbf{c}(\ell_{625}) \end{aligned}$$

# Color matching demo



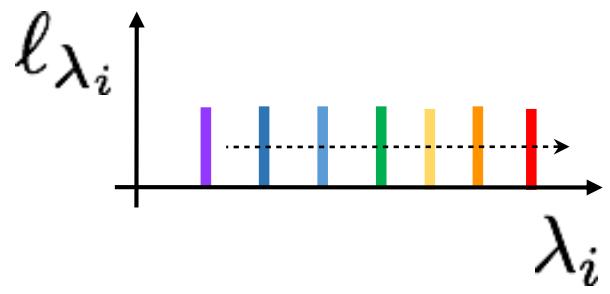
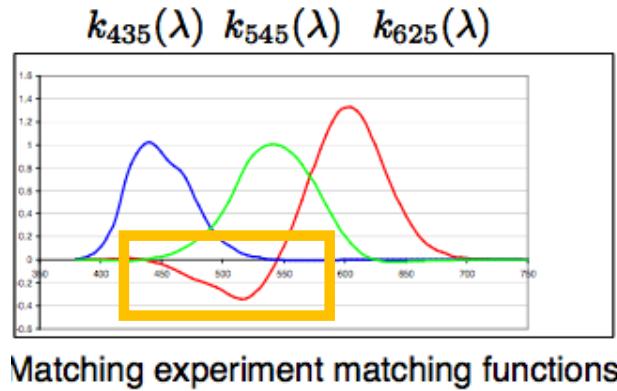
# CIE color matching



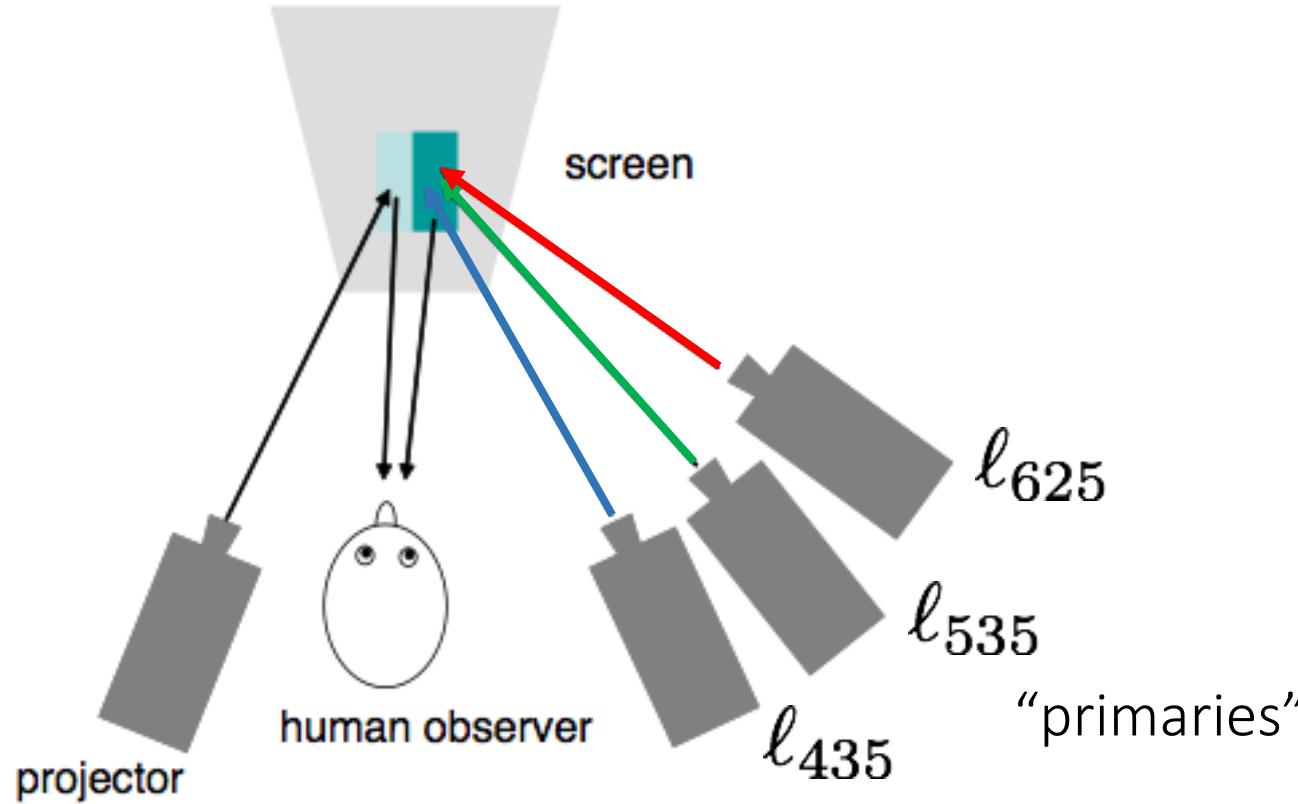
Repeat this matching experiments for pure test beams at wavelengths  $\lambda_i$  and keep track of the coefficients (negative or positive) required to reproduce each pure test beam.

$$\mathbf{c}(\lambda_i) = k_{435}(\lambda) \mathbf{c}(\ell_{435}) + k_{535}(\lambda) \mathbf{c}(\ell_{535}) + k_{625}(\lambda) \mathbf{c}(\ell_{625})$$

note the  
negative values



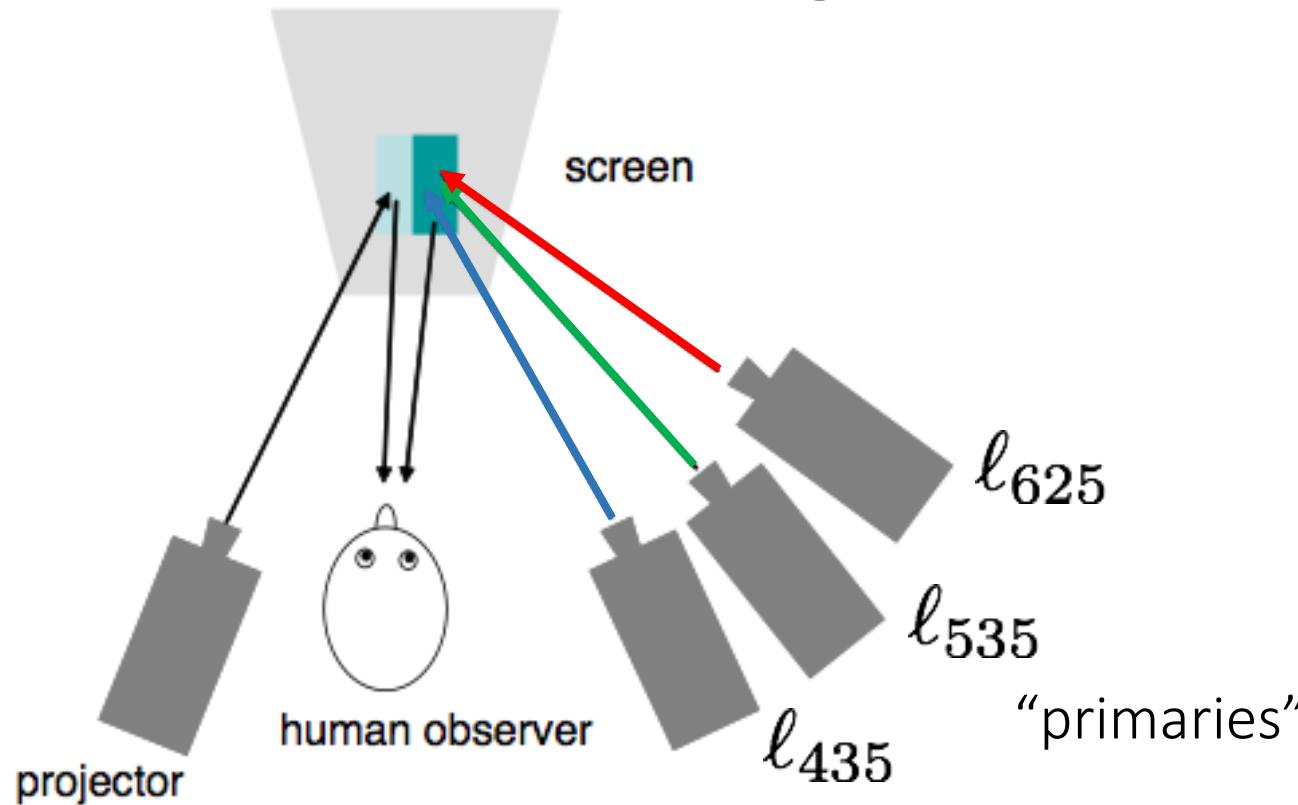
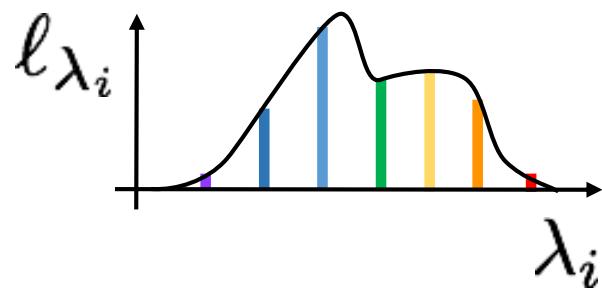
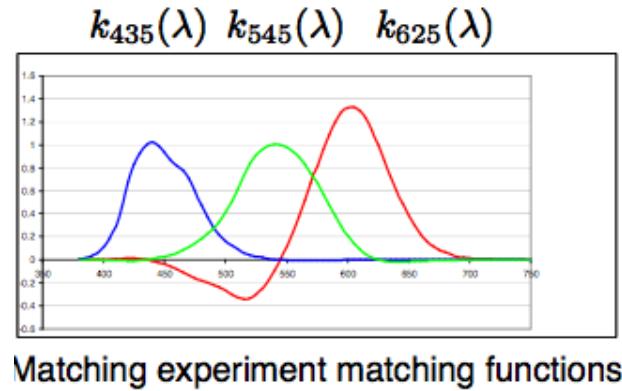
# CIE color matching



Repeat this matching experiments for pure test beams at wavelengths  $\lambda_i$  and keep track of the coefficients (negative or positive) required to reproduce each pure test beam.

$$\mathbf{c}(\lambda_i) = k_{435}(\lambda) \mathbf{c}(\ell_{435}) + k_{535}(\lambda) \mathbf{c}(\ell_{535}) + k_{625}(\lambda) \mathbf{c}(\ell_{625})$$

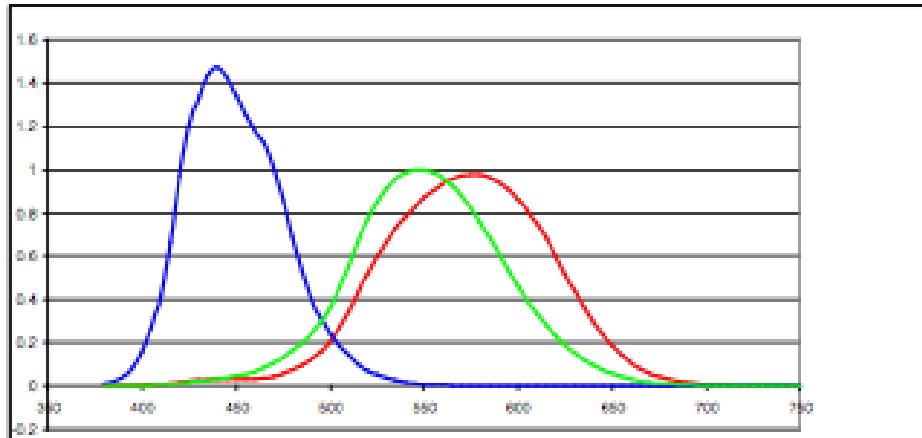
# CIE color matching



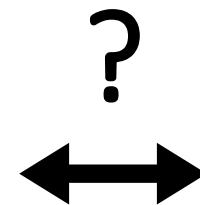
What about “mixed beams”?

# Two views of retinal color

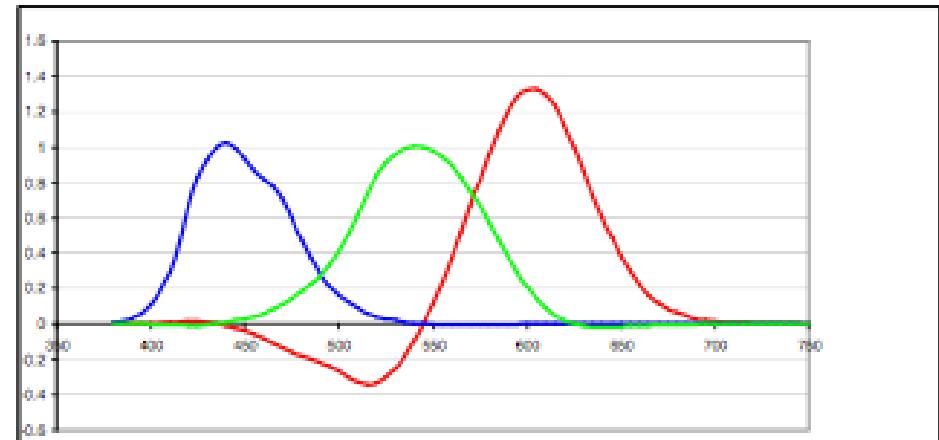
$$k_s(\lambda) \ k_m(\lambda) \ k_l(\lambda)$$



LMS sensitivity functions



$$k_{435}(\lambda) \ k_{545}(\lambda) \ k_{625}(\lambda)$$



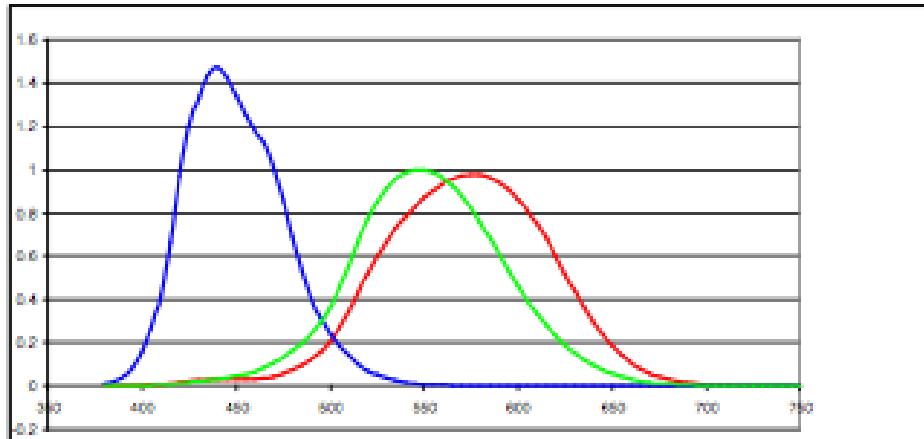
Matching experiment matching functions

Analytic: Retinal color is produced by analyzing spectral power distributions using the color sensitivity functions.

Synthetic: Retinal color is produced by synthesizing color primaries using the color matching functions.

# Two views of retinal color

$k_s(\lambda)$   $k_m(\lambda)$   $k_l(\lambda)$

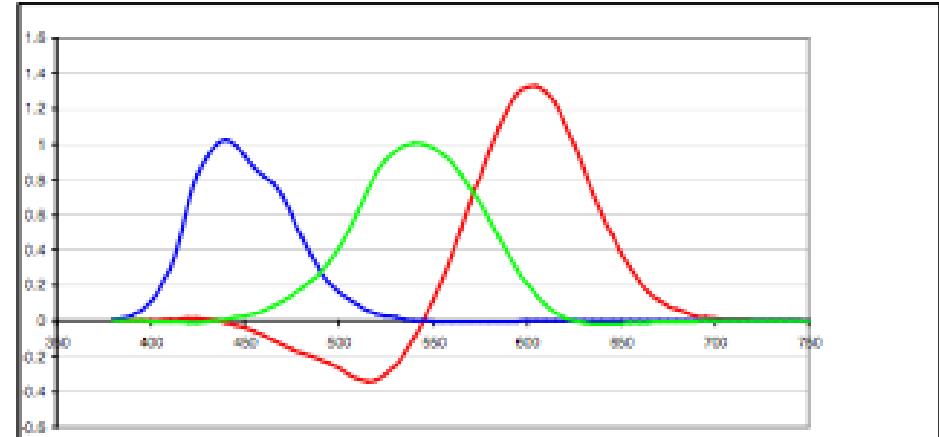


LMS sensitivity functions

Analytic: Retinal color is produced by analyzing spectral power distributions using the color sensitivity functions.

The two views are equivalent: Color matching functions are also color sensitivity functions. For each set of color sensitivity functions, there are corresponding color primaries.

$k_{435}(\lambda)$   $k_{545}(\lambda)$   $k_{625}(\lambda)$



Matching experiment matching functions

Synthetic: Retinal color is produced by synthesizing color primaries using the color matching functions.

# Linear color spaces

# Linear color spaces

1) Color matching experimental outcome:

$$\mathbf{c}(\lambda_i) = k_{435}(\lambda)\mathbf{c}(\ell_{435}) + k_{535}(\lambda)\mathbf{c}(\ell_{535}) + k_{625}(\lambda)\mathbf{c}(\ell_{625})$$

same in matrix form:

$$\begin{bmatrix} \mathbf{c}(\lambda_i) \\ | \\ \mathbf{c}(\lambda_i) \end{bmatrix} = \begin{bmatrix} \mathbf{c}(\ell_{435}) & \mathbf{c}(\ell_{535}) & \mathbf{c}(\ell_{625}) \\ | & | & | \end{bmatrix} \begin{bmatrix} k_{435} \\ k_{535} \\ k_{625} \end{bmatrix}$$



how is this matrix formed?

# Linear color spaces

1) Color matching experimental outcome:

$$\mathbf{c}(\lambda_i) = k_{435}(\lambda) \mathbf{c}(\ell_{435}) + k_{535}(\lambda) \mathbf{c}(\ell_{535}) + k_{625}(\lambda) \mathbf{c}(\ell_{625})$$

same in matrix form:

$$\begin{bmatrix} \mathbf{c}(\lambda_i) \\ | \\ | \end{bmatrix} = \begin{bmatrix} \mathbf{c}(\ell_{435}) & \mathbf{c}(\ell_{535}) & \mathbf{c}(\ell_{625}) \\ | & | & | \end{bmatrix} \begin{bmatrix} k_{435} \\ k_{535} \\ k_{625} \end{bmatrix}$$

2) Implication for arbitrary mixed beams:

$$\begin{bmatrix} \mathbf{c}(\ell(\lambda)) \\ | \\ | \end{bmatrix} = \begin{bmatrix} \mathbf{c}(\ell_{435}) & \mathbf{c}(\ell_{535}) & \mathbf{c}(\ell_{625}) \\ | & | & | \end{bmatrix} \begin{bmatrix} \int k_{435}(\lambda) \ell(\lambda) d\lambda \\ \int k_{535}(\lambda) \ell(\lambda) d\lambda \\ \int k_{625}(\lambda) \ell(\lambda) d\lambda \end{bmatrix}$$



where do these terms come from?

# Linear color spaces

1) Color matching experimental outcome:

$$\mathbf{c}(\lambda_i) = k_{435}(\lambda) \mathbf{c}(\ell_{435}) + k_{535}(\lambda) \mathbf{c}(\ell_{535}) + k_{625}(\lambda) \mathbf{c}(\ell_{625})$$

same in matrix form:

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what is this similar to?

# Linear color spaces

1) Color matching experimental outcome:

$$\mathbf{c}(\lambda_i) = k_{435}(\lambda) \mathbf{c}(\ell_{435}) + k_{535}(\lambda) \mathbf{c}(\ell_{535}) + k_{625}(\lambda) \mathbf{c}(\ell_{625})$$

same in matrix form:

$$\begin{bmatrix} \mathbf{c}(\lambda_i) \\ | \\ | \end{bmatrix} = \begin{bmatrix} \mathbf{c}(\ell_{435}) & \mathbf{c}(\ell_{535}) & \mathbf{c}(\ell_{625}) \\ | & | & | \end{bmatrix} \begin{bmatrix} k_{435} \\ k_{535} \\ k_{625} \end{bmatrix}$$

2) Implication for arbitrary mixed beams:

$$\begin{bmatrix} \mathbf{c}(\ell(\lambda)) \\ | \\ | \end{bmatrix} = \begin{bmatrix} \mathbf{c}(\ell_{435}) & \mathbf{c}(\ell_{535}) & \mathbf{c}(\ell_{625}) \\ | & | & | \end{bmatrix} \begin{bmatrix} \int k_{435}(\lambda) \ell(\lambda) d\lambda \\ \int k_{535}(\lambda) \ell(\lambda) d\lambda \\ \int k_{625}(\lambda) \ell(\lambda) d\lambda \end{bmatrix}$$

representation of retinal  
color in LMS space

change of basis matrix

representation of retinal  
color in space of primaries

# Linear color spaces

1) Color matching experimental outcome:

$$\mathbf{c}(\lambda_i) = k_{435}(\lambda) \mathbf{c}(\ell_{435}) + k_{535}(\lambda) \mathbf{c}(\ell_{535}) + k_{625}(\lambda) \mathbf{c}(\ell_{625})$$

same in matrix form:

$$\begin{bmatrix} \mathbf{c}(\lambda_i) \\ | \\ | \end{bmatrix} = \begin{bmatrix} \mathbf{c}(\ell_{435}) & \mathbf{c}(\ell_{535}) & \mathbf{c}(\ell_{625}) \\ | & | & | \end{bmatrix} \begin{bmatrix} k_{435} \\ k_{535} \\ k_{625} \end{bmatrix}$$

2) Implication for arbitrary mixed beams:

$$\begin{bmatrix} \mathbf{c}(\ell(\lambda)) \\ | \\ | \end{bmatrix} = \begin{bmatrix} \mathbf{c}(\ell_{435}) & \mathbf{c}(\ell_{535}) & \mathbf{c}(\ell_{625}) \\ | & | & | \end{bmatrix} \begin{bmatrix} \int k_{435}(\lambda) \ell(\lambda) d\lambda \\ \int k_{535}(\lambda) \ell(\lambda) d\lambda \\ \int k_{625}(\lambda) \ell(\lambda) d\lambda \end{bmatrix}$$

representation of retinal  
color in LMS space

change of basis matrix

representation of retinal  
color in space of primaries

# Linear color spaces

basis for retinal color  $\Leftrightarrow$  color matching functions  $\Leftrightarrow$  primary colors  $\Leftrightarrow$  color space

$$\begin{bmatrix} \mathbf{c}(\ell(\lambda)) \\ \vdots \end{bmatrix} = \underbrace{\begin{bmatrix} \mathbf{c}_1 & \mathbf{c}_2 & \mathbf{c}_3 \\ \vdots & \vdots & \vdots \end{bmatrix}}_{\mathbf{M}^{-1}} \begin{bmatrix} \int k_1(\lambda) \ell(\lambda) d\lambda \\ \int k_2(\lambda) \ell(\lambda) d\lambda \\ \int k_3(\lambda) \ell(\lambda) d\lambda \end{bmatrix}$$
$$\begin{bmatrix} \mathbf{c}(\ell_{435}) & \mathbf{c}(\ell_{545}) & \mathbf{c}(\ell_{625}) \\ \vdots & \vdots & \vdots \end{bmatrix} \mathbf{M}^{-1} \quad \begin{bmatrix} k_1(\lambda) \\ k_2(\lambda) \\ k_3(\lambda) \end{bmatrix} = \mathbf{M} \begin{bmatrix} k_{435}(\lambda) \\ k_{545}(\lambda) \\ k_{625}(\lambda) \end{bmatrix}$$

$$\begin{bmatrix} \mathbf{c}(\ell(\lambda)) \\ \vdots \end{bmatrix} = \begin{bmatrix} \mathbf{c}(\ell_{435}) & \mathbf{c}(\ell_{545}) & \mathbf{c}(\ell_{625}) \\ \vdots & \vdots & \vdots \end{bmatrix} \begin{bmatrix} \int k_{435}(\lambda) \ell(\lambda) d\lambda \\ \int k_{535}(\lambda) \ell(\lambda) d\lambda \\ \int k_{625}(\lambda) \ell(\lambda) d\lambda \end{bmatrix}$$

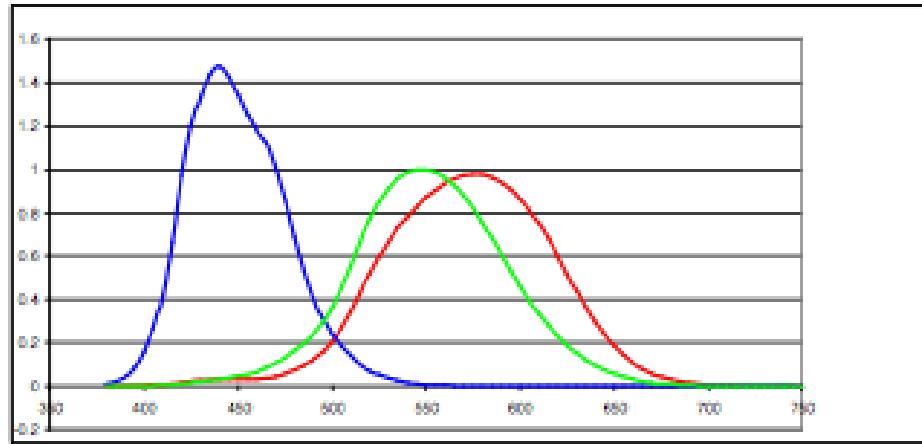
representation of retinal color in LMS space

change of basis matrix

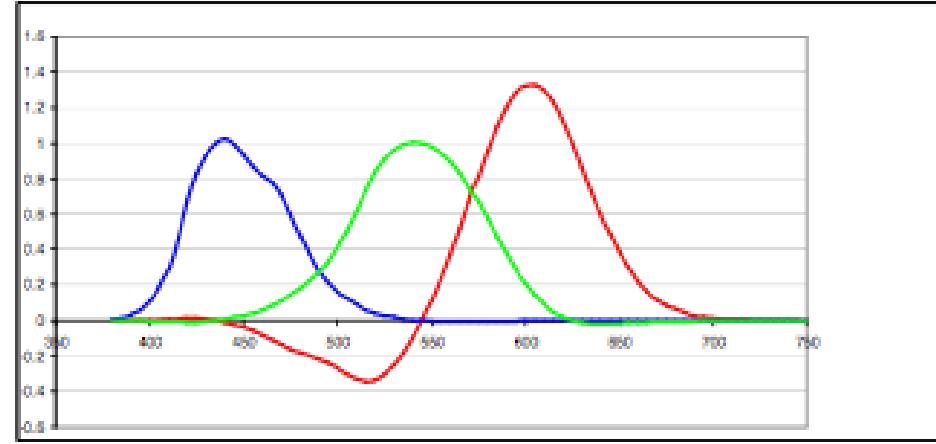
$\mathbf{M}^{-1}\mathbf{M}$  can insert any invertible  $\mathbf{M}$

representation of retinal color in space of primaries

# A few important color spaces



LMS color space



CIE RGB color space

not the “usual” RGB color space encountered in practice

# Two views of retinal color

Analytic: Retinal color is three numbers formed by taking the dot product of a power spectral distribution with three color matching/sensitivity functions.

Synthetic: Retinal color is three numbers formed by assigning weights to three color primaries to match the perception of a power spectral distribution.

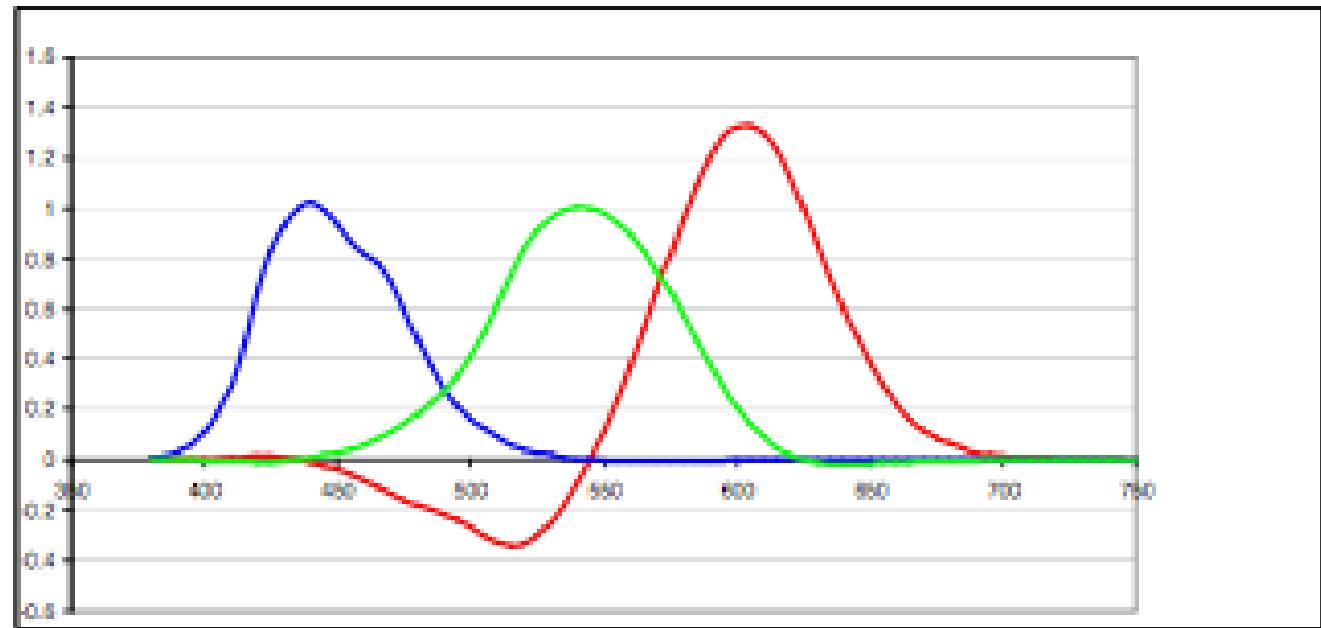
How would you make a color measurement device?

# How would you make a color measurement device?

Do what the eye does:

- Select three spectral filters (i.e., three color matching functions.).
- Capture three measurements.

Can we use the CIE RGB color matching functions?



CIE RGB color space

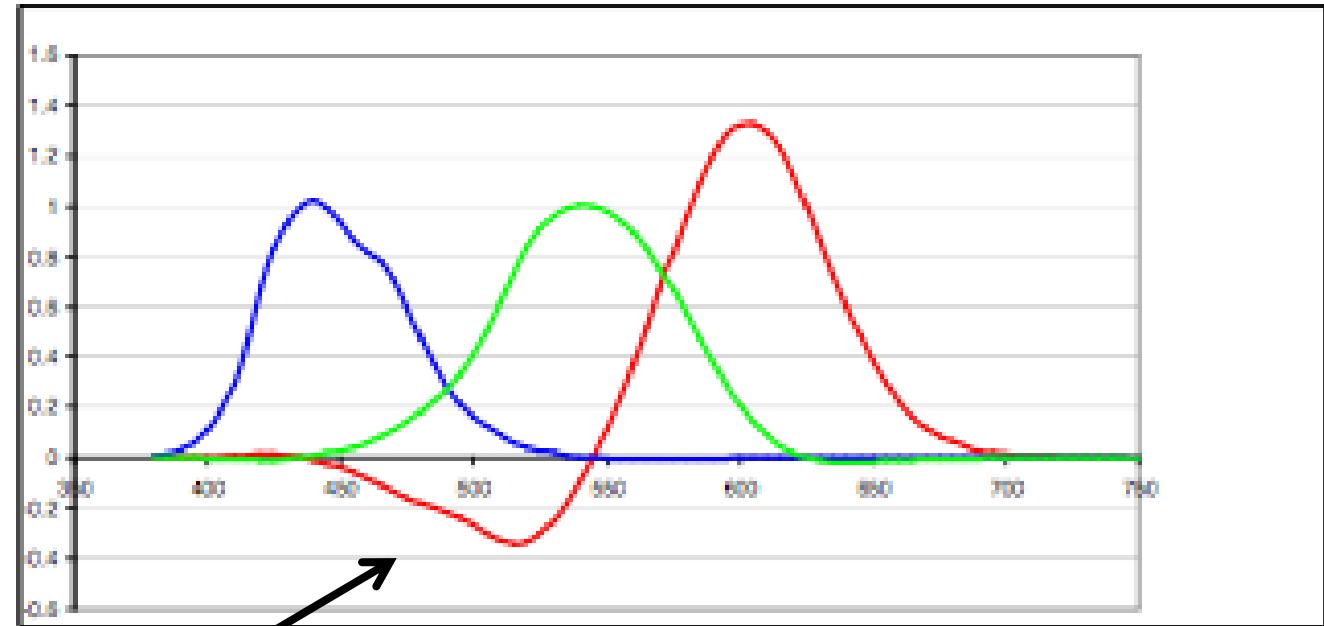
# How would you make a color measurement device?

Do what the eye does:

- Select three spectral filters (i.e., three color matching functions.).
- Capture three measurements.

Can we use the CIE RGB color matching functions?

Negative values are an issue (we can't "subtract" light at a sensor)



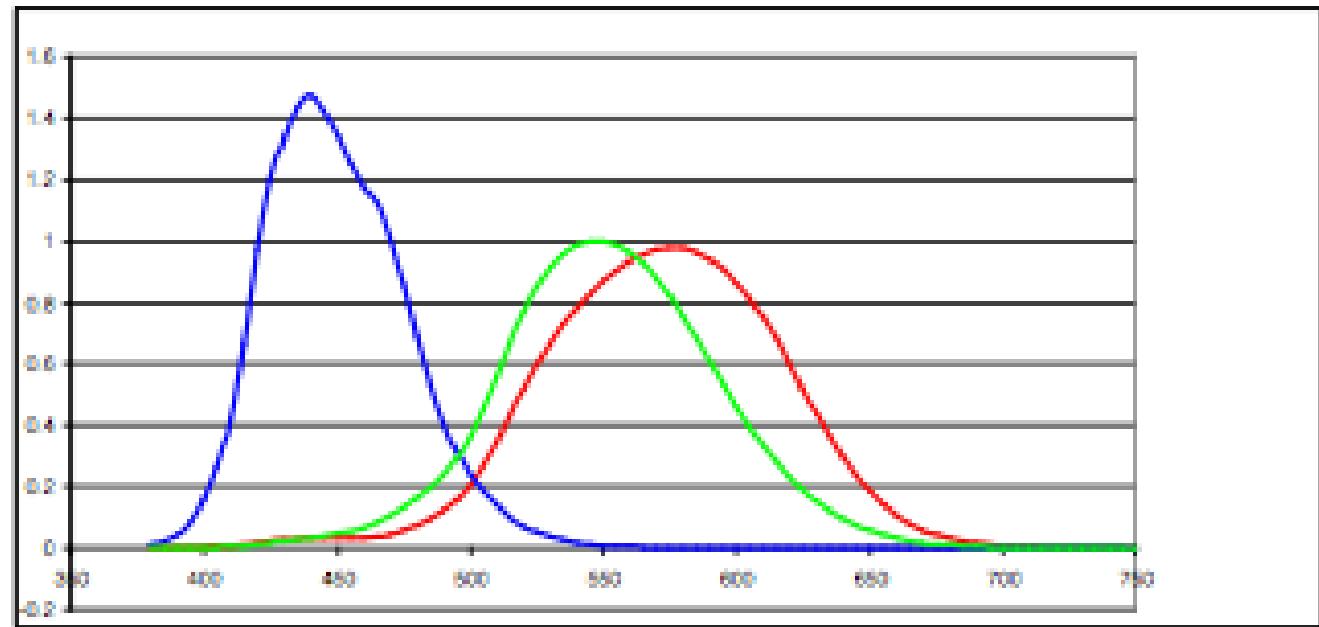
CIE RGB color space

# How would you make a color measurement device?

Do what the eye does:

- Select three spectral filters (i.e., three color matching functions.).
- Capture three measurements.

Can we use the LMS color matching functions?



LMS color space

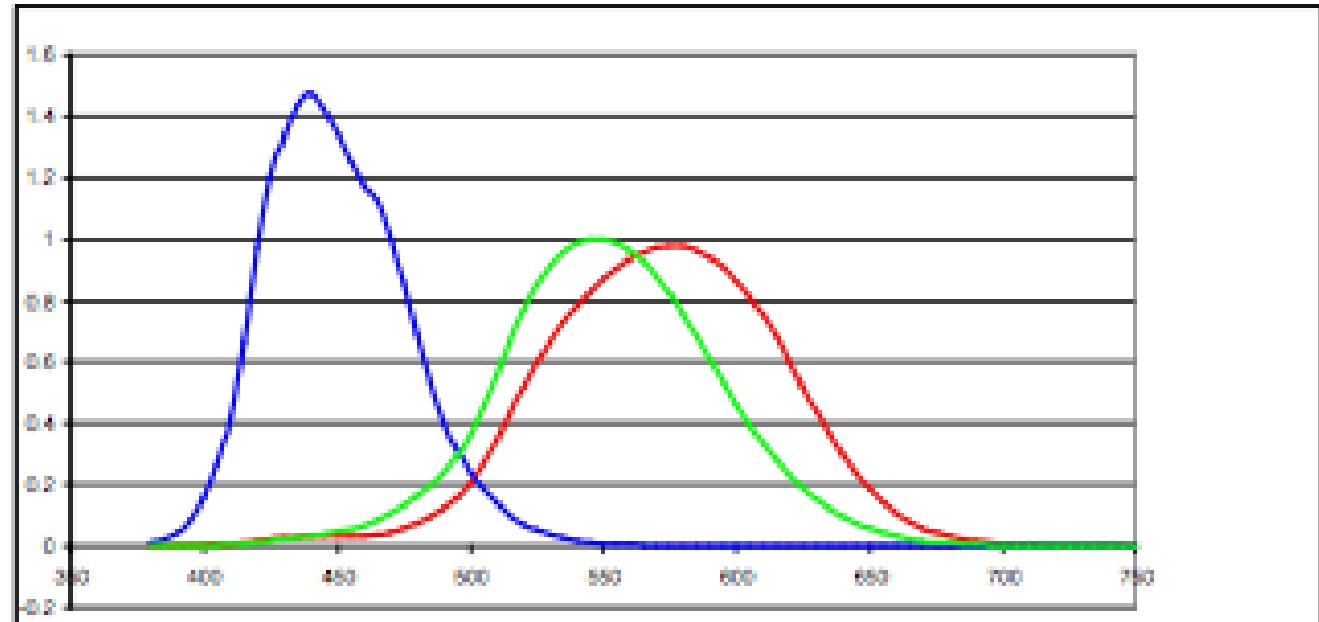
# How would you make a color measurement device?

Do what the eye does:

- Select three spectral filters (i.e., three color matching functions.).
- Capture three measurements.

Can we use the LMS color matching functions?

- They weren't known when CIE was doing their color matching experiments.
- We'll see later they create other issues.



LMS color space

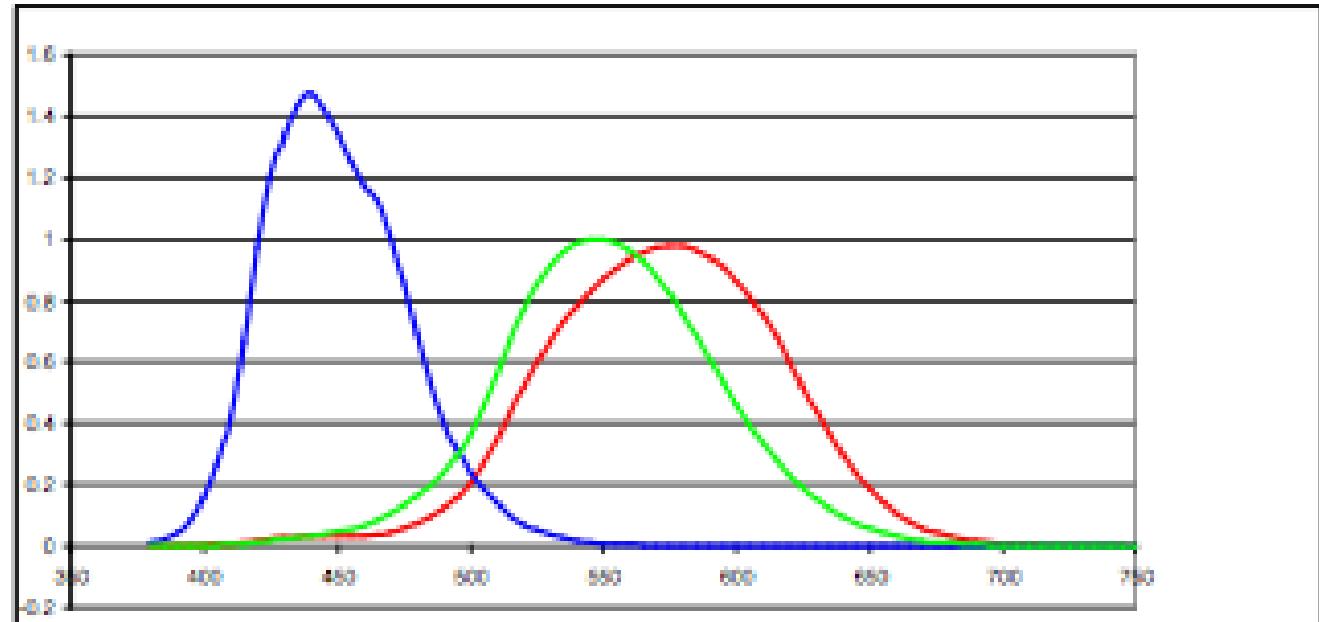
# How would you make a color measurement device?

Do what the eye does:

- Select three spectral filters (i.e., three color matching functions).
- Capture three measurements.

Can we use the LMS color matching functions?

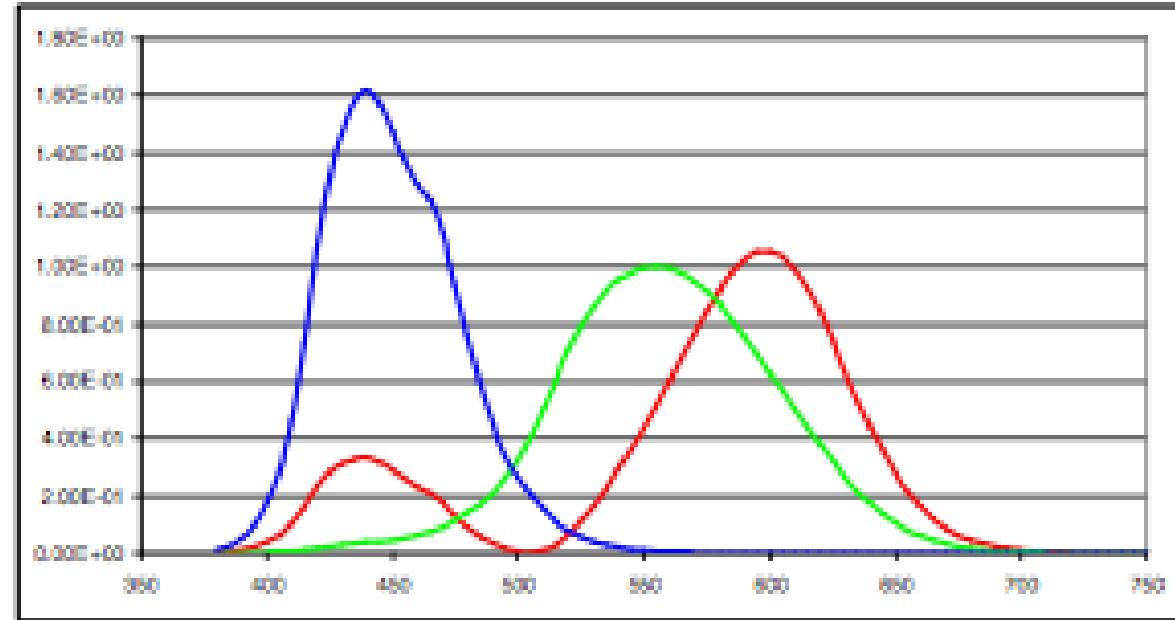
- They weren't known when CIE was doing their color matching experiments.
- We'll see later they create other issues.



LMS color space

# The CIE XYZ color space

- Derived from CIE RGB by adding enough blue and green to make the red positive.
- Probably the most important *reference* (i.e., device independent) color space.



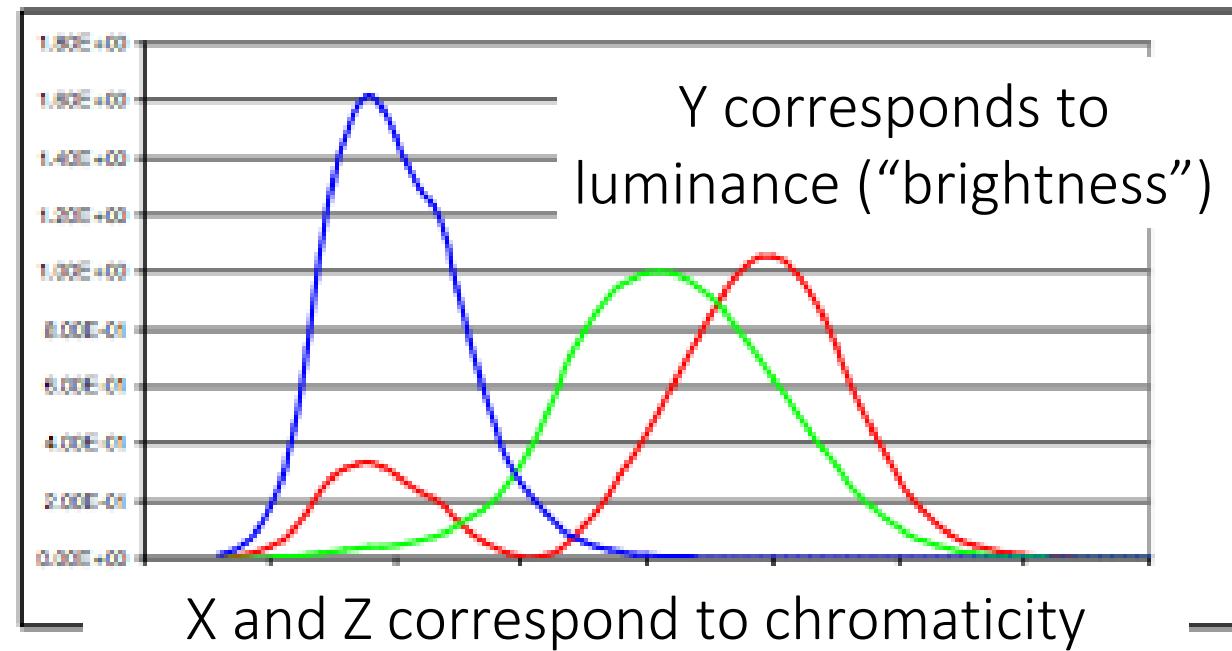
Remarkable and/or scary: 80+ years of CIE XYZ is all down to color matching experiments done with 12 “standard observers”.

CIE XYZ color space

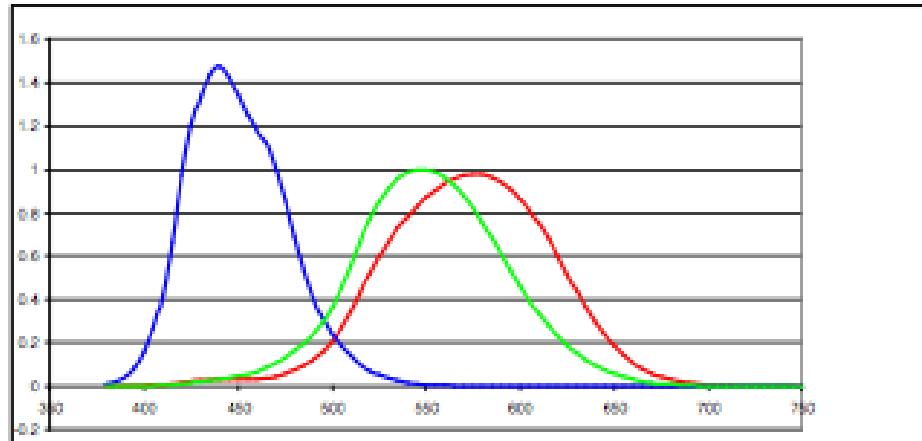
# The CIE XYZ color space

- Derived from CIE RGB by adding enough blue and green to make the red positive.
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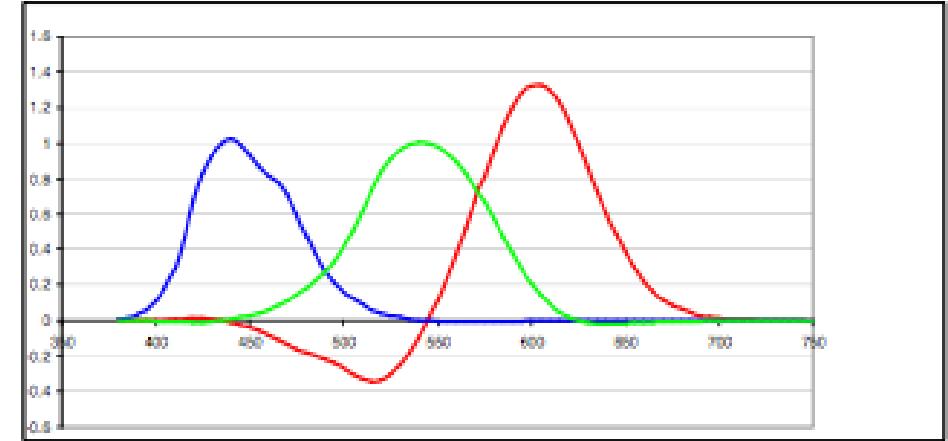
How would you convert a color image to grayscale?



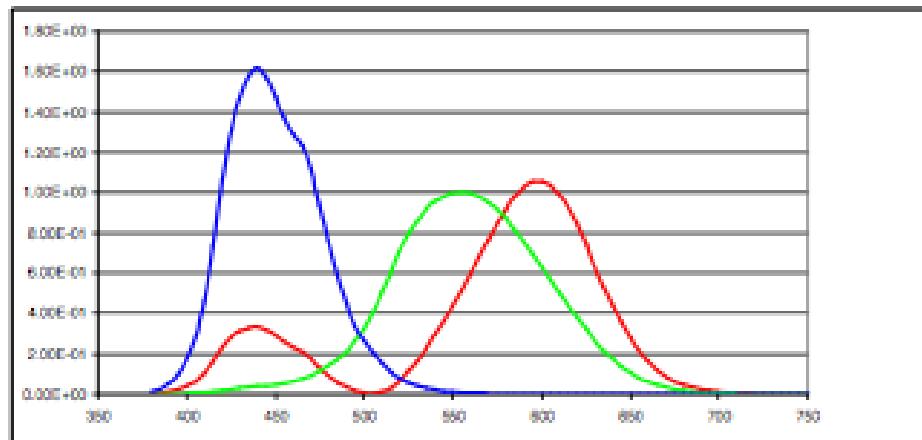
# A few important color spaces



LMS color space



CIE RGB color space



CIE XYZ color space

# Two views of retinal color

Analytic: Retinal color is three numbers formed by taking the dot product of a power spectral distribution with three color matching/sensitivity functions.

Synthetic: Retinal color is three numbers formed by assigning weights to three color primaries to match the perception of a power spectral distribution.

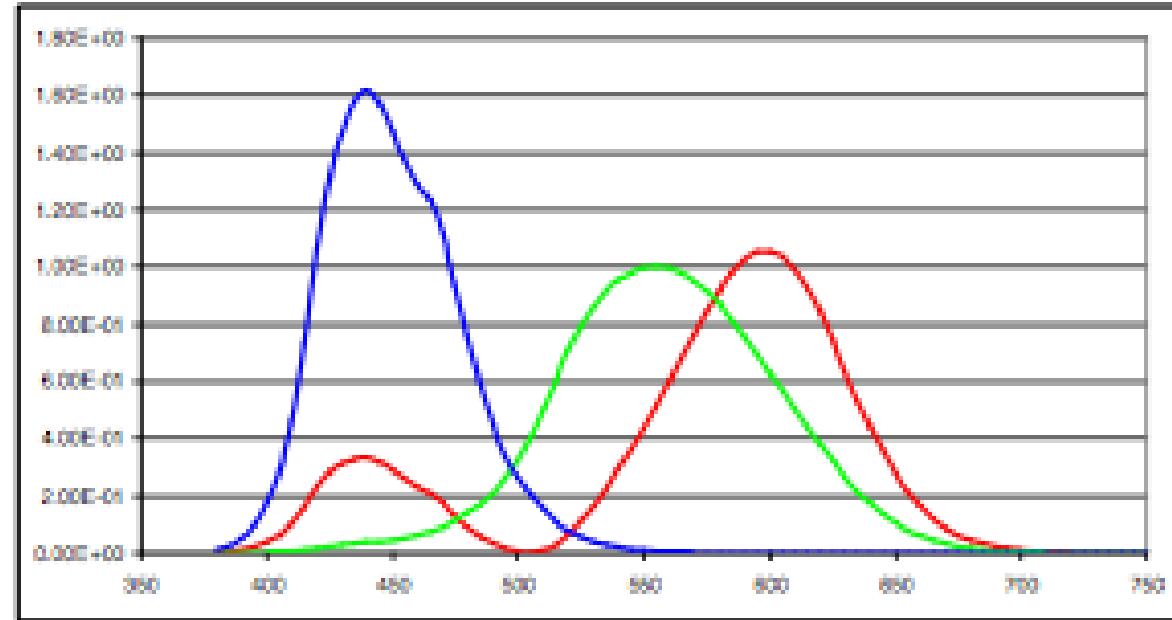
How would you make a color reproduction device?

# How would you make a color reproduction device?

Do what color matching does:

- Select three color primaries.
- Represent all colors as mixtures of these three primaries.

Can we use the XYZ color primaries?



CIE XYZ color space

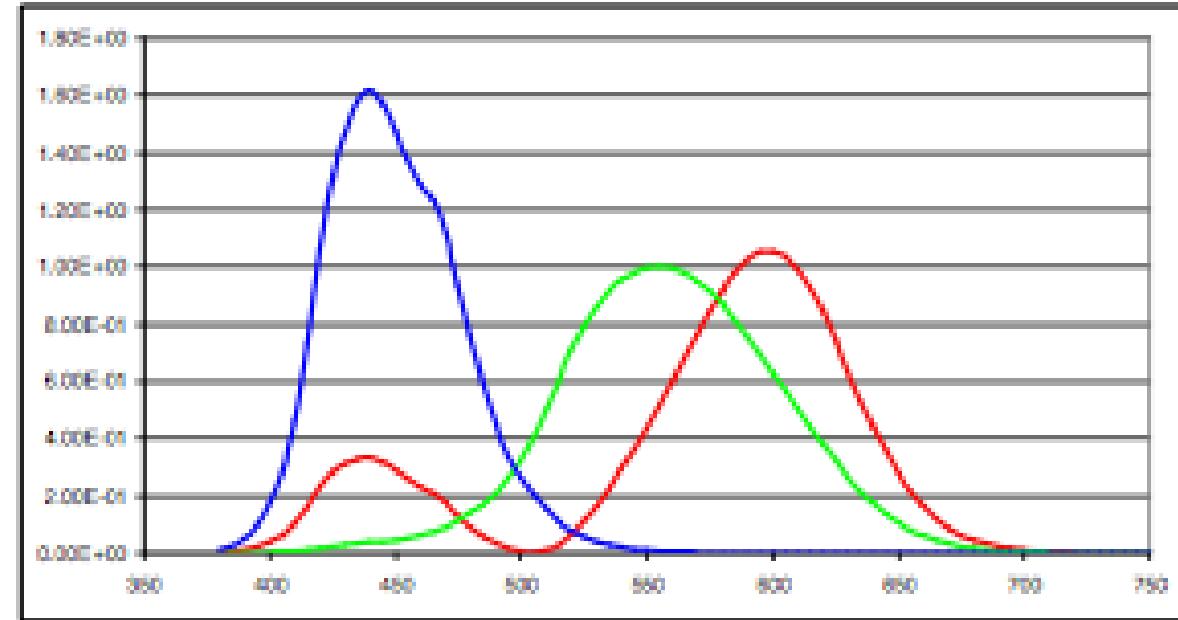
# How would you make a color reproduction device?

Do what color matching does:

- Select three color primaries.
- Represent all colors as mixtures of these three primaries.

Can we use the XYZ color primaries?

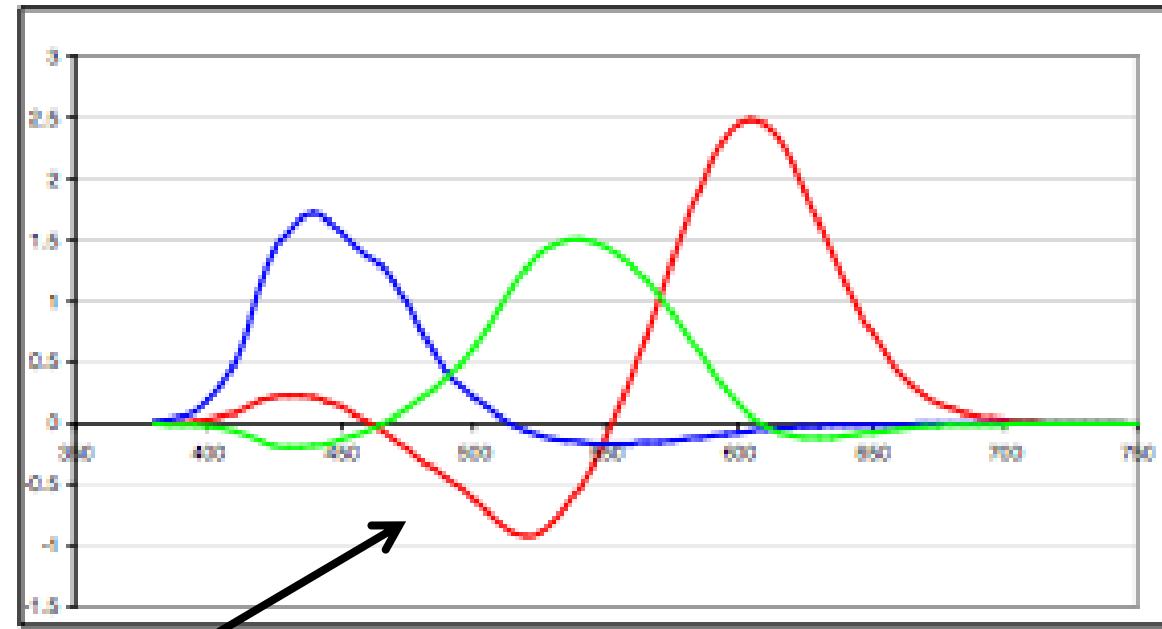
- No, because they are not “real” colors (they require an SPD with negative values).
- Same goes for LMS color primaries.



CIE XYZ color space

# The Standard RGB (sRGB) color space

- Derived by Microsoft and HP in 1996, based on CRT displays used at the time.
- Similar but not equivalent to CIE RGB.

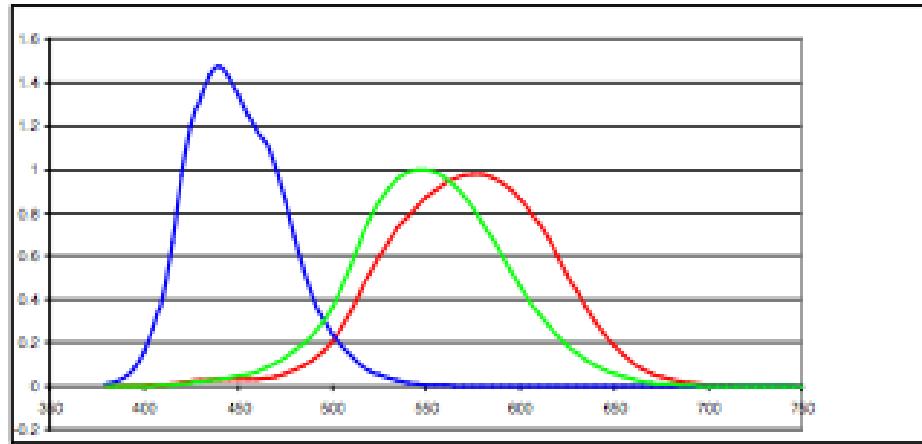


Note the negative values

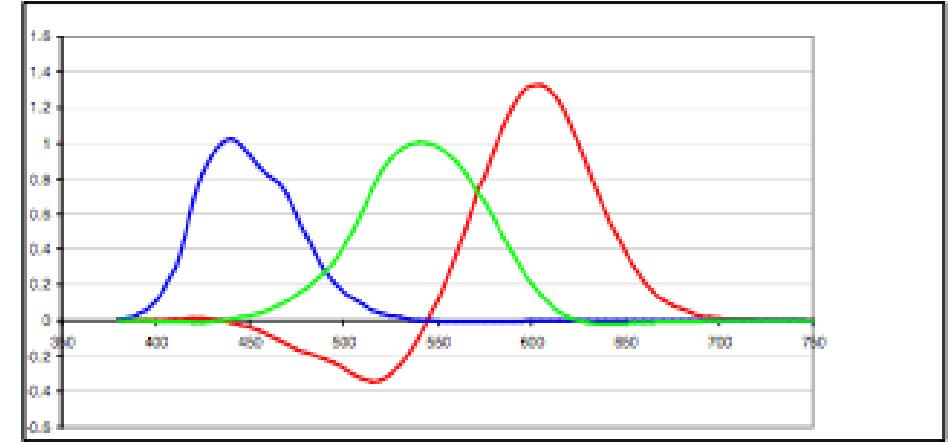
sRGB color space

While it is called “standard”, when you grab an “RGB” image, it is highly likely it is in a different RGB color space...

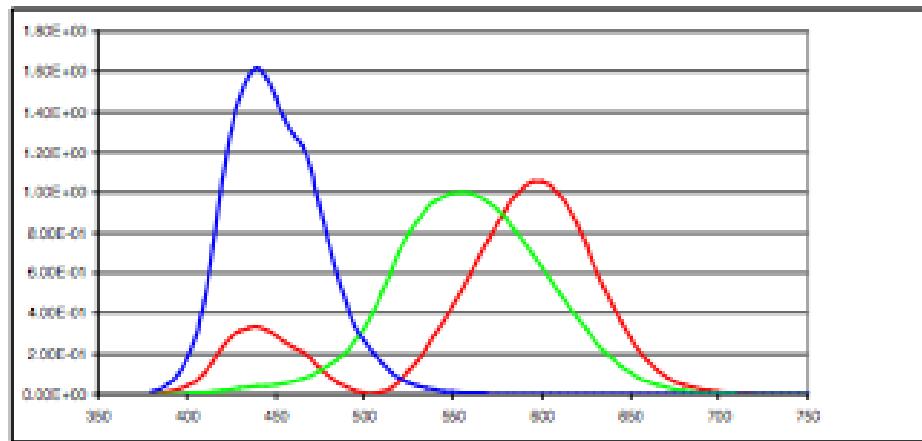
# A few important color spaces



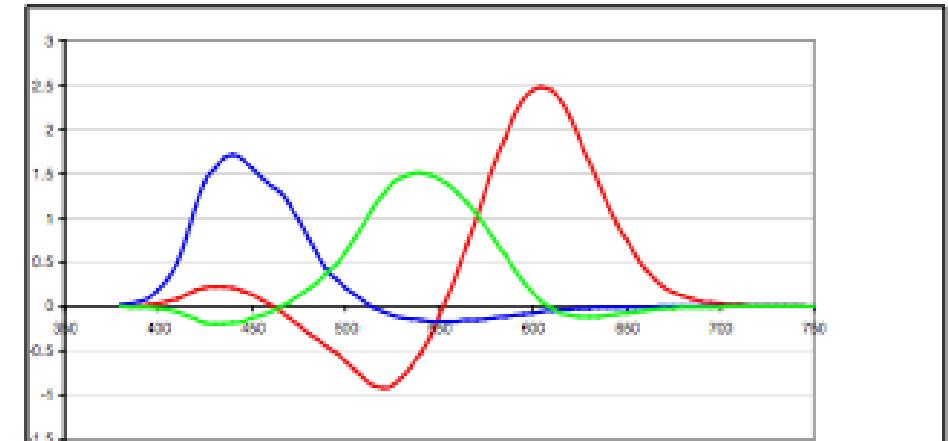
LMS color space



CIE RGB color space

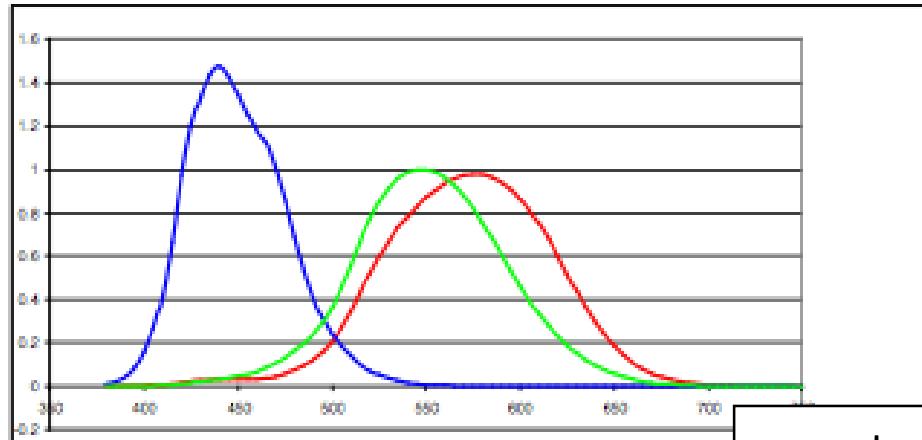


CIE XYZ color space

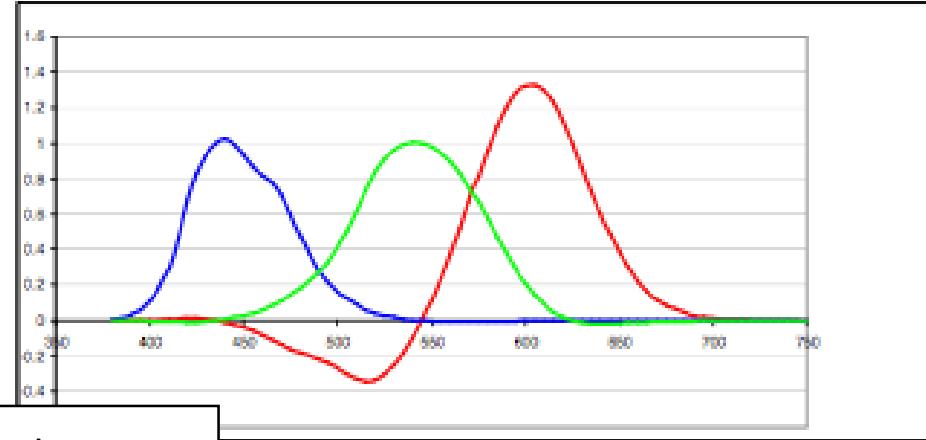


sRGB color space

# A few important color spaces

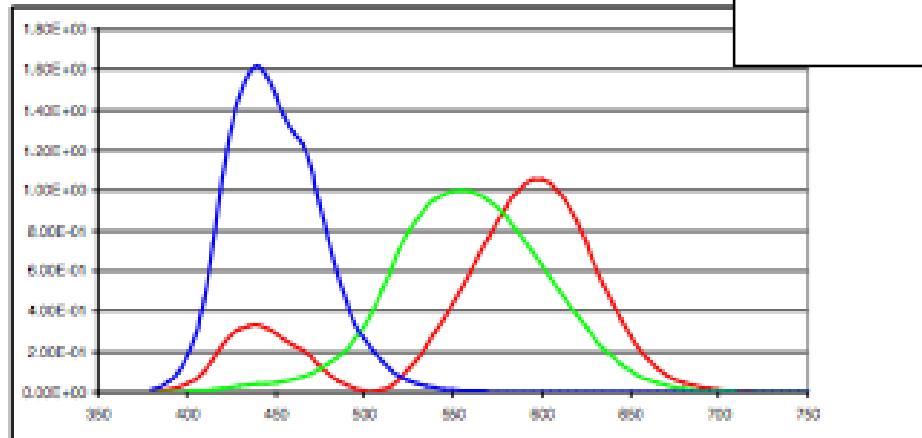


LMS color space

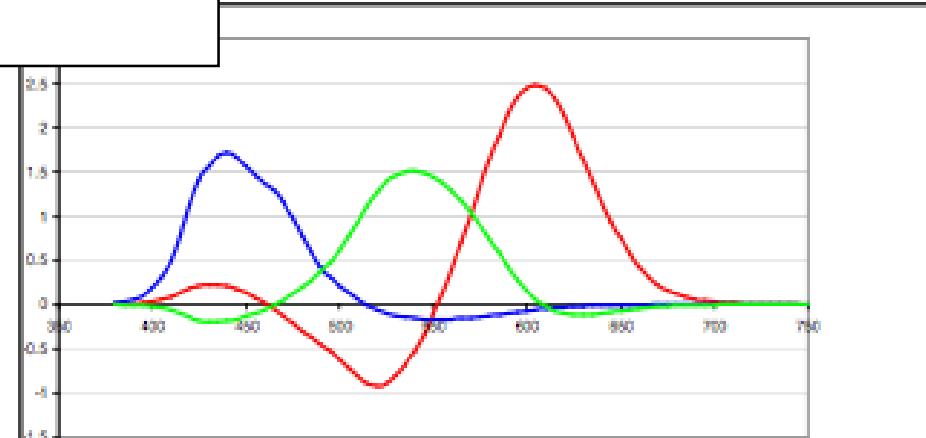


CIE RGB color space

Is there a way to  
“compare” all these color  
spaces?



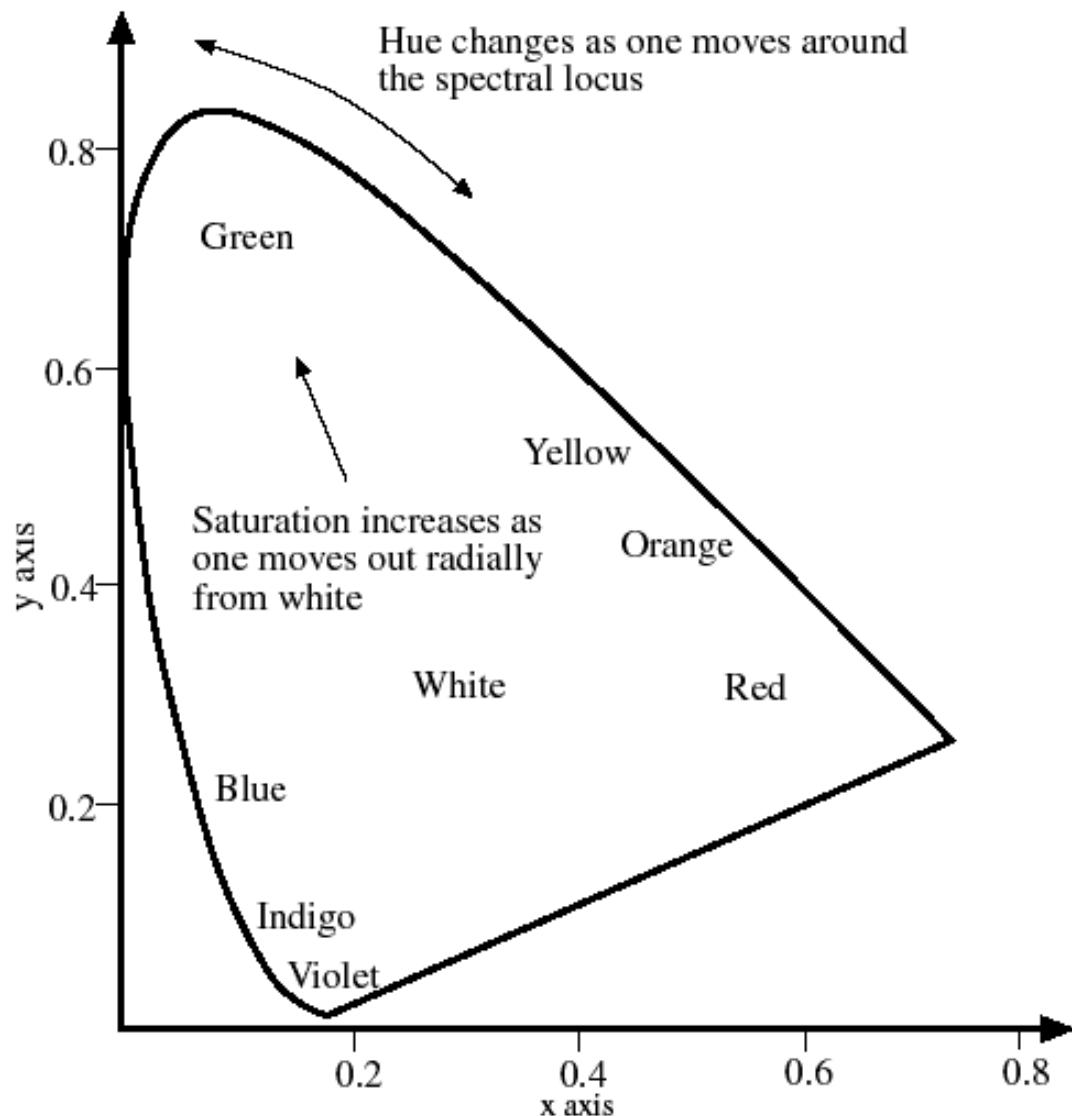
CIE XYZ color space



sRGB color space

# Chromaticity

# CIE xy (chromaticity)



$$x = \frac{X}{X + Y + Z}$$

$$y = \frac{Y}{X + Y + Z}$$

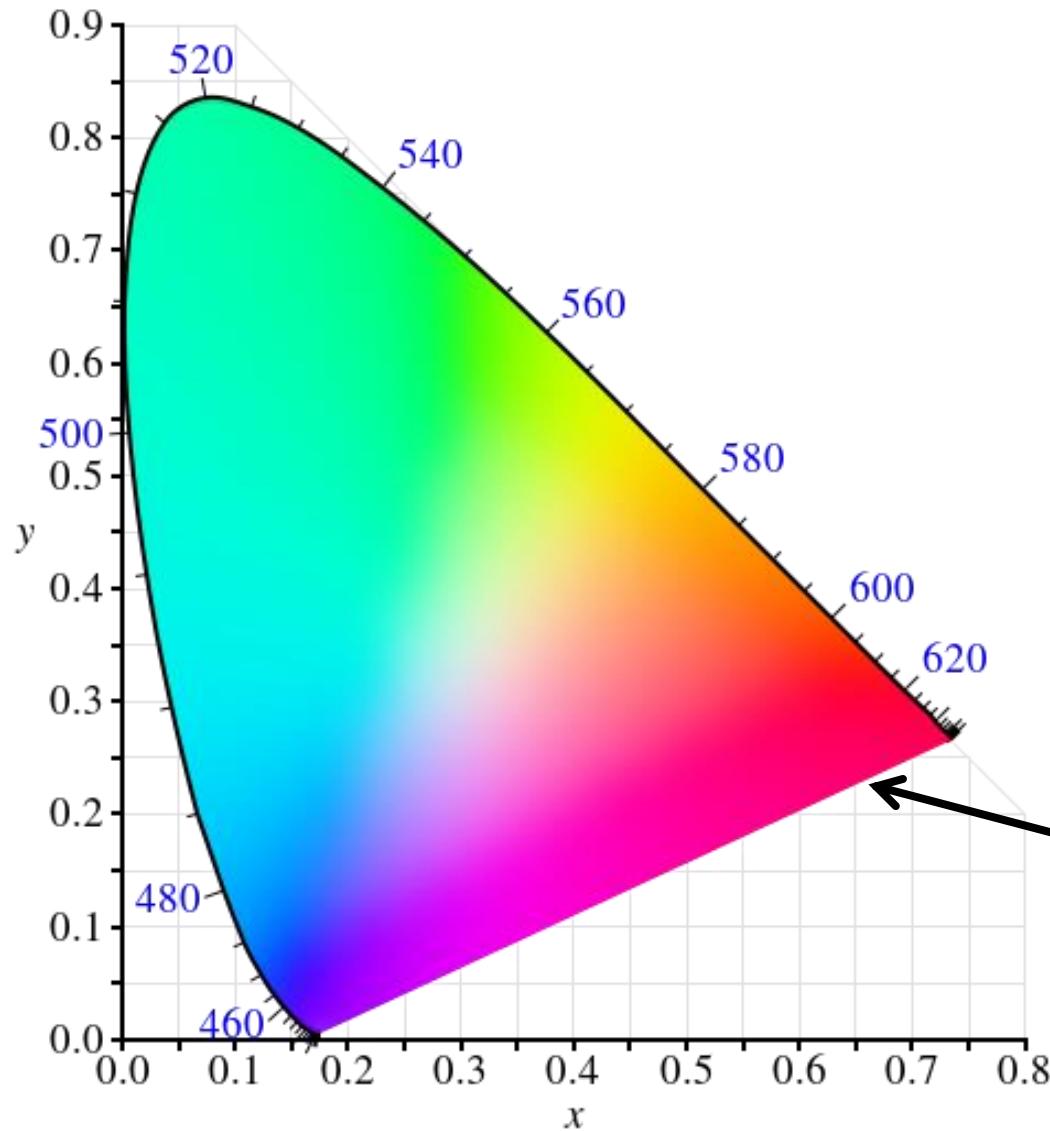
$$(X, Y, Z) \longleftrightarrow (\underline{x}, \underline{y}, Y)$$

chromaticity

luminance/brightness

Perspective projection of 3D retinal color space to two dimensions.

# CIE xy (chromaticity)



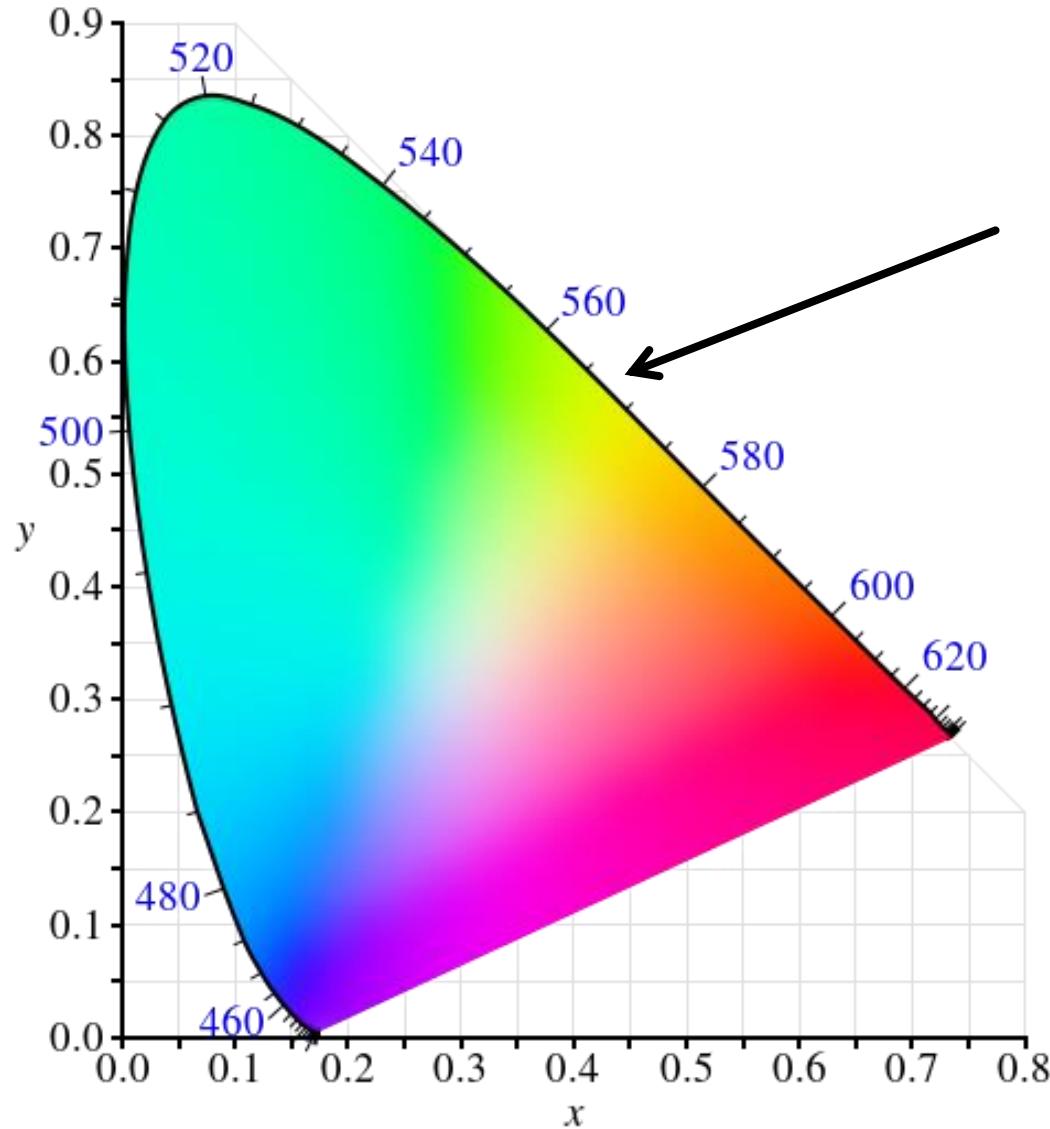
$$x = \frac{X}{X + Y + Z}$$

$$y = \frac{Y}{X + Y + Z}$$

$$(X, Y, Z) \longleftrightarrow (x, y, Y)$$

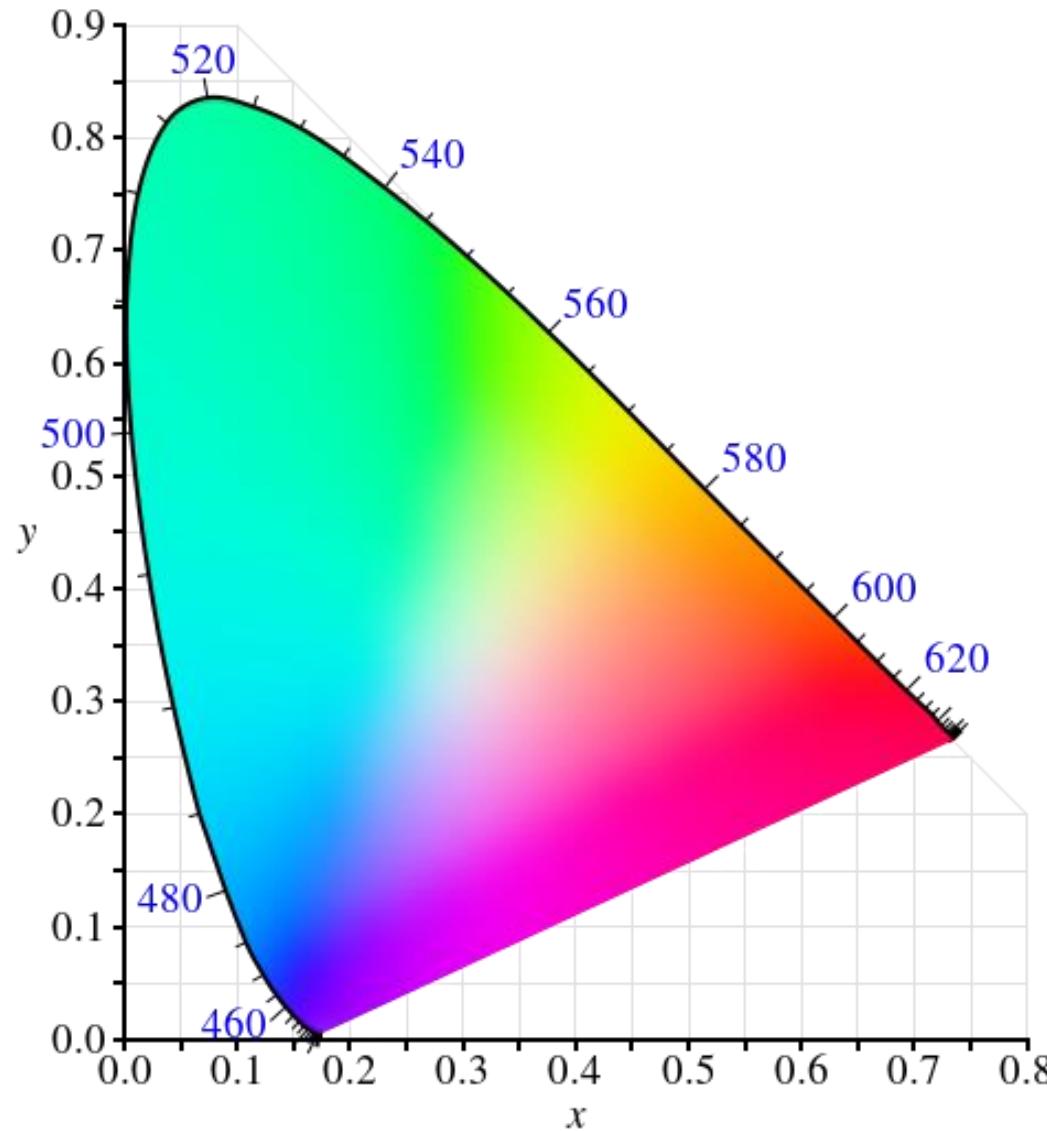
Note: These colors can be extremely misleading depending on the file origin and the display you are using

# CIE xy (chromaticity)



What does the boundary of the chromaticity diagram correspond to?

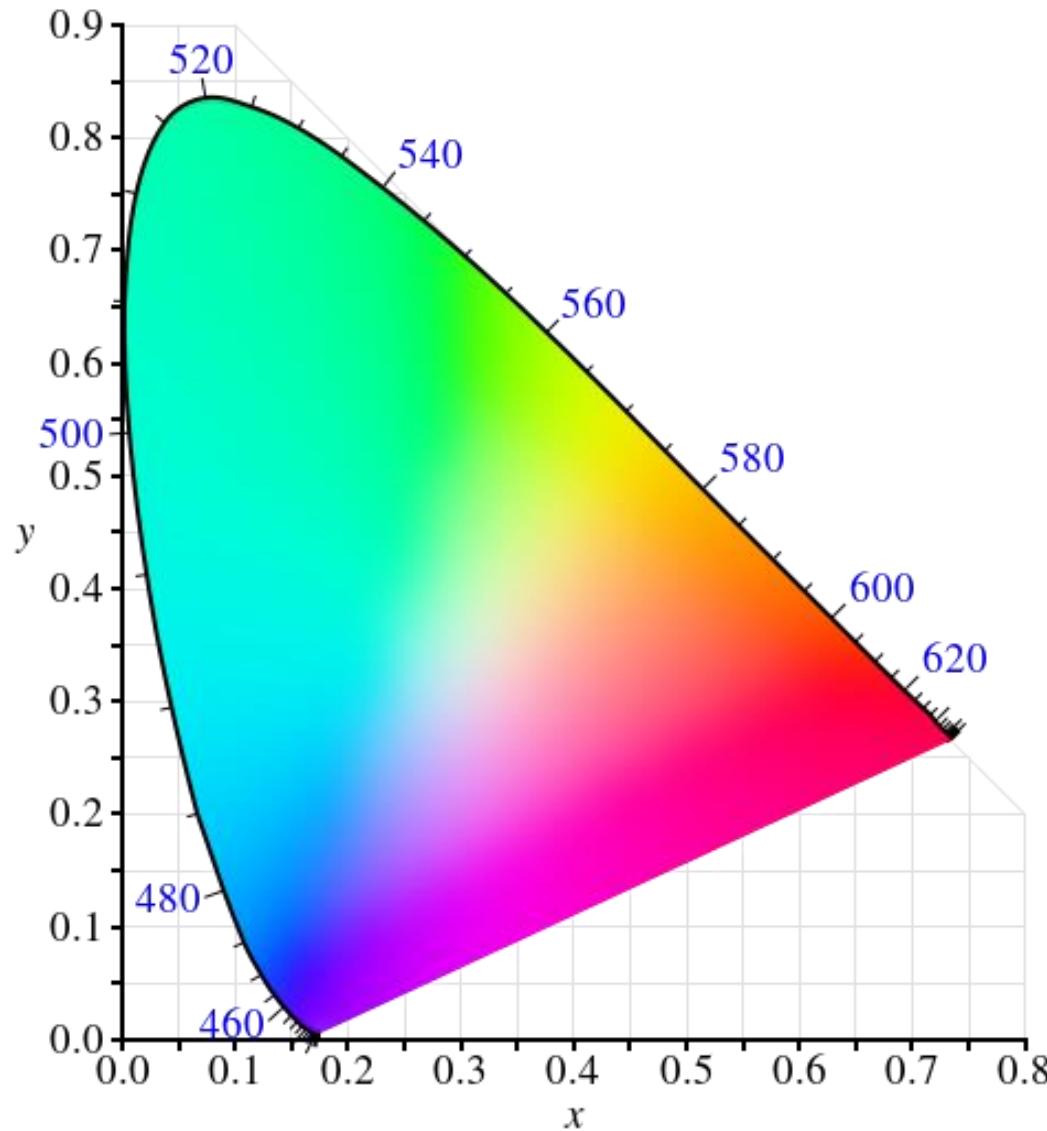
# Color gamuts



We can compare color spaces by looking at what parts of the chromaticity space they can reproduce with their primaries.

But why would a color space not be able to reproduce all of the chromaticity space?

# Color gamuts

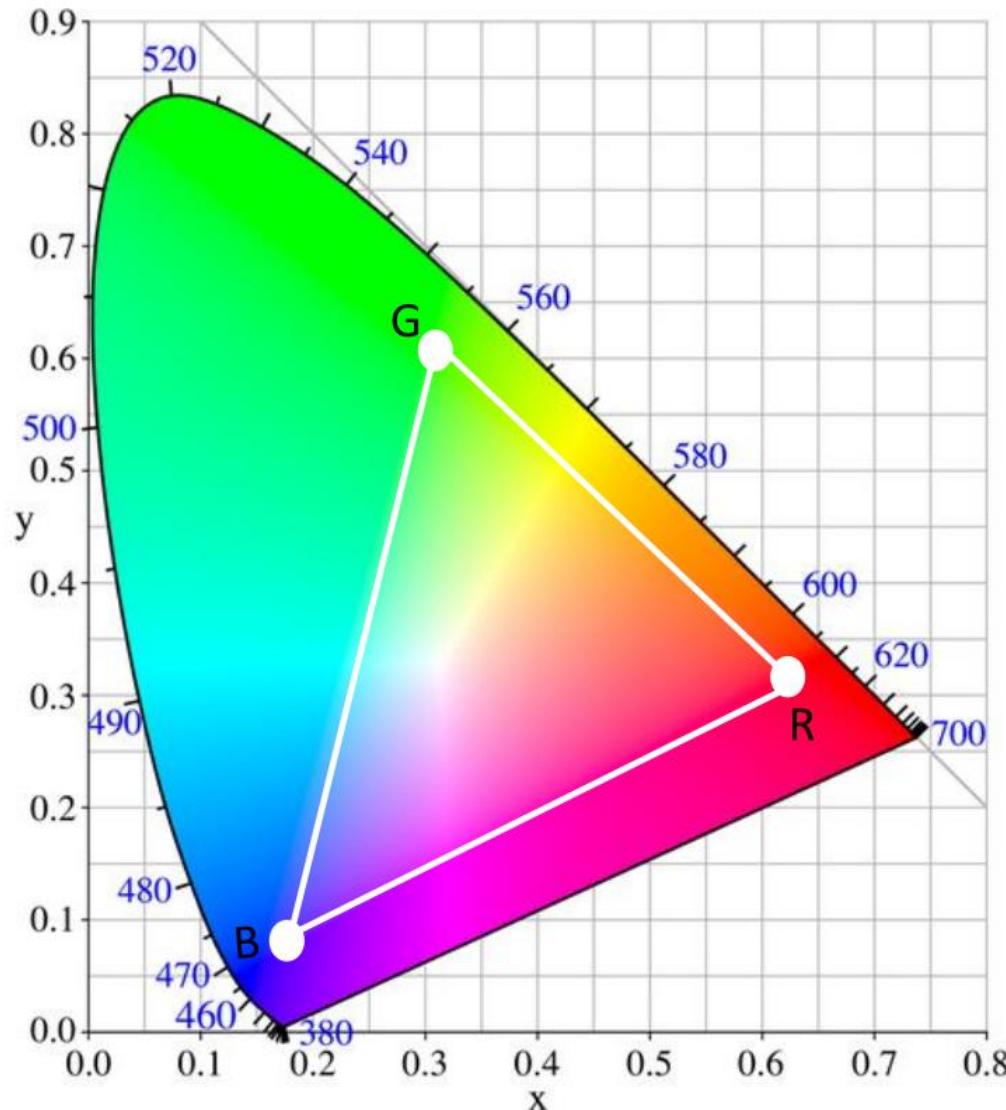


We can compare color spaces by looking at what parts of the chromaticity space they can reproduce with their primaries.

But why would a color space not be able to reproduce all of the chromaticity space?

- Many colors require negative weights to be reproduced, which are not realizable.

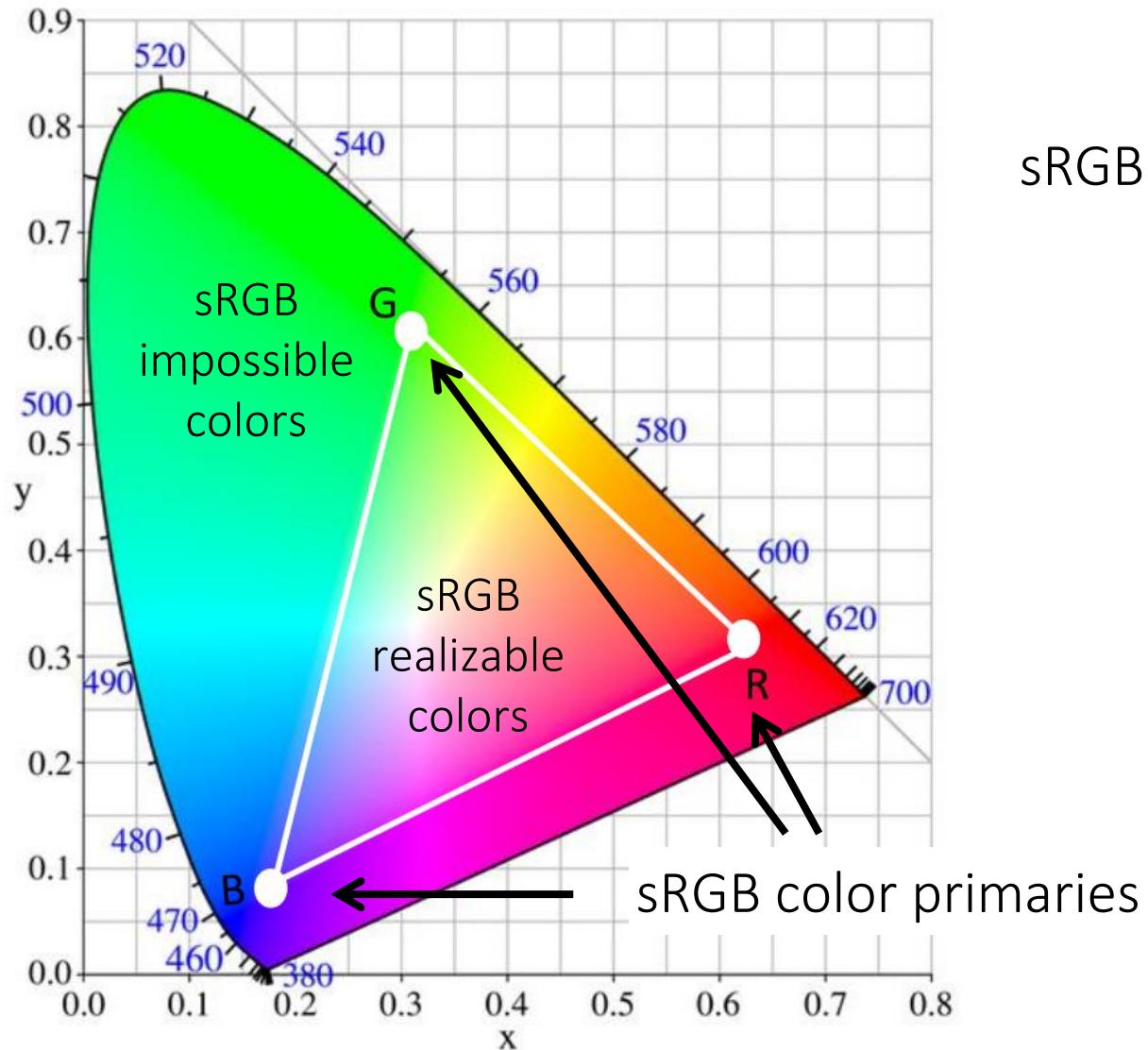
# Color gamuts



sRGB color gamut:

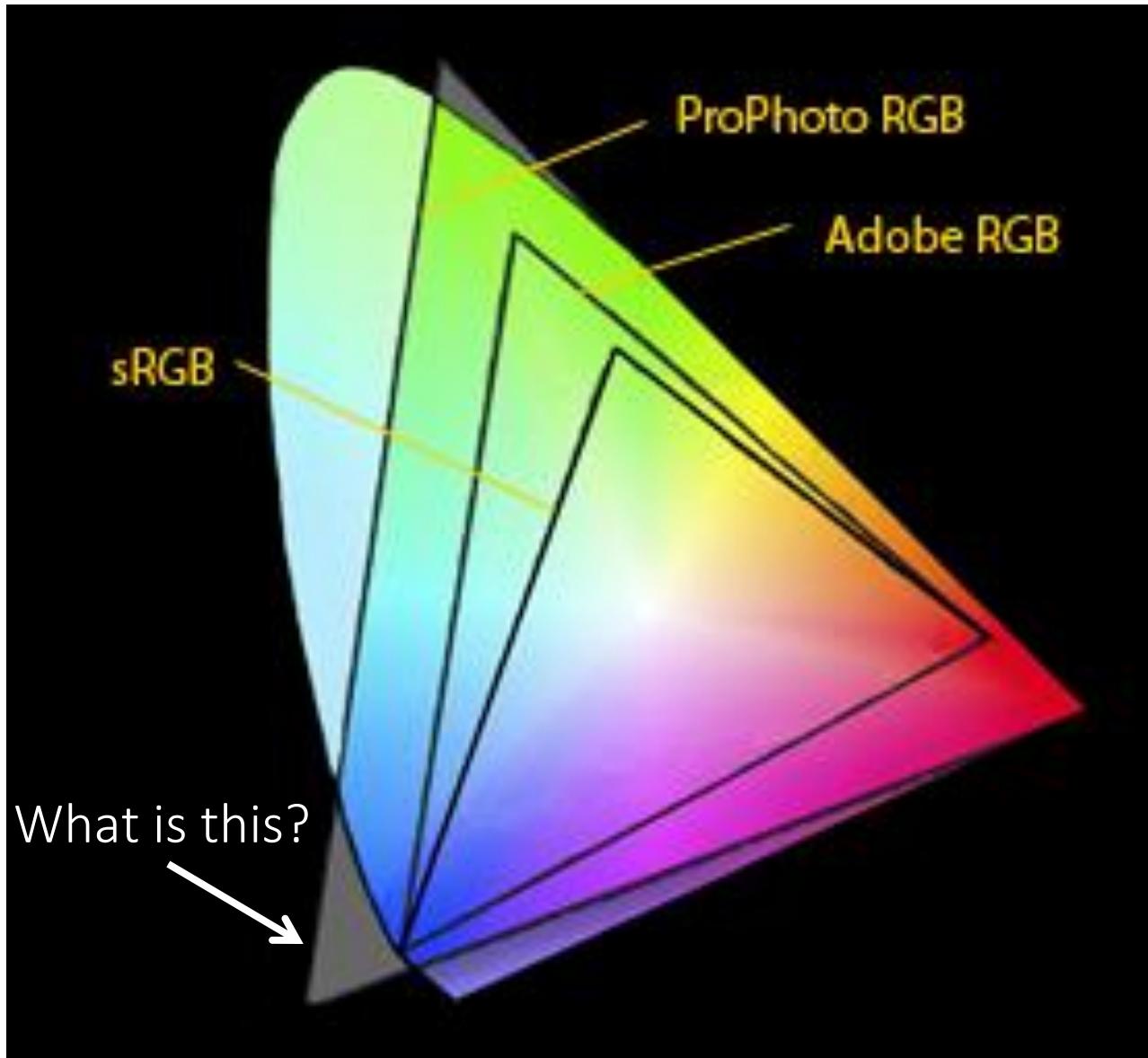
- What are the three triangle corners?
- What is the interior of the triangle?
- What is the exterior of the triangle?

# Color gamuts



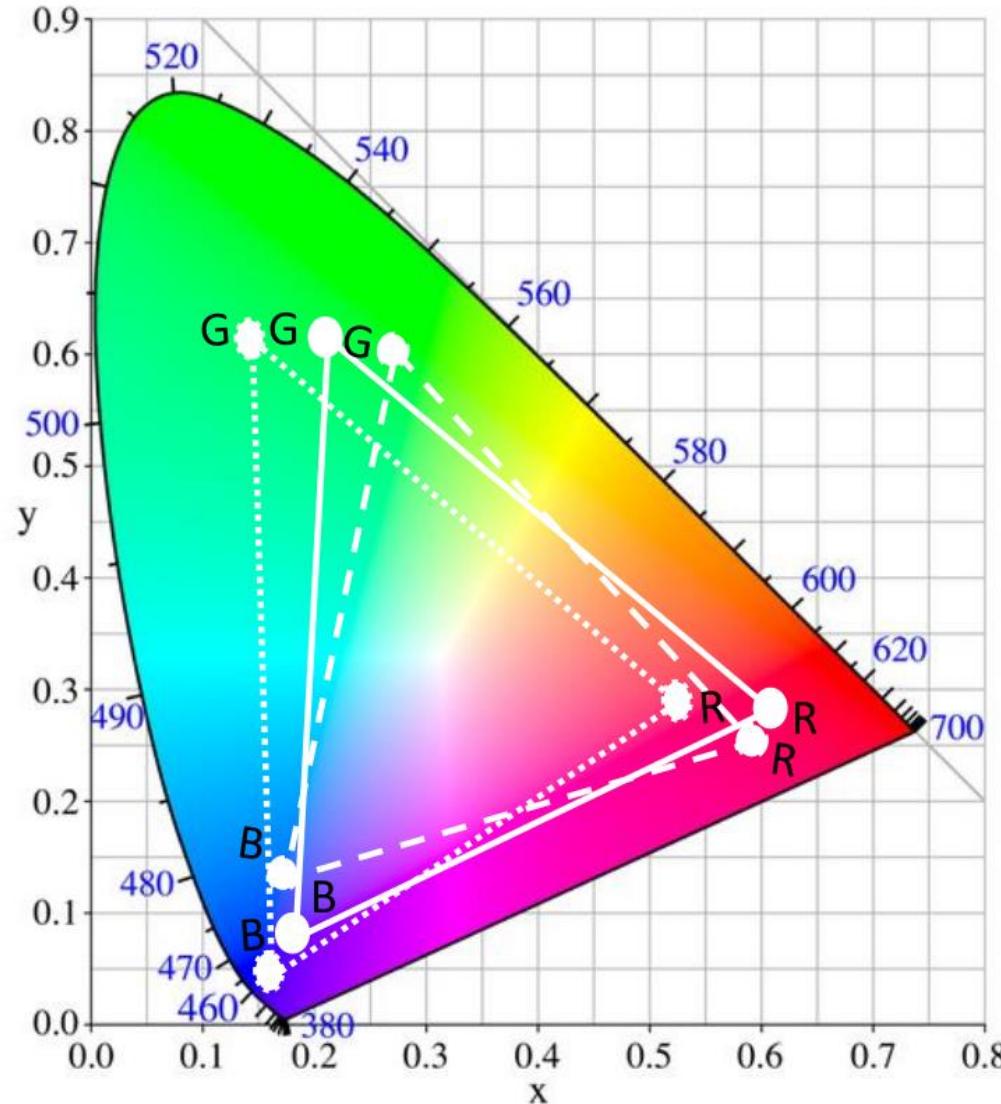
sRGB color gamut

# Color gamuts



Gamuts of various common industrial RGB spaces

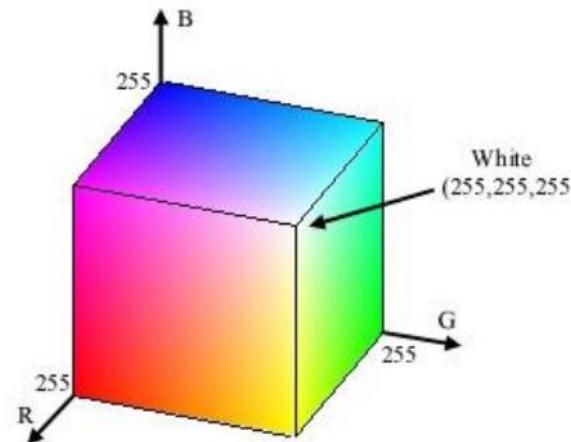
# The problem with RGBs visualized in chromaticity space



Device 1 —

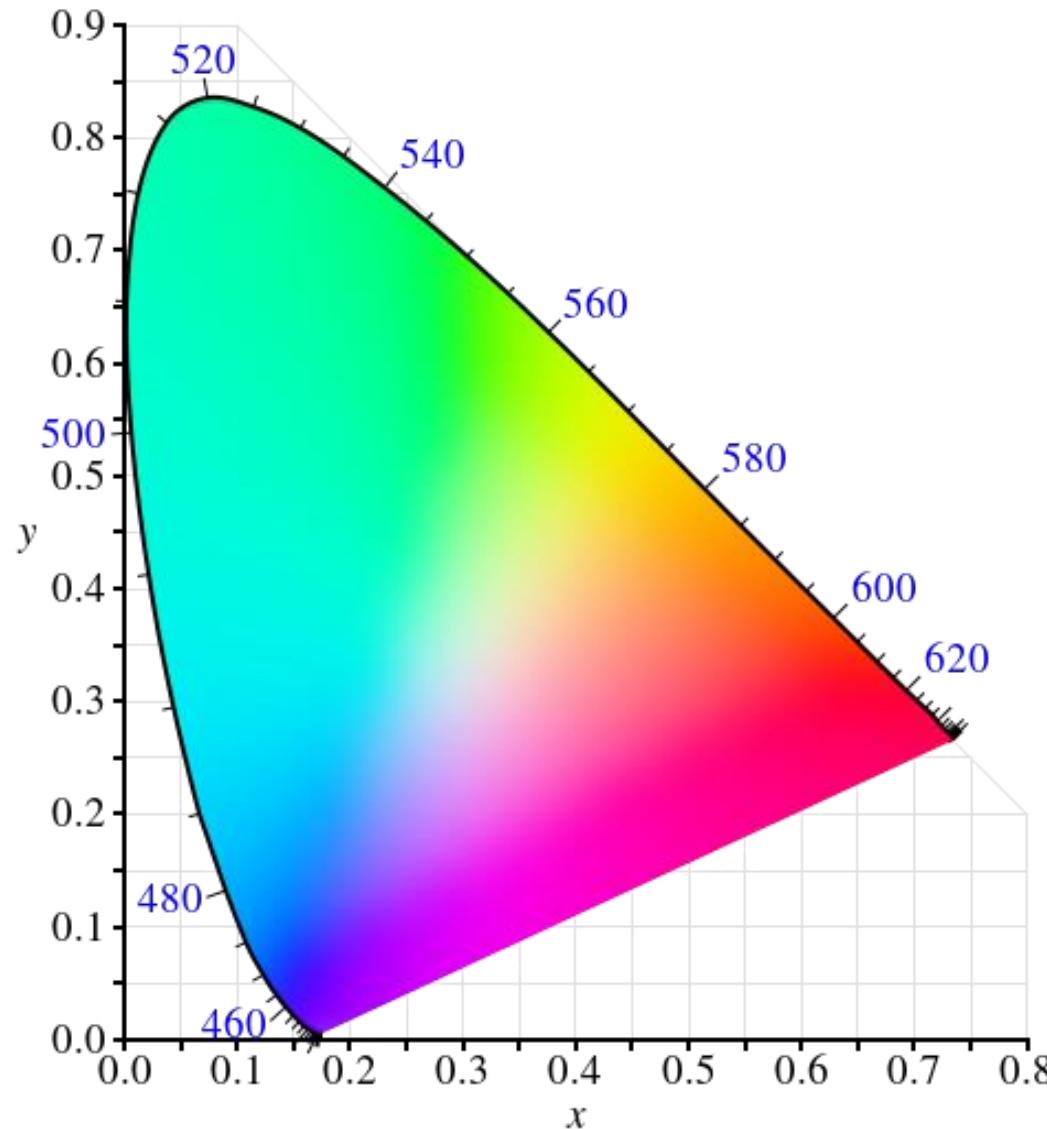
Device 2 .....

Device 3 - -



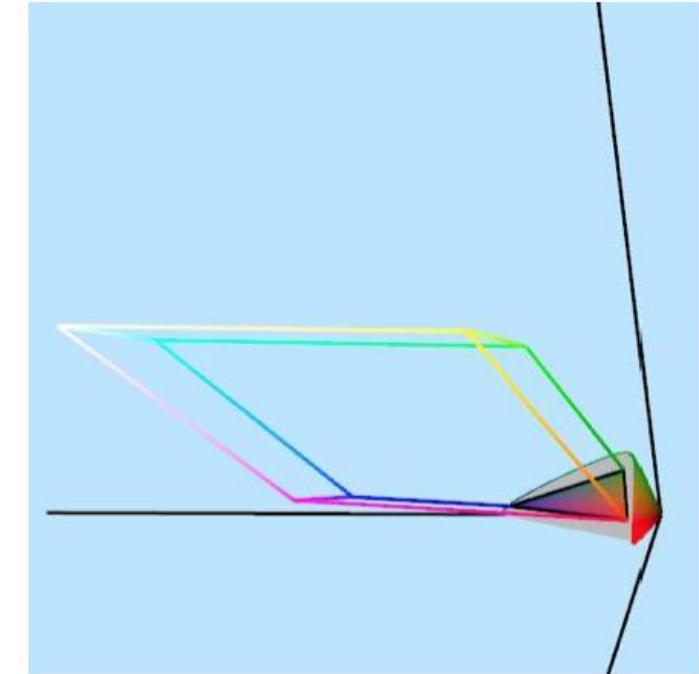
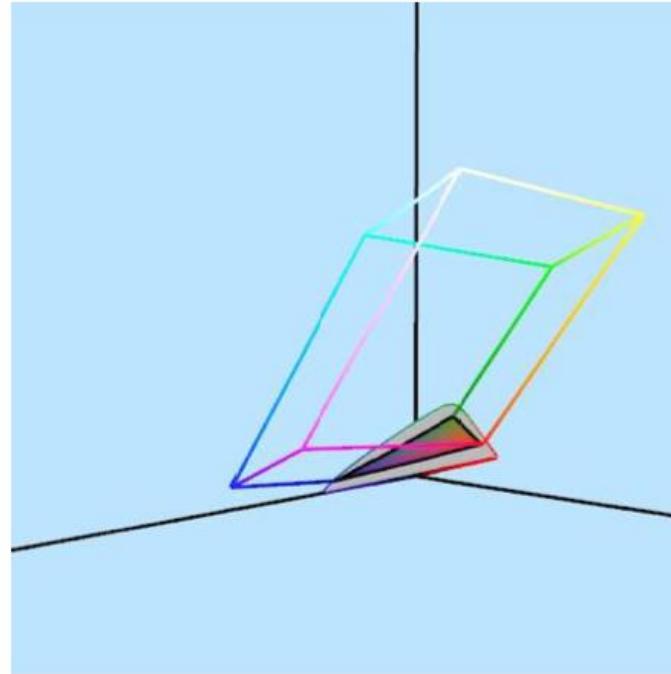
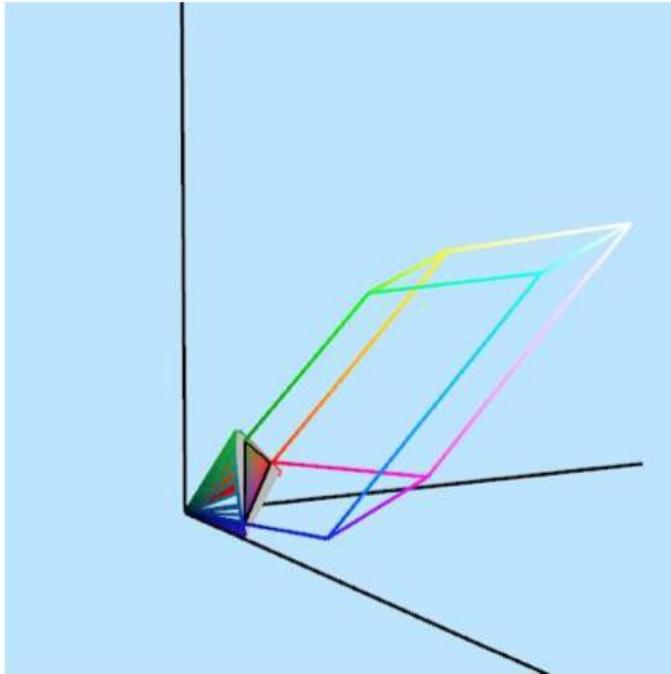
RGB values have no meaning if the primaries between devices are not the same!

# Color gamuts



- Can we create an RGB color space that reproduces the entire chromaticity diagram?
- What would be the pros and cons of such a color space?
- What devices would you use it for?

# Chromaticity diagrams can be misleading



Different gamuts may compare very differently when seen in full 3D retinal color space.

# Two views of retinal color

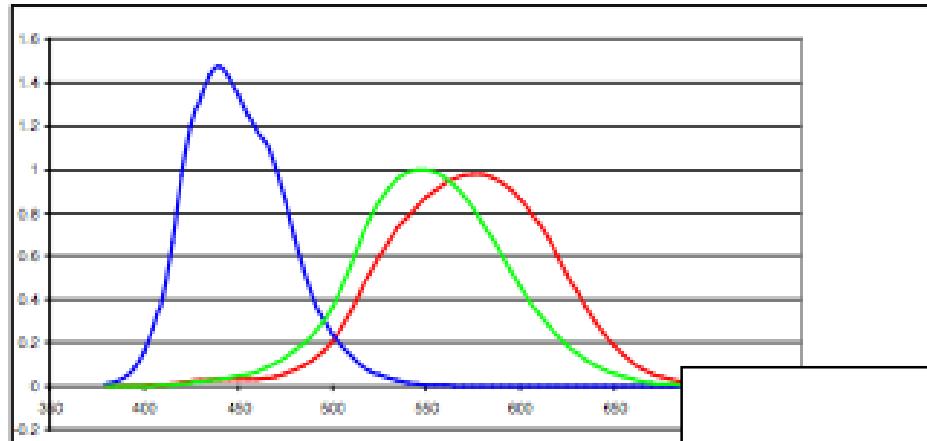
Analytic: Retinal color is three numbers formed by taking the dot product of a power spectral distribution with three color matching/sensitivity functions.

Synthetic: Retinal color is three numbers formed by assigning weights to three color primaries to match the perception of a power spectral distribution.

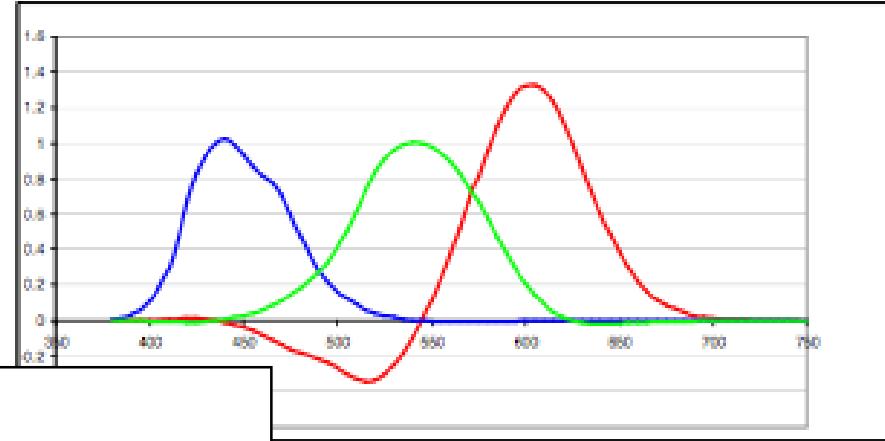
How would you make a color reproduction device?

# Non-linear color spaces

# A few important linear color spaces

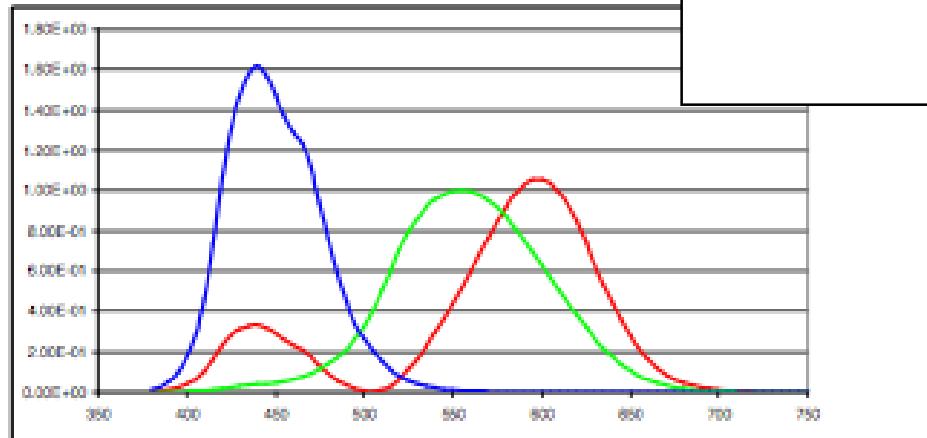


LMS color space

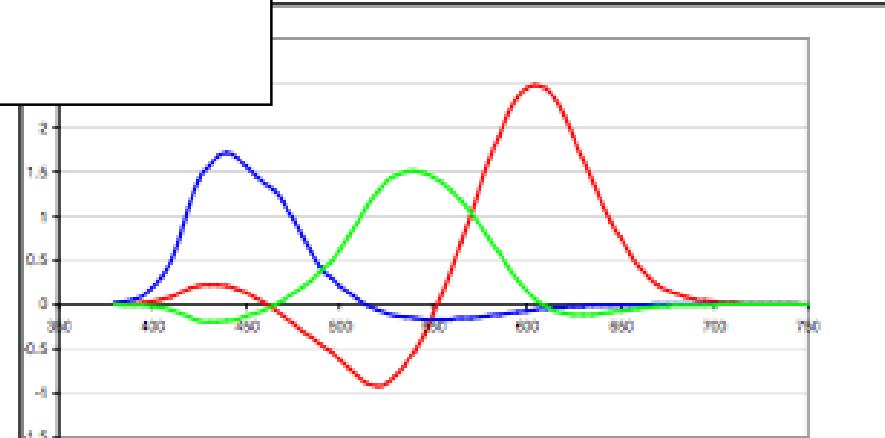


E RGB color space

What about non-linear color spaces?

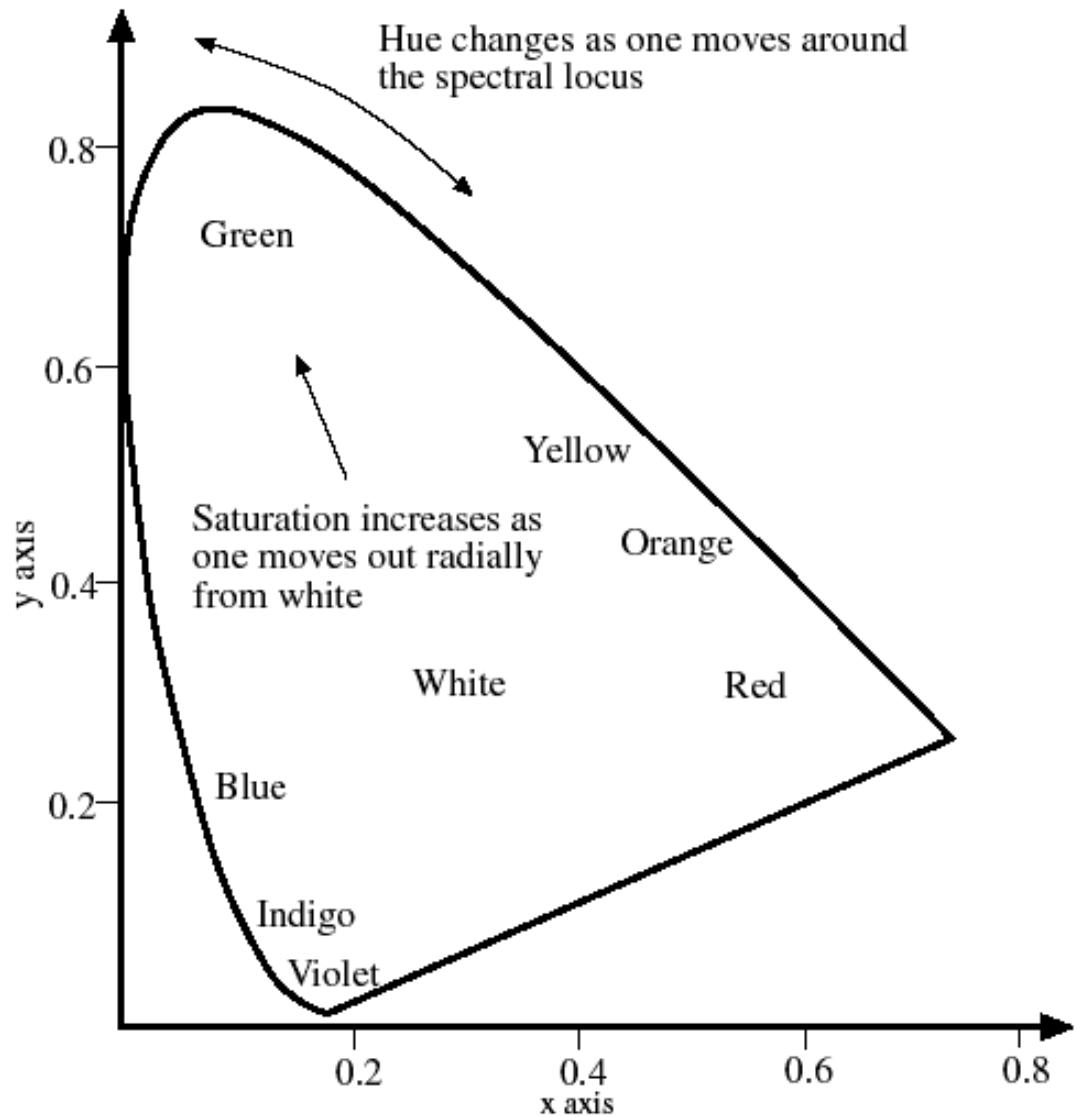


CIE XYZ color space



sRGB color space

# CIE xy (chromaticity)



$$x = \frac{X}{X + Y + Z}$$

$$y = \frac{Y}{X + Y + Z}$$

$$(X, Y, Z) \longleftrightarrow (\underline{x}, \underline{y}, \underline{Y})$$

chromaticity

luminance/brightness

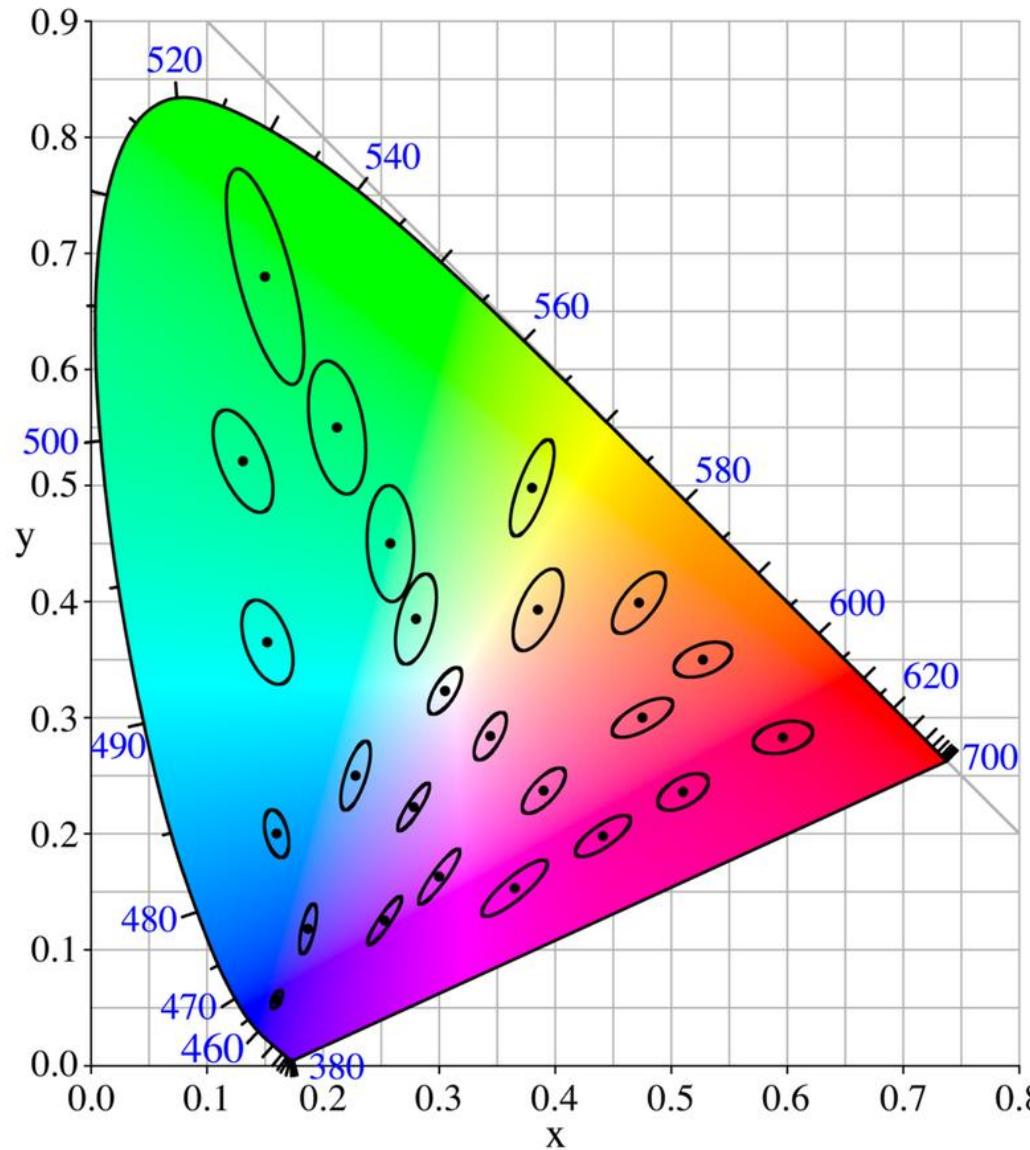
CIE xyY is a non-linear color space.

# Uniform color spaces

Find map  $F : \mathbb{R}^3 \rightarrow \mathbb{R}^3$  such that perceptual distance can be well approximated using Euclidean distance:

$$d(\vec{c}, \vec{c}') \approx \|F(\vec{c}) - F(\vec{c}')\|_2$$

# MacAdam ellipses

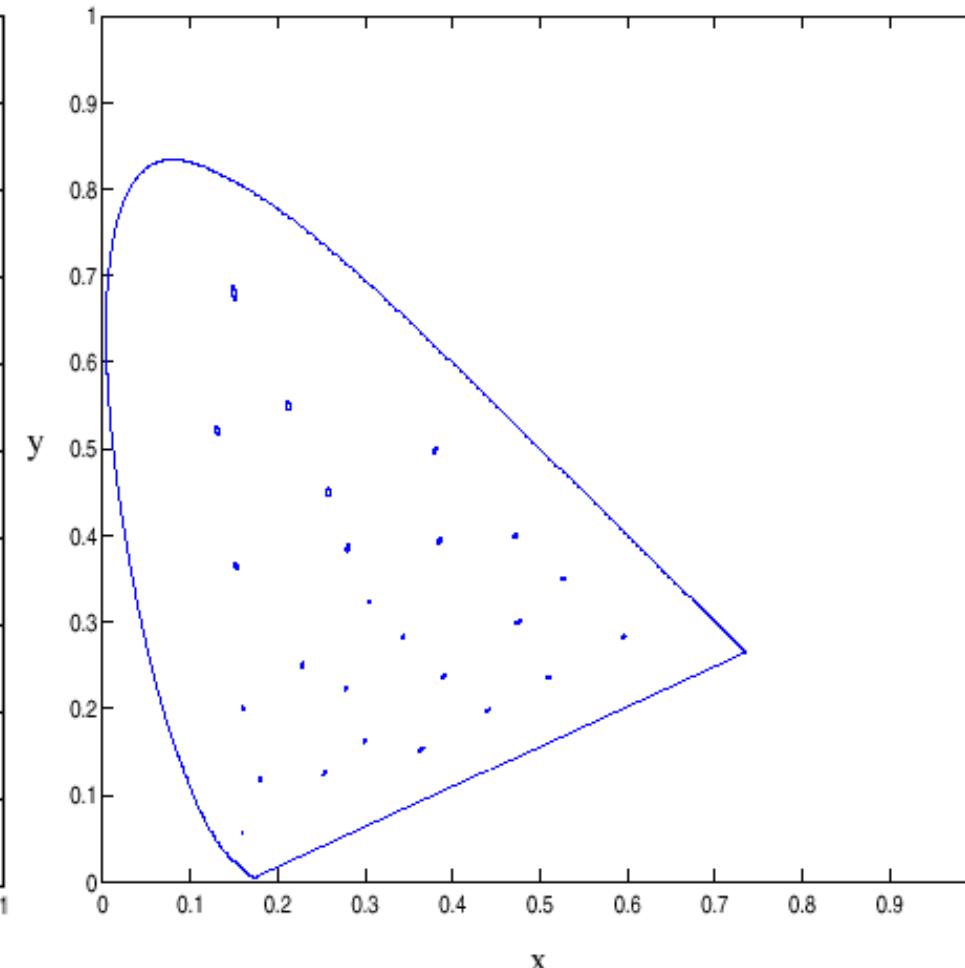
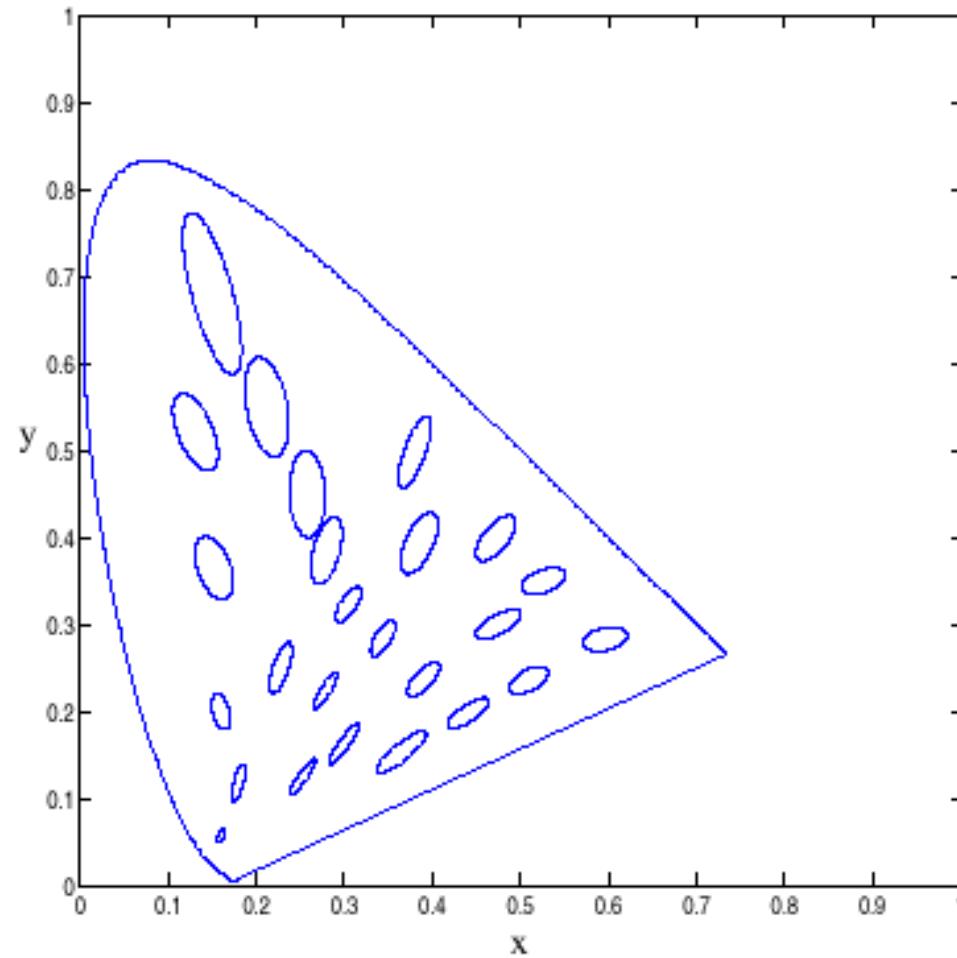


Areas in chromaticity space of imperceptible change:

- They are ellipses instead of circles.
- They change scale and direction in different parts of the chromaticity space.

# MacAdam ellipses

Note: MacAdam ellipses are almost always shown at 10x scale for visualization. In reality, the areas of imperceptible difference are much smaller.



# The Lab (aka L\*ab, aka L\*a\*b\*) color space

The L\* component of *lightness* is defined as

$$L^* = 116f\left(\frac{Y}{Y_n}\right), \quad (2.105)$$

where  $Y_n$  is the luminance value for nominal white (Fairchild 2005) and

$$f(t) = \begin{cases} t^{1/3} & t > \delta^3 \\ t/(3\delta^2) + 2\delta/3 & \text{else,} \end{cases} \quad (2.106)$$

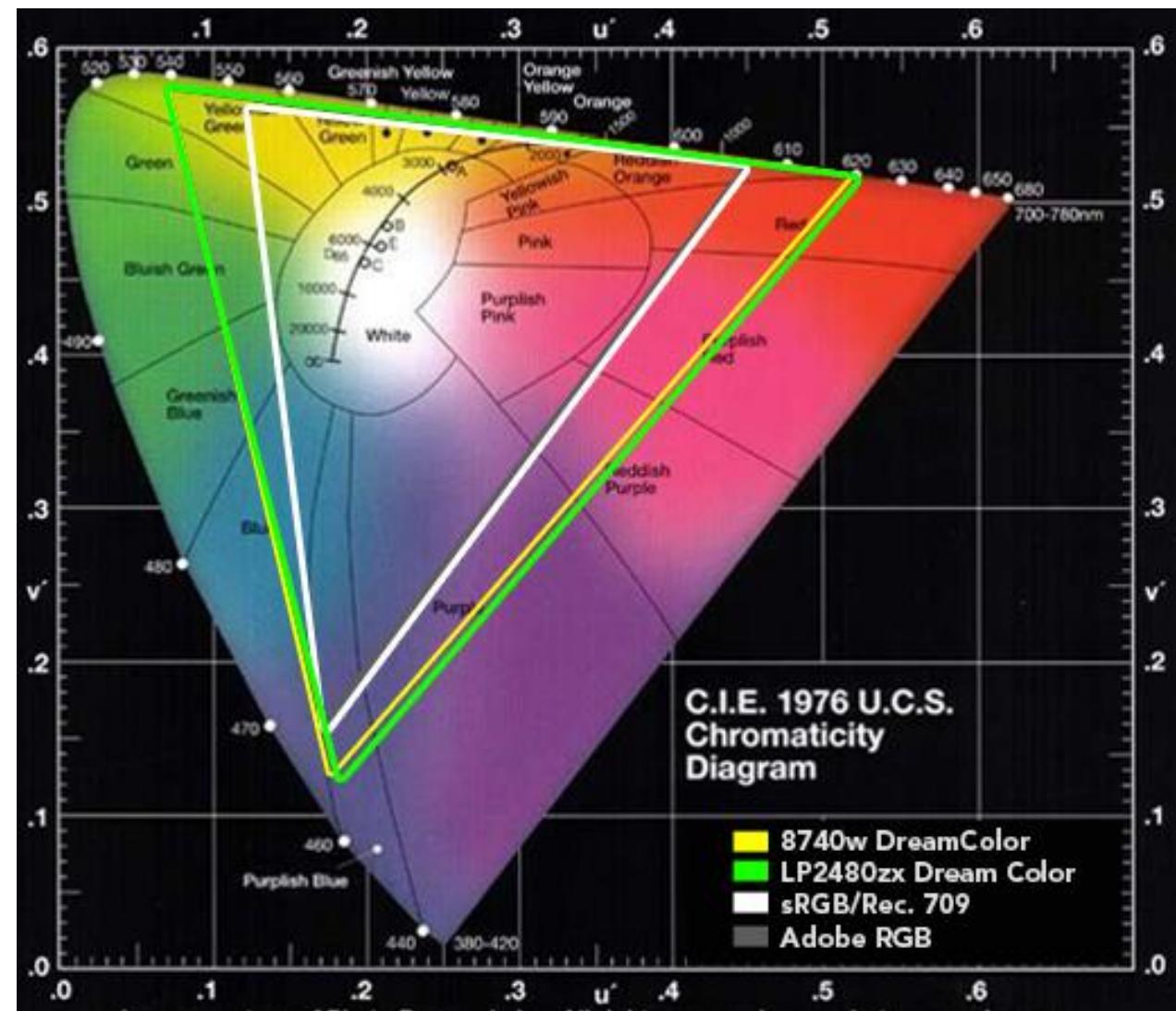
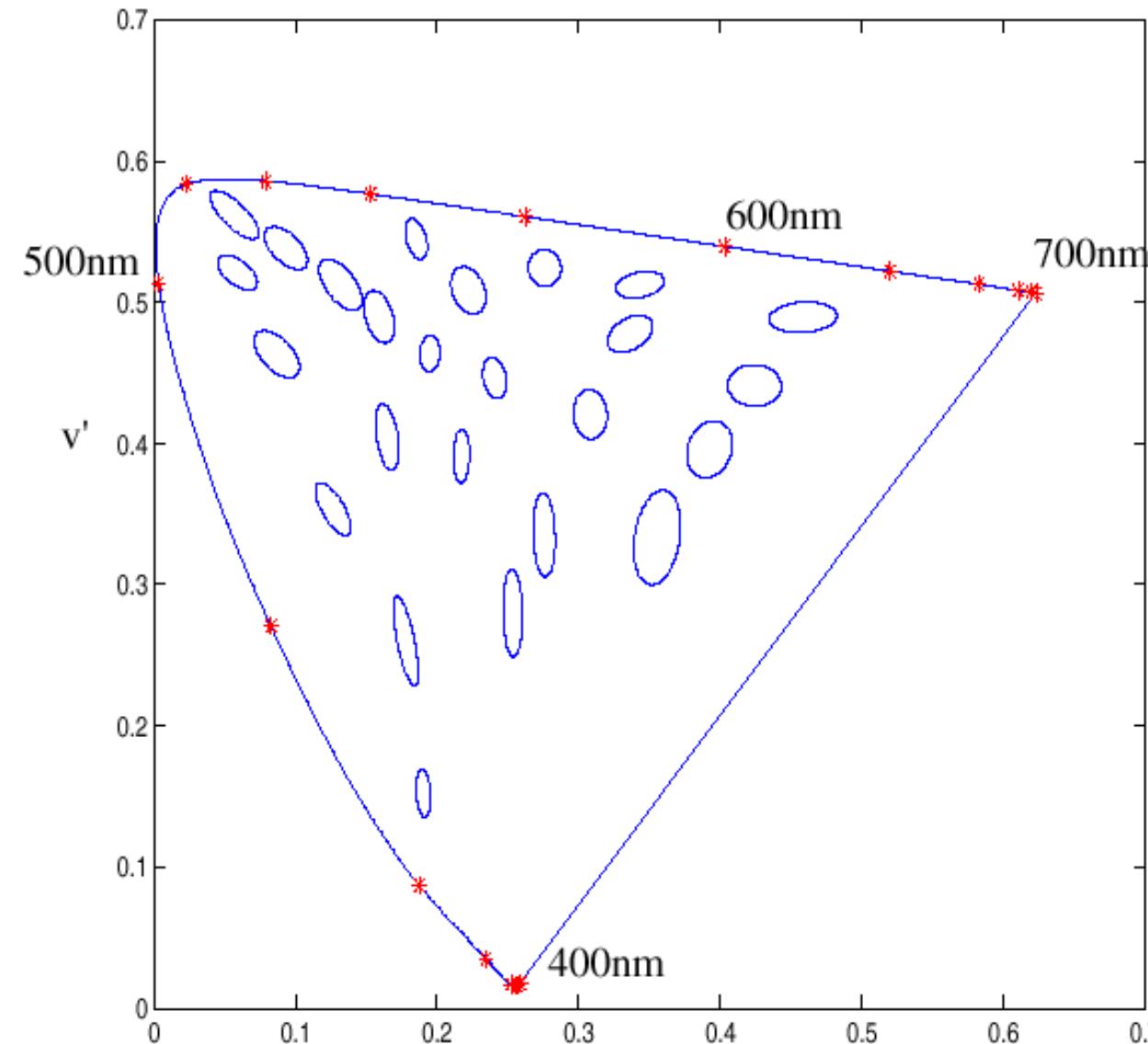
is a finite-slope approximation to the cube root with  $\delta = 6/29$ . The resulting 0...100 scale roughly measures equal amounts of lightness perceptibility.

In a similar fashion, the a\* and b\* components are defined as

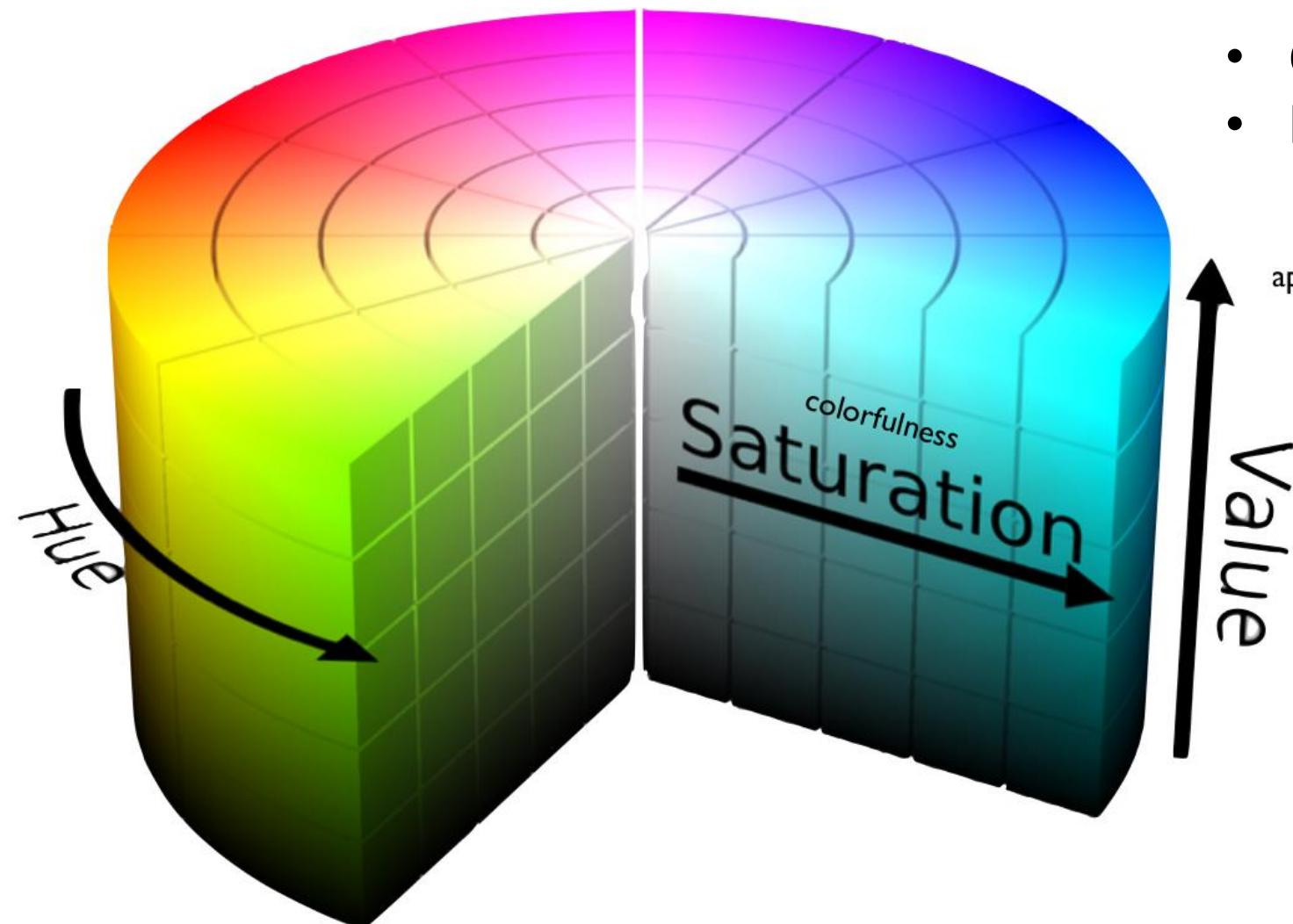
$$a^* = 500 \left[ f\left(\frac{X}{X_n}\right) - f\left(\frac{Y}{Y_n}\right) \right] \text{ and } b^* = 200 \left[ f\left(\frac{Y}{Y_n}\right) - f\left(\frac{Z}{Z_n}\right) \right], \quad (2.107)$$

where again,  $(X_n, Y_n, Z_n)$  is the measured white point. Figure 2.32i–k show the L\*a\*b\* representation for a sample color image.

# The Lab (aka L\*ab, aka L\*a\*b\*) color space



# Hue, saturation, and value



Do not use color space HSV! Use LCh:

- $L^*$  for “value”.
- $C = \sqrt{a^2 + b^2}$  for “saturation” (chroma).
- $h = \text{atan}(b / a)$  for “hue”.

How could you make an image like this from a color image?

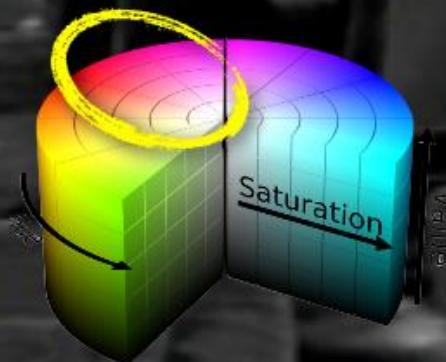


How could you make an image like this from a color image?

Zero saturation

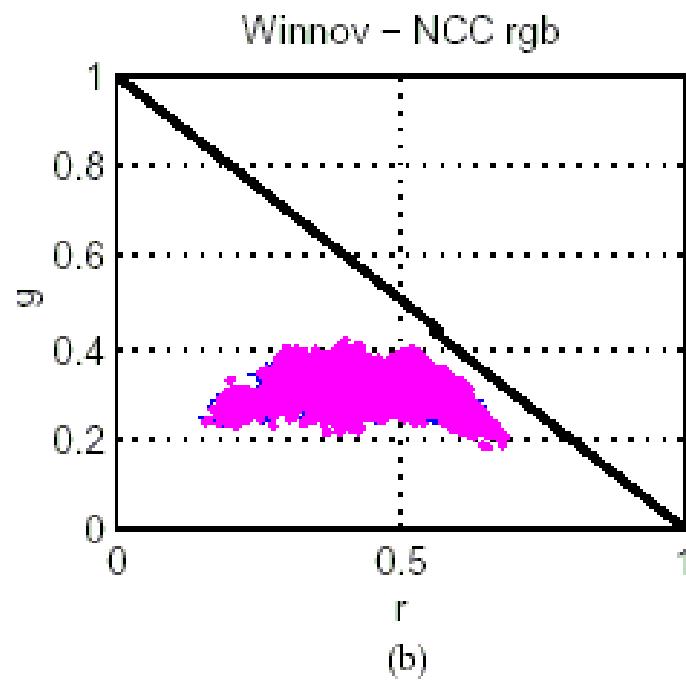
Higher saturation

Control saturation with red-pass filter

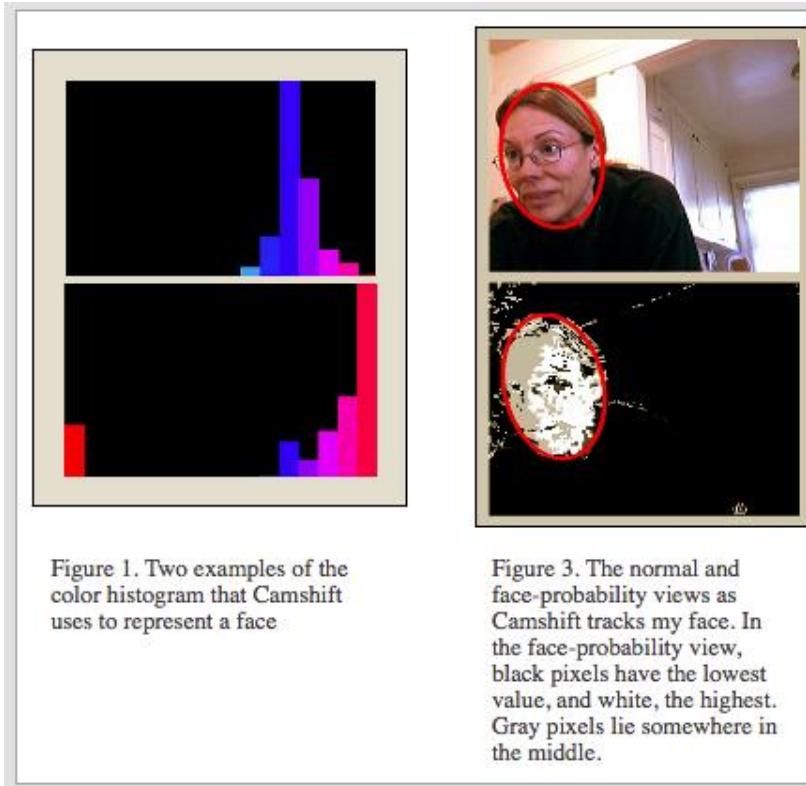


LCh  
Easier to do color processing in ~~HSV~~

# Chromaticity: Human skin



# Useful for detecting faces

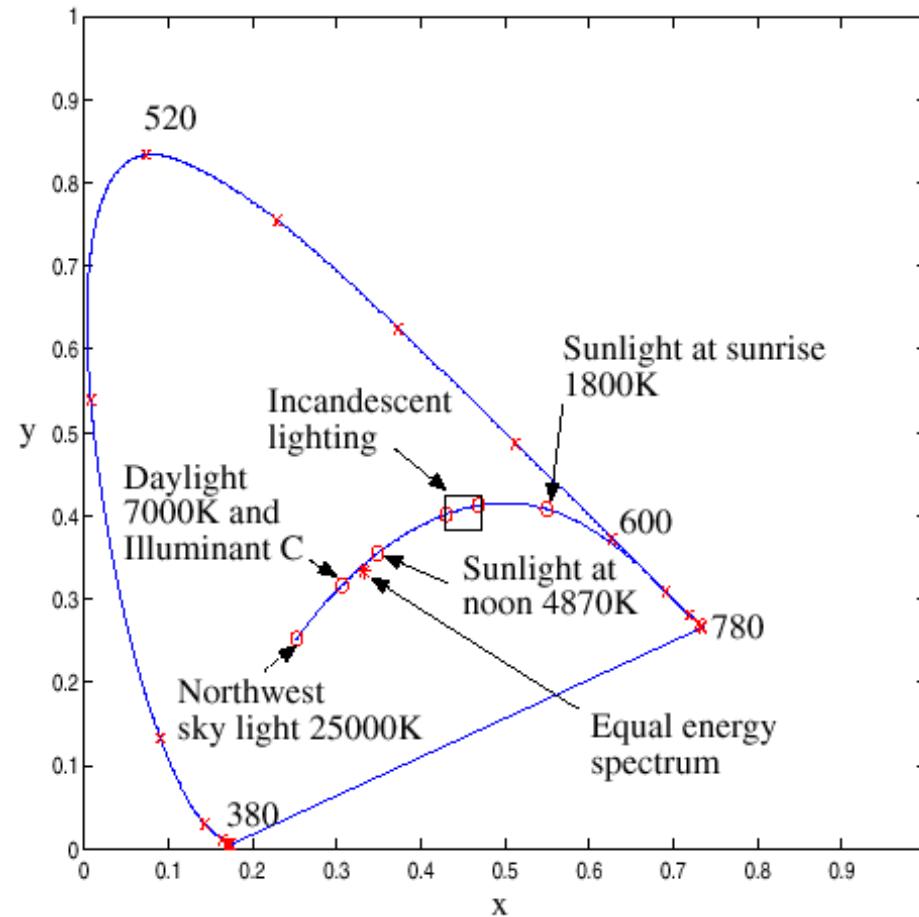


“How OpenCV's Face Tracker Works”  
-SERVO Magazine, March 2007

# Application: Shadow removal

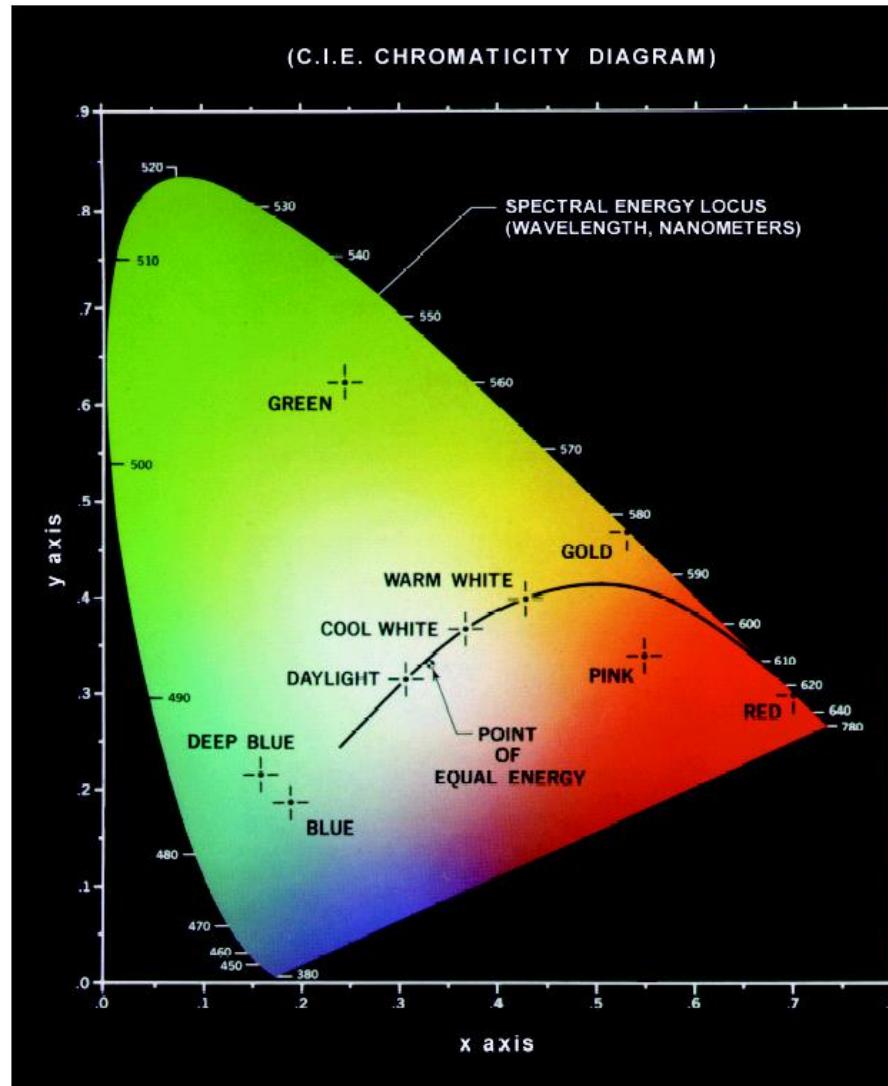


# Application: Shadow removal



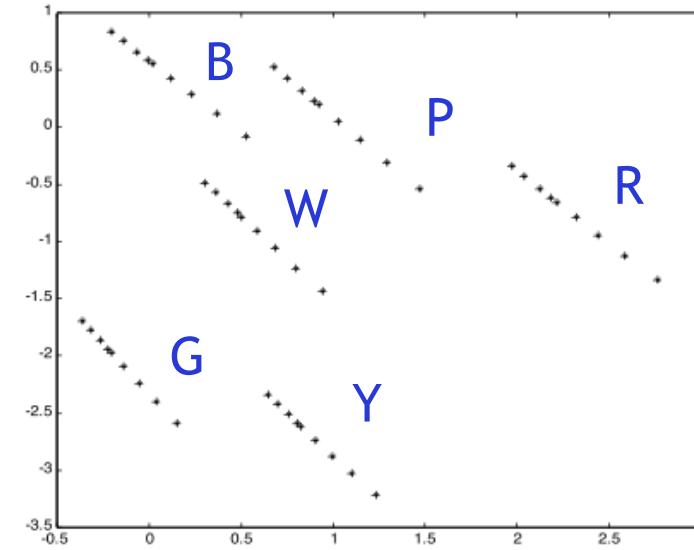
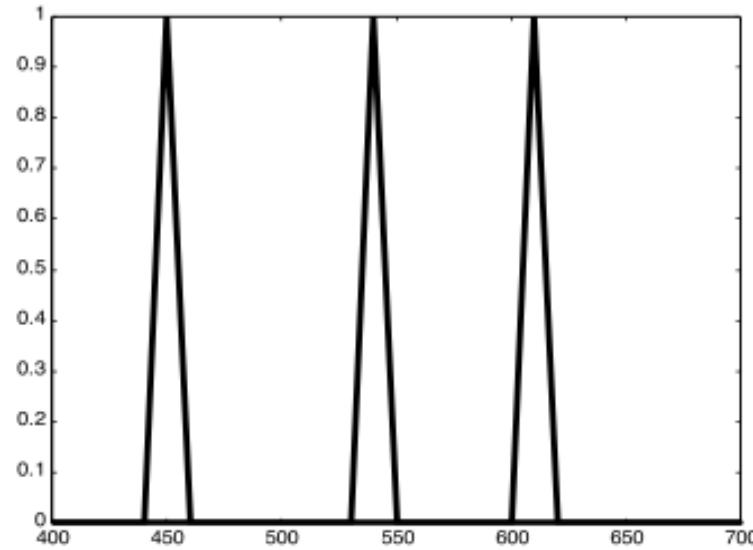
# Application: Shadow removal

**FIGURE 6.5**  
Chromaticity  
diagram.  
(Courtesy of the  
General Electric  
Co., Lamp  
Business  
Division.)



# Application: Shadow removal

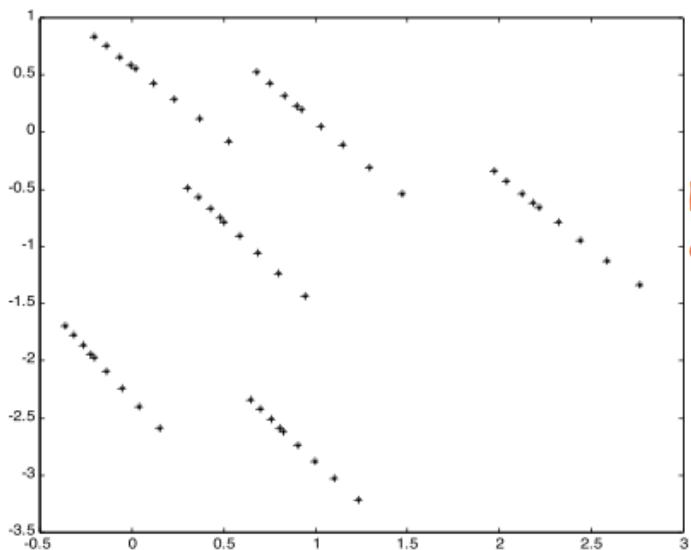
Narrow-band  
(delta-function  
sensitivities)



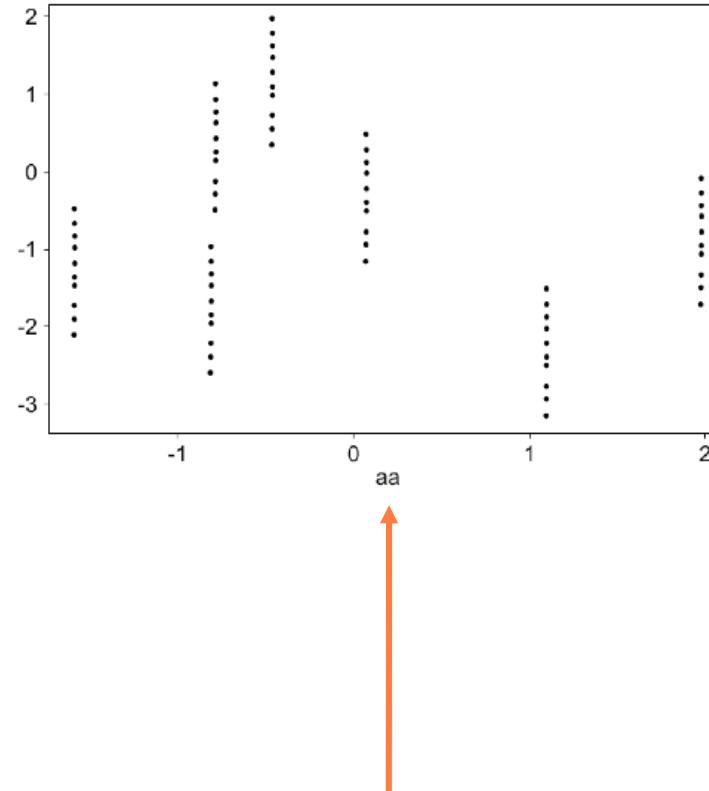
Log-opponent  
chromaticities for 6  
surfaces under 9  
lights

# Application: Shadow removal

Log-opponent  
chromaticities for 6  
surfaces under 9  
lights



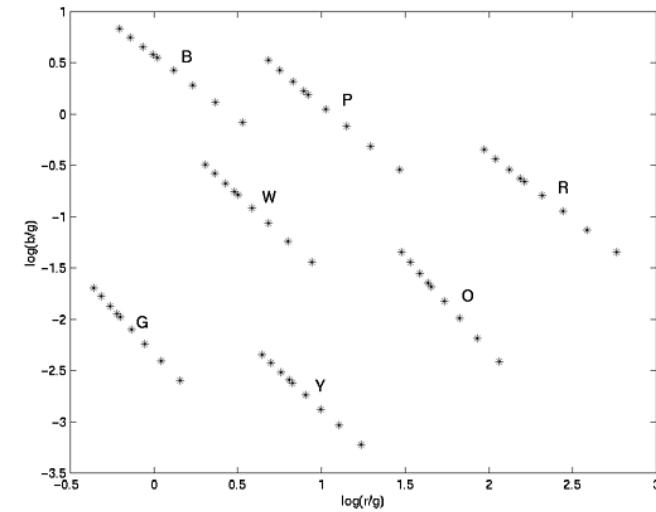
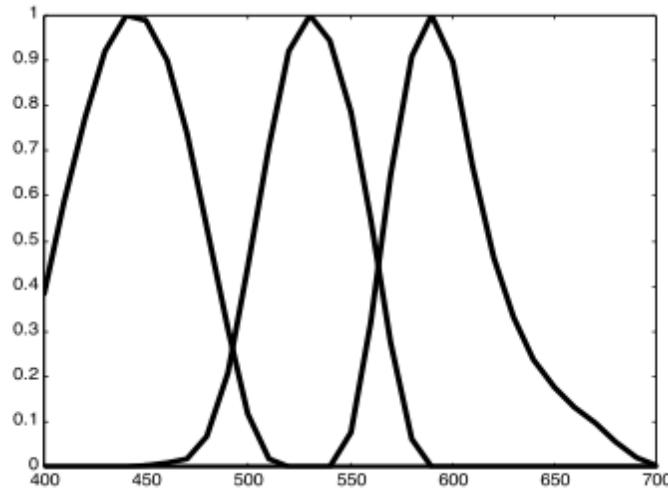
Rotate  
chromaticities



This axis is invariant to  
illuminant colour

# Application: Shadow removal

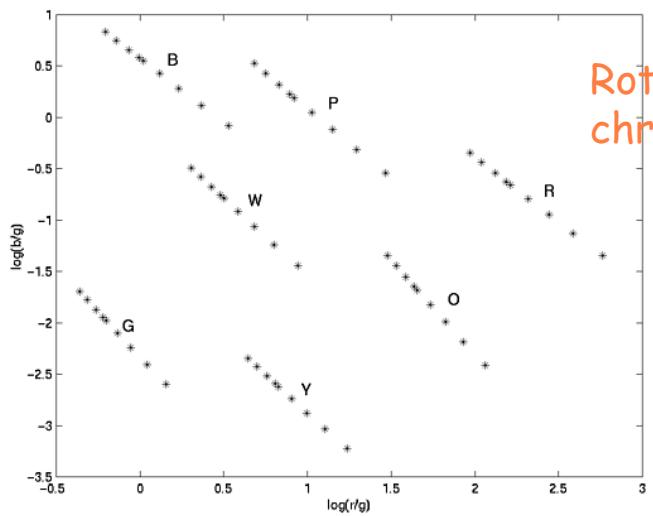
Normalized sensitivities of a SONY DXC-930 video camera



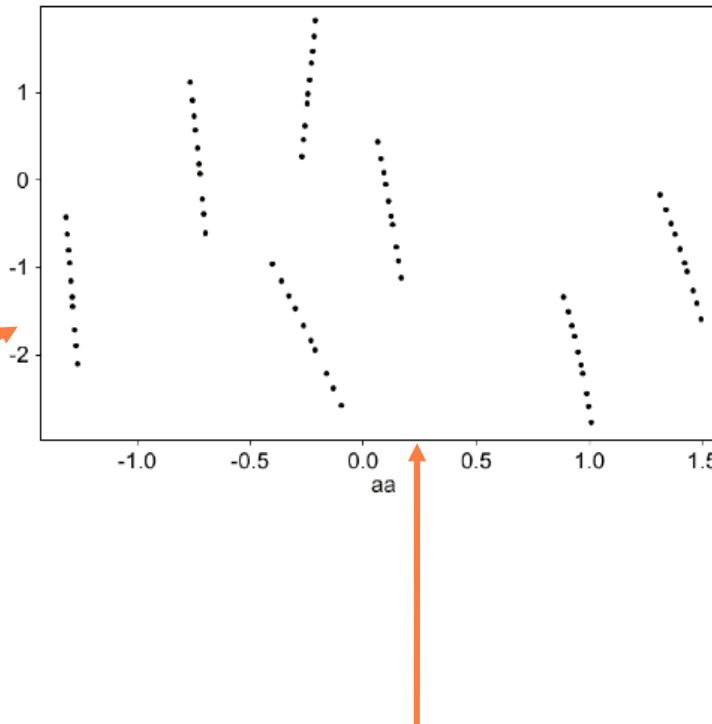
Log-opponent chromaticities for 6 surfaces under 9 different lights

# Application: Shadow removal

Log-opponent chromaticities for 6 surfaces under 9 different lights



Rotate chromaticities



The invariant axis is now only approximately illuminant invariant (but hopefully good enough)

# Application: Shadow removal

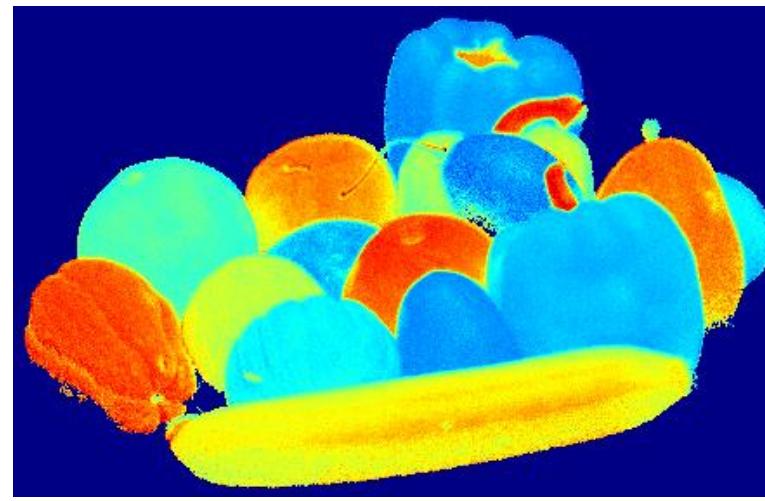


## Application: Invariance for material segmentation

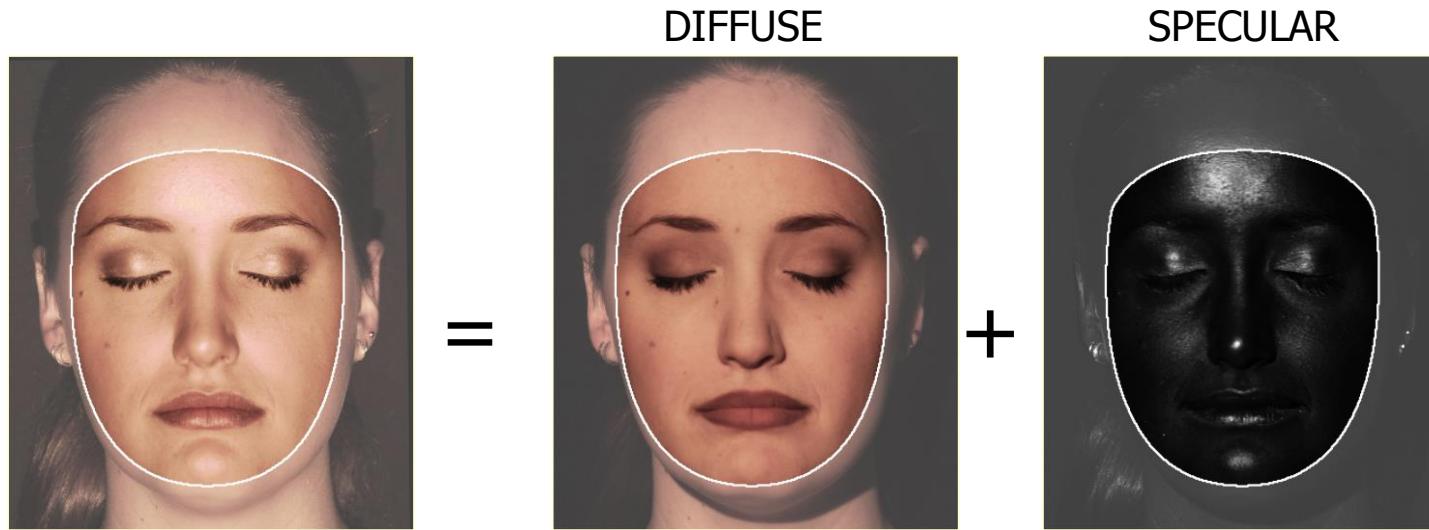
Input image



Hue



# Application: highlight removal



Problem: This is hard when the diffuse color is spatially-varying

$$C_{RGB}(x) = g_d(x)D_{RGB}(x) + g_s(x)S_{RGB}$$

# Teaser for Homework 4



# References

Basic reading:

- Szeliski textbook, Section 2.3.2, 3.1.2.
- Gortler textbook, Chapter 19.
- Michael Brown, “Understanding the In-Camera Image Processing Pipeline for Computer Vision,” CVPR 2016, very detailed discussion of issues relating to color photography and management, slides available at: [http://www.comp.nus.edu.sg/~brown/CVPR2016\\_Brown.html](http://www.comp.nus.edu.sg/~brown/CVPR2016_Brown.html)

Additional reading:

- Reinhard et al., “Color Imaging: Fundamentals and Applications,” A.K Peters/CRC Press 2008.
- Koenderink, “Color Imaging: Fundamentals and Applications,” MIT Press 2010.
- Fairchild, “Color Appearance Models,” Wiley 2013.  
all of the above books are great references on color photography, reproduction, and management.