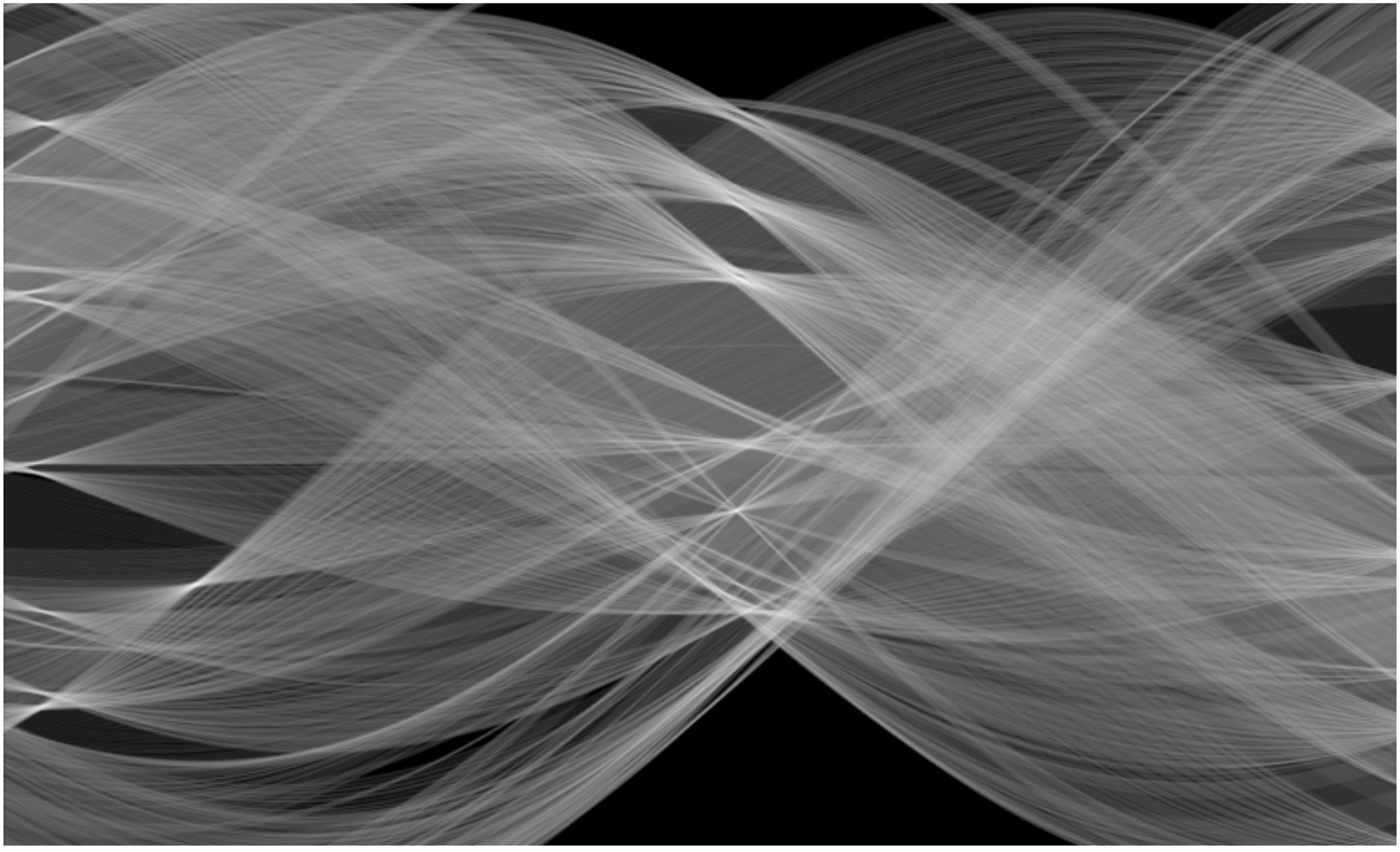


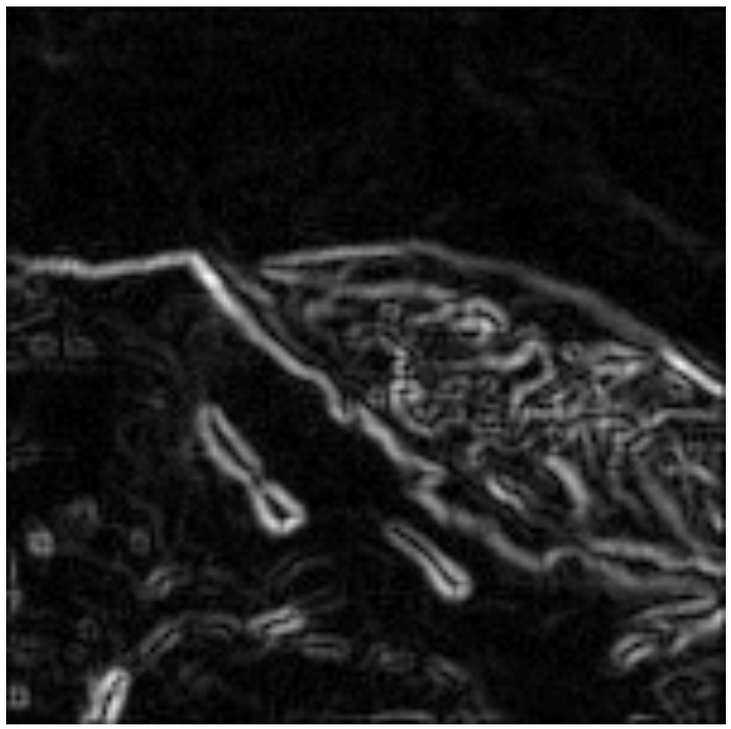
Hough Transform

16-385 Computer Vision (Kris Kitani)

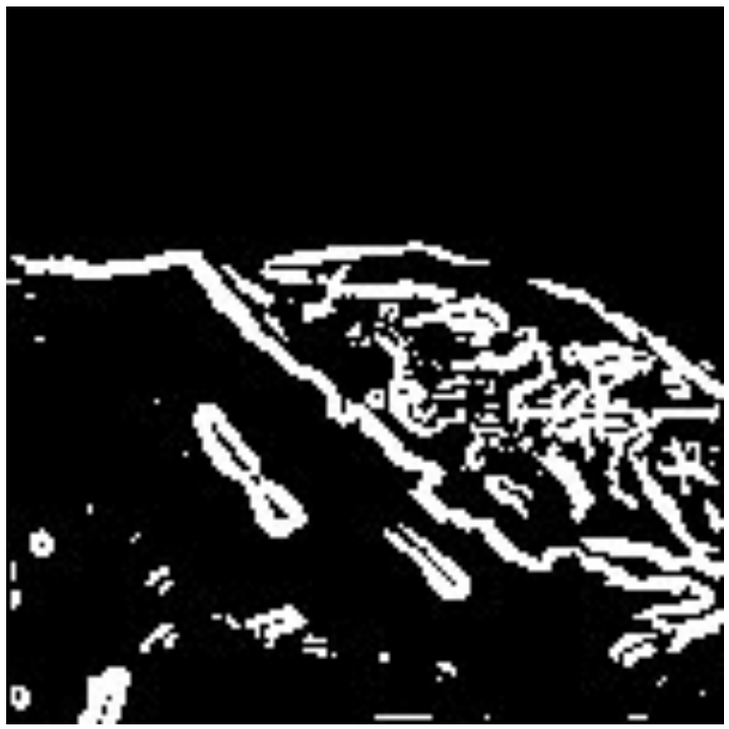
Carnegie Mellon University

motivation

Original image



Edge detection



Thresholding

How do we find image boundaries (lines)?

Hough Transform

- Generic framework for detecting a shape/object
- Edges don’t have to be connected
- Lines can be occluded
- Key idea: edges **vote** for the possible models

Image and parameter space

variables

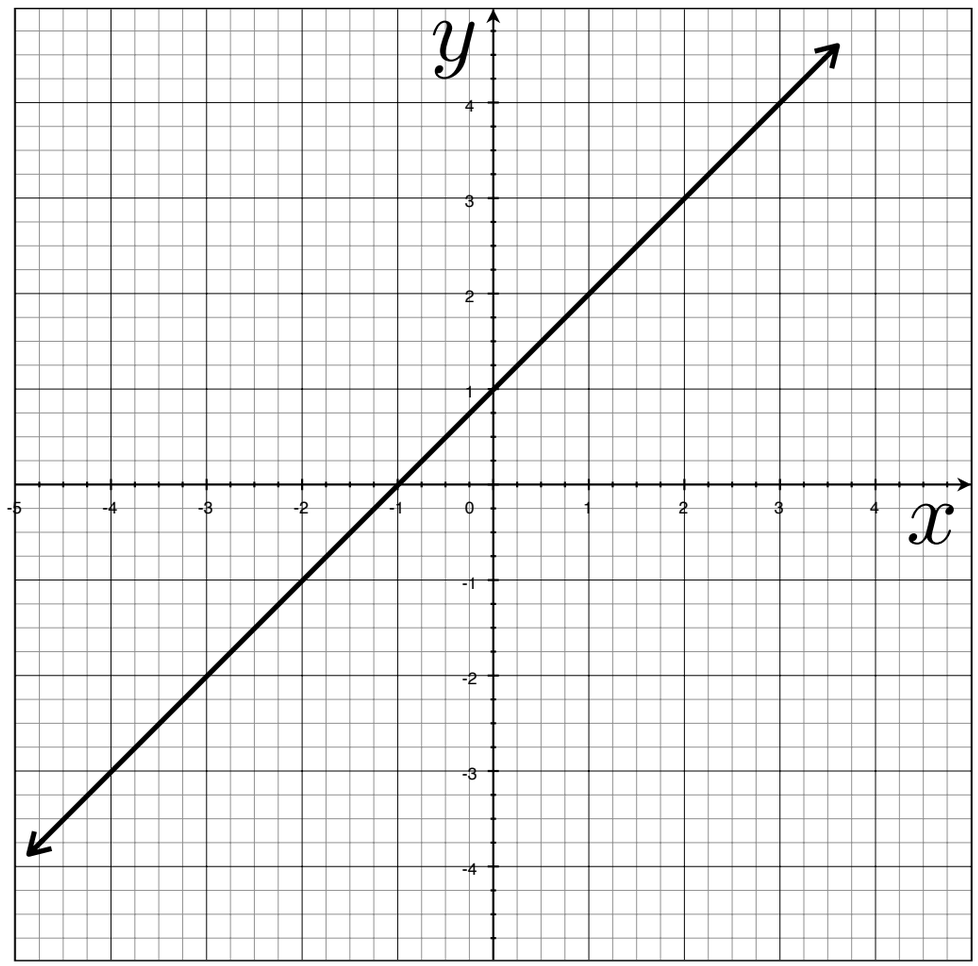
$$y = mx + b$$

parameters

variables

$$y - mx = b$$

parameters



a line becomes a point

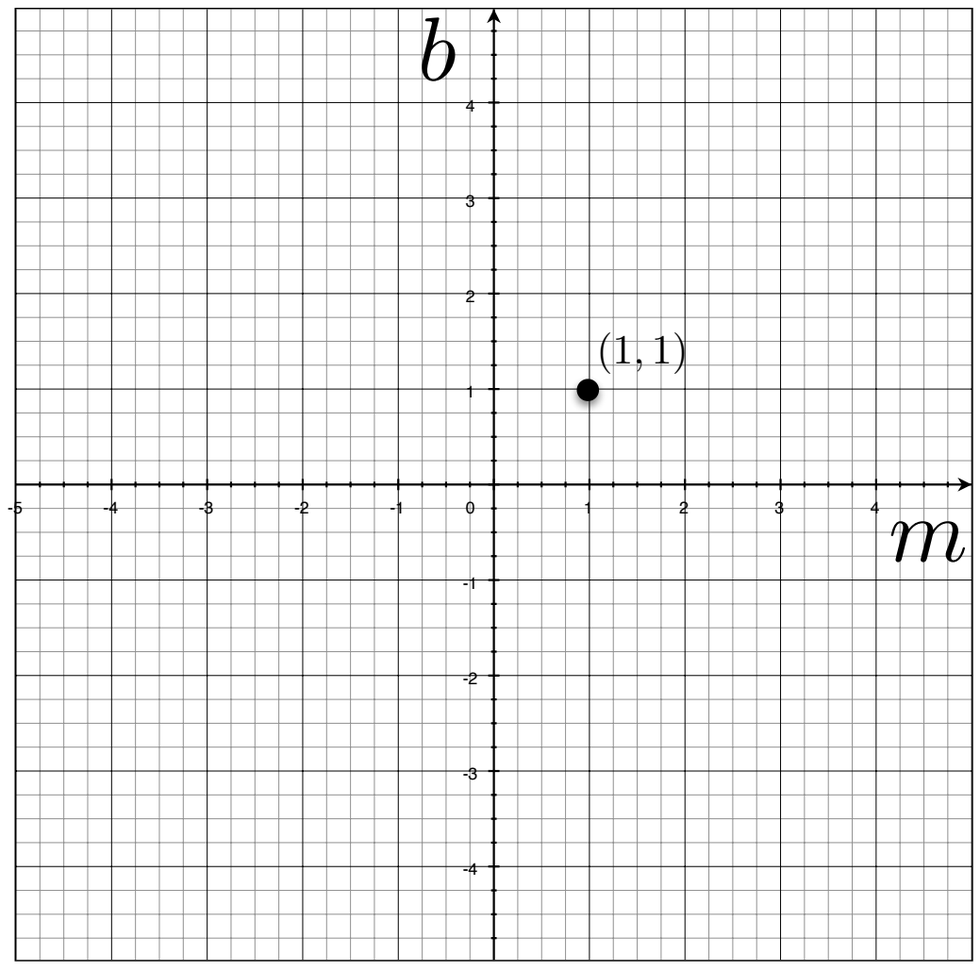


Image space

Parameter space

Image and parameter space

variables

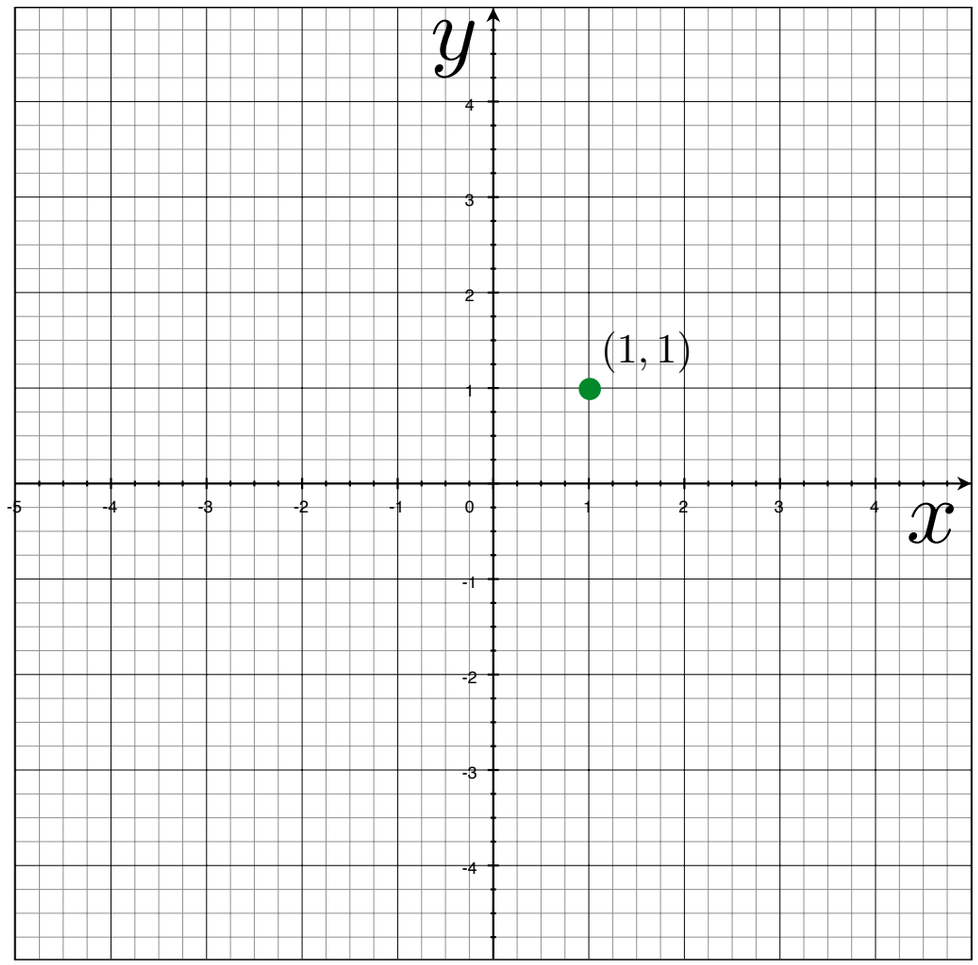
$$y = mx + b$$

parameters

variables

$$y - mx = b$$

parameters



a point becomes a line

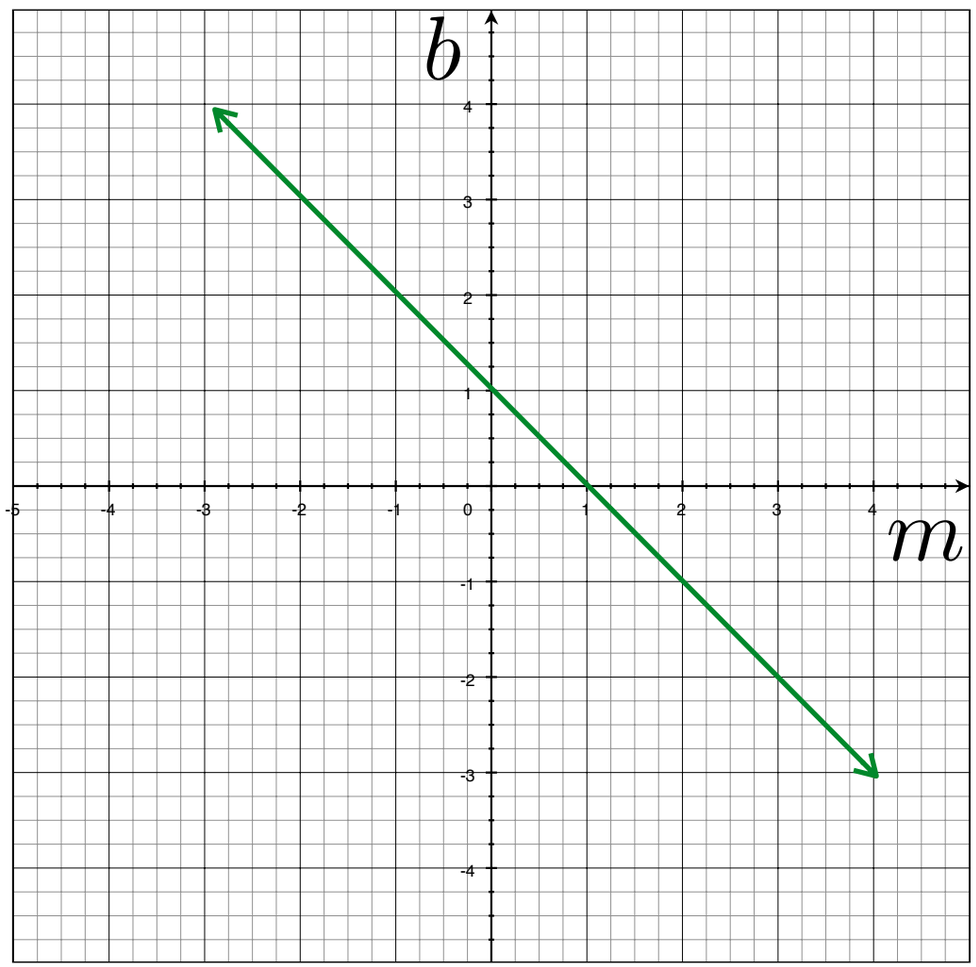


Image space

Parameter space

Image and parameter space

variables

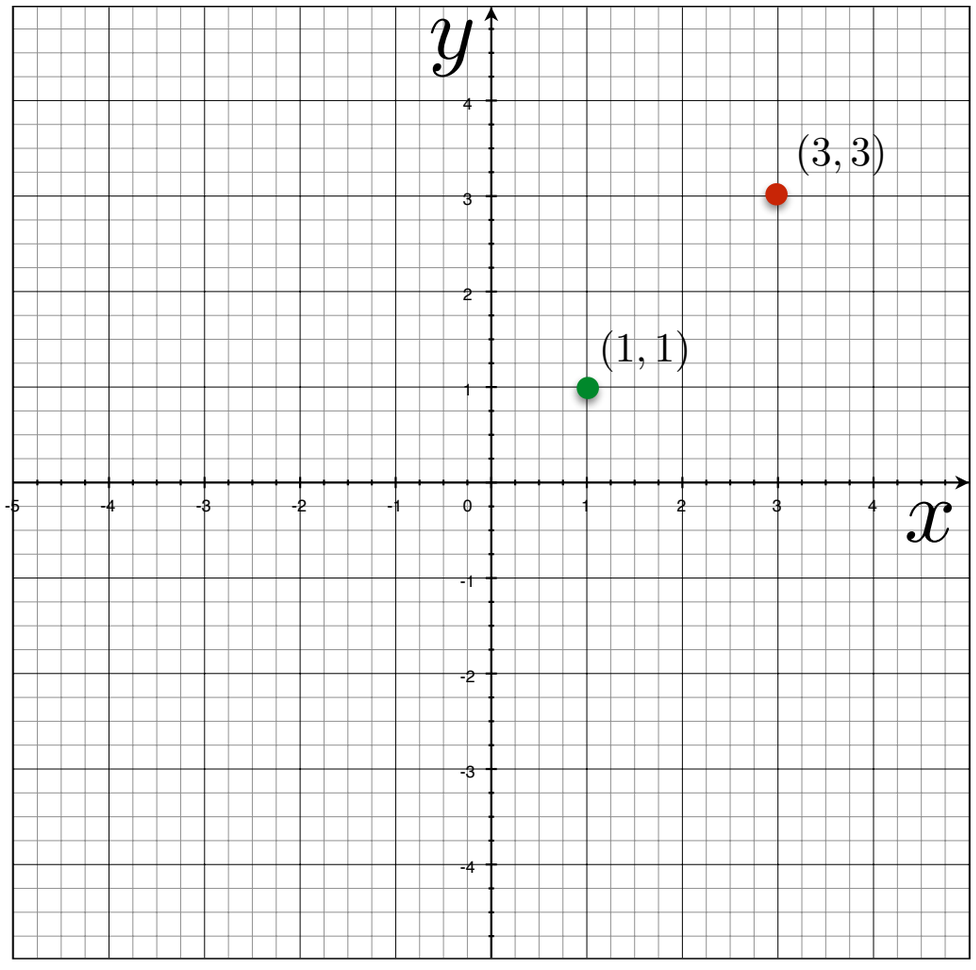
$$y = mx + b$$

parameters

variables

$$y - mx = b$$

parameters



two points become ?

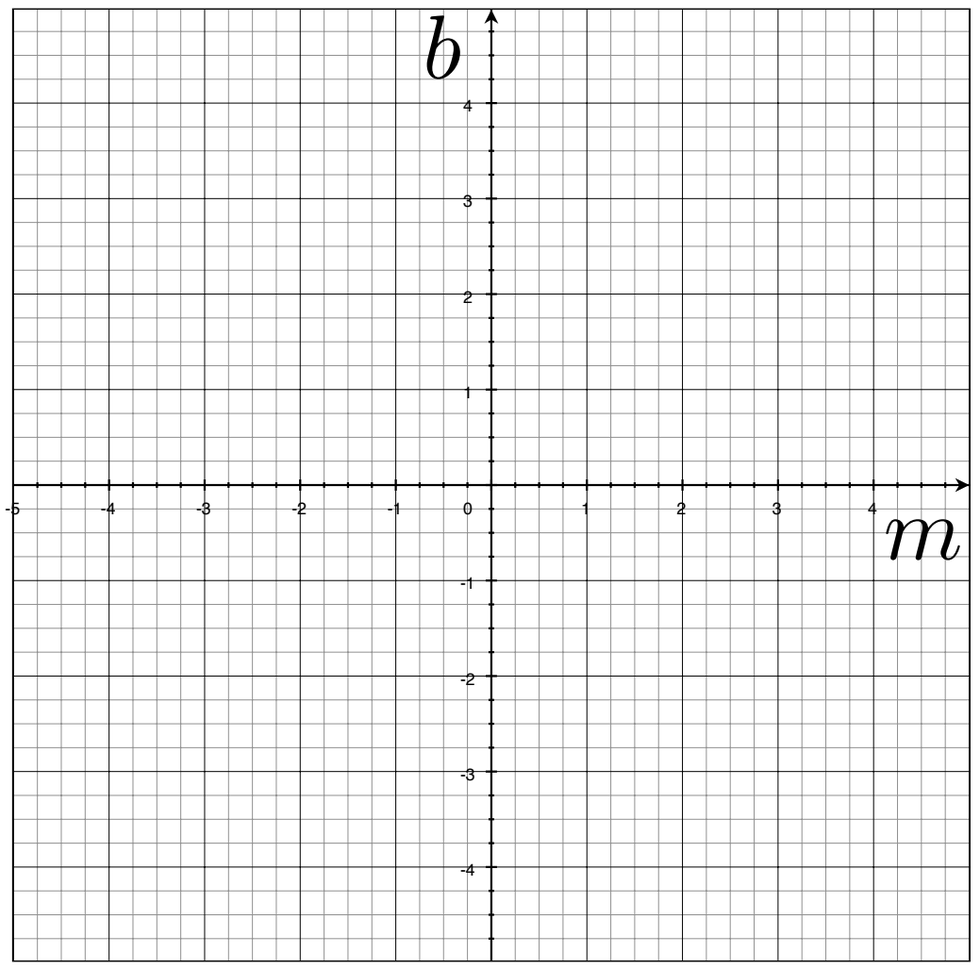


Image space

Parameter space

Image and parameter space

variables

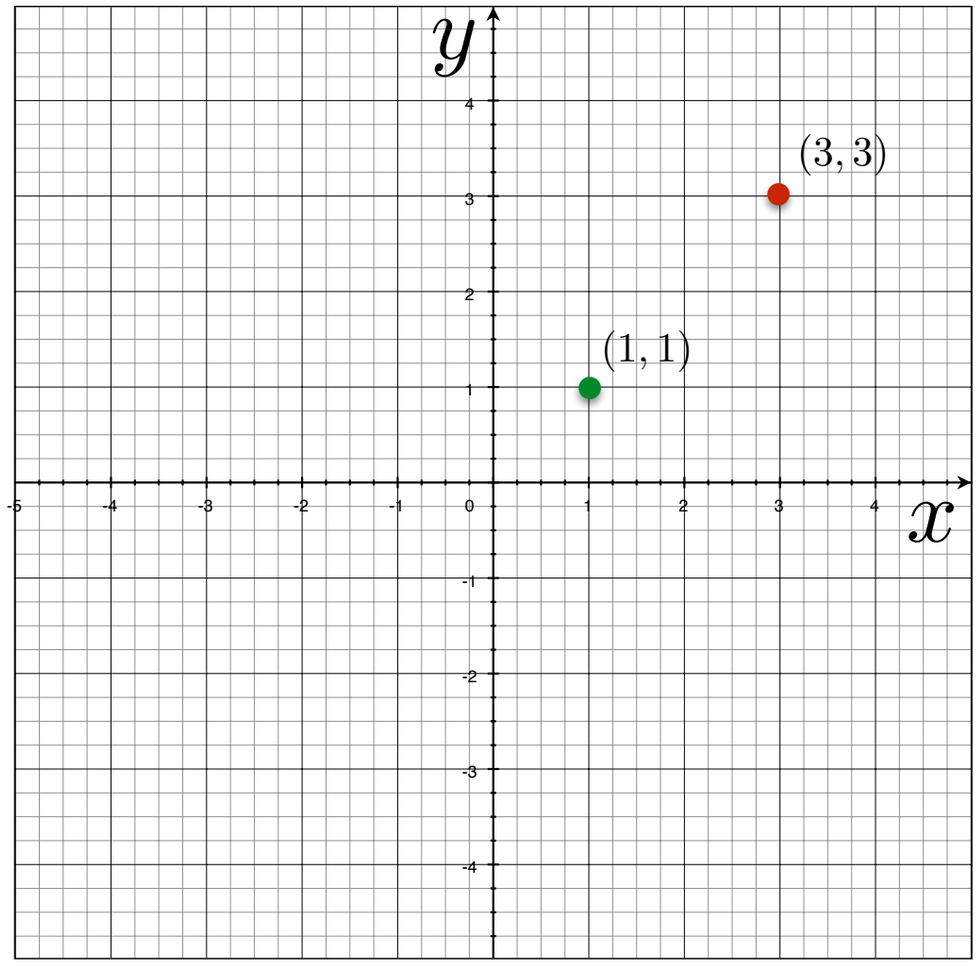
$$y = mx + b$$

parameters

variables

$$y - mx = b$$

parameters



two points become ?

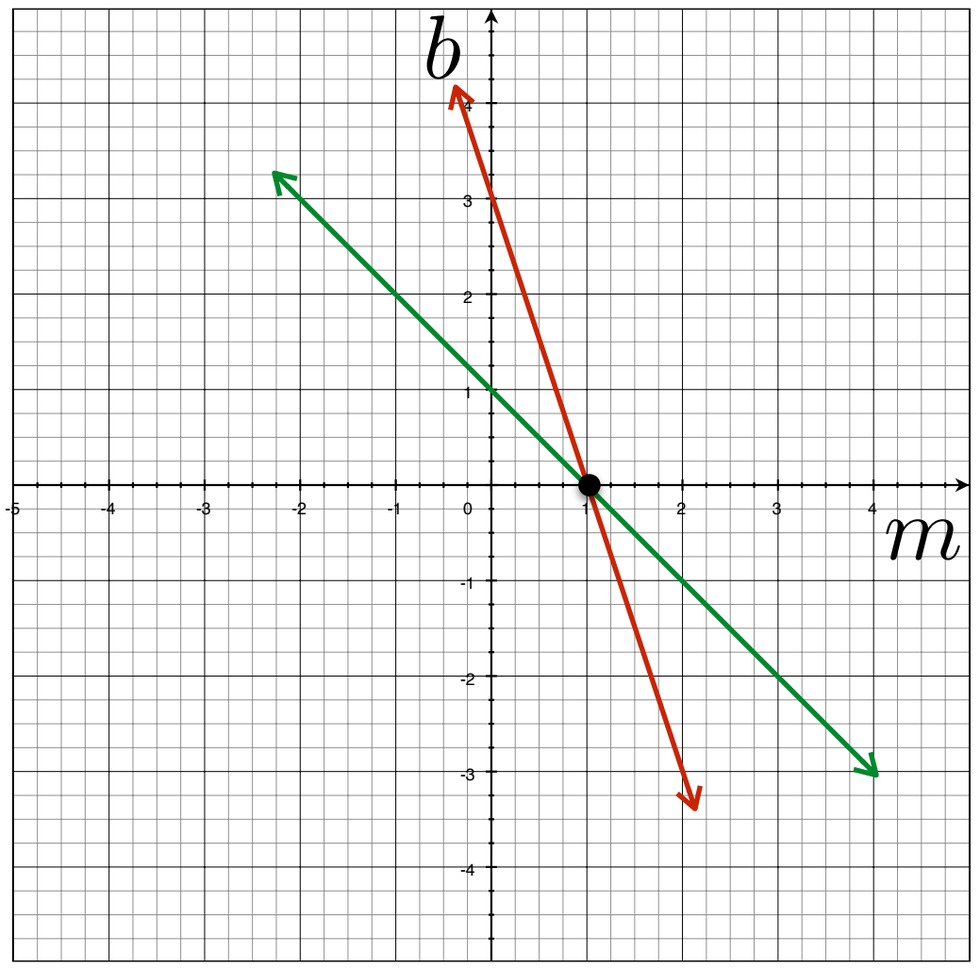


Image space

Parameter space

Image and parameter space

variables

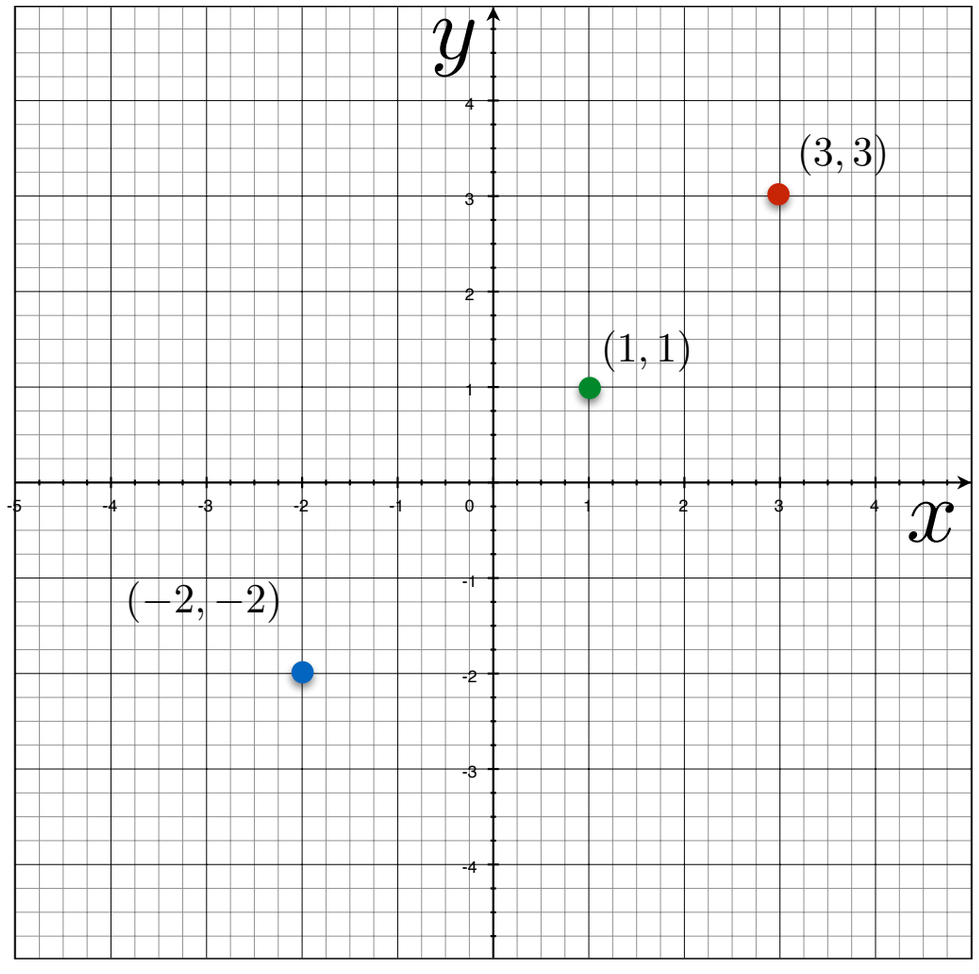
$$y = mx + b$$

parameters

variables

$$y - mx = b$$

parameters



three points become ?

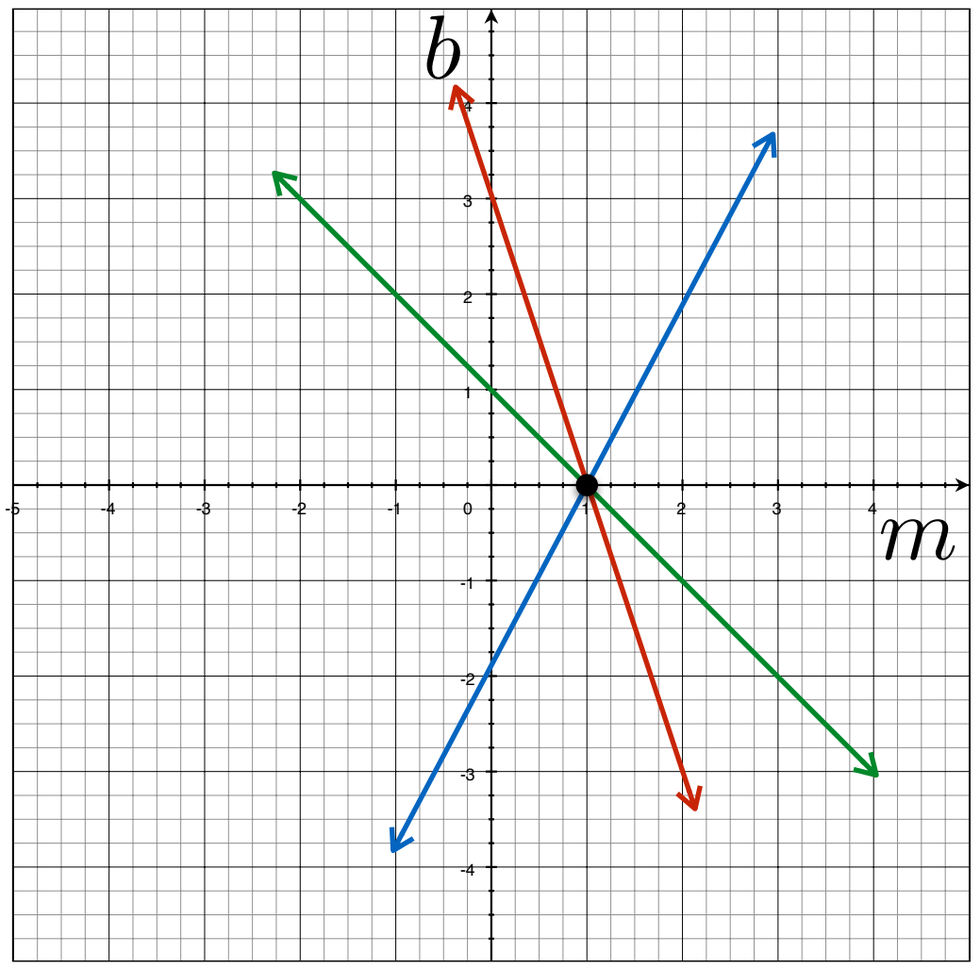


Image space

Parameter space

Image and parameter space

variables

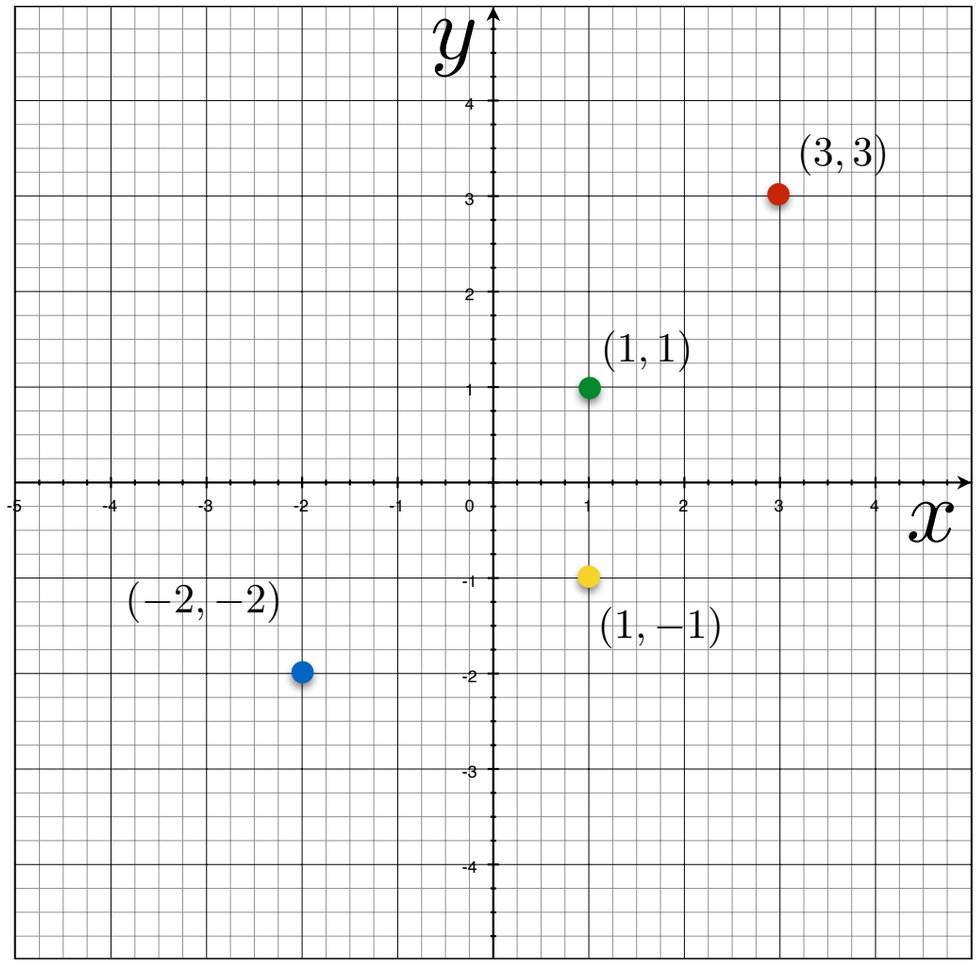
$$y = mx + b$$

parameters

variables

$$y - mx = b$$

parameters



four points become ?

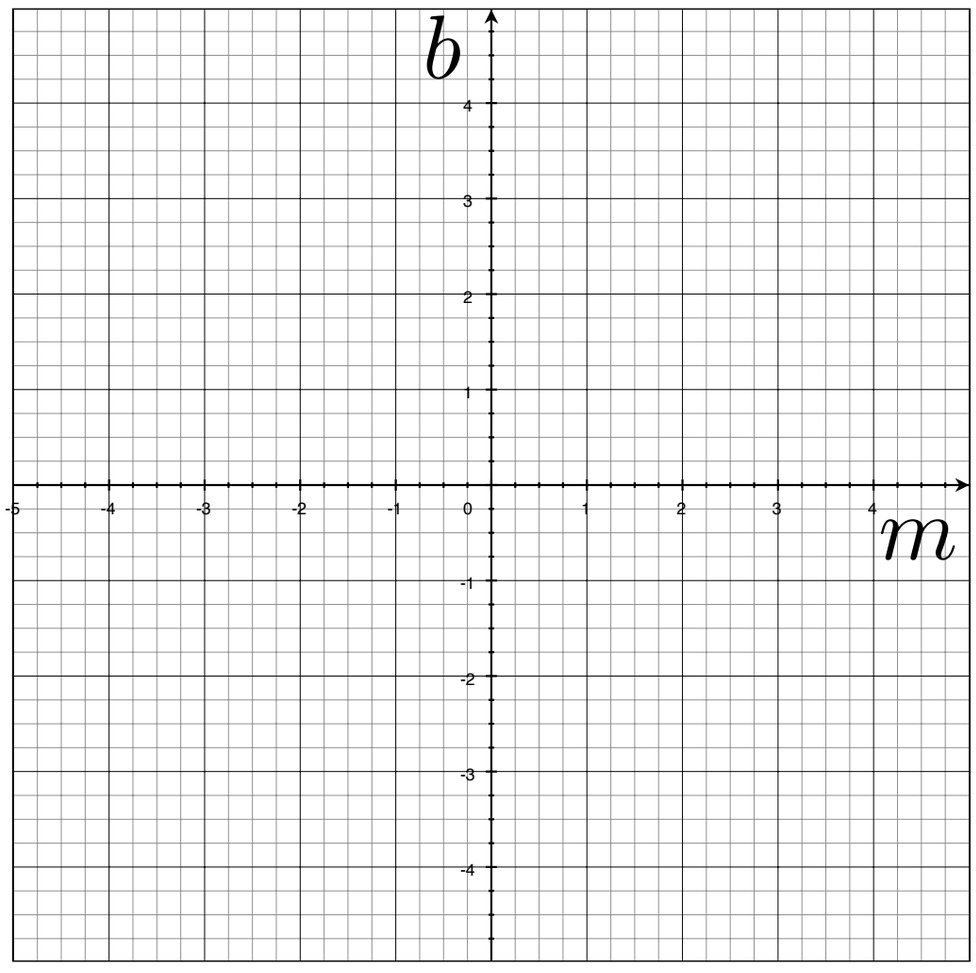


Image space

Parameter space

Image and parameter space

variables

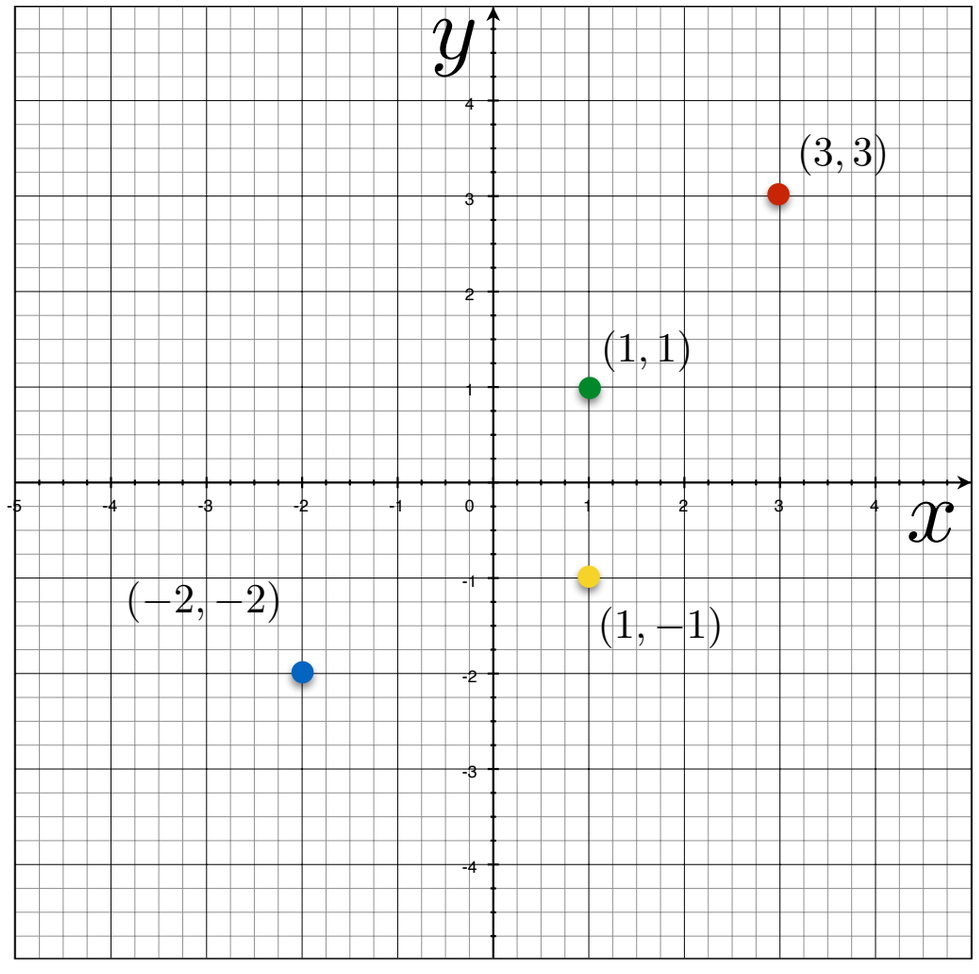
$$y = mx + b$$

parameters

variables

$$y - mx = b$$

parameters



four points become ?

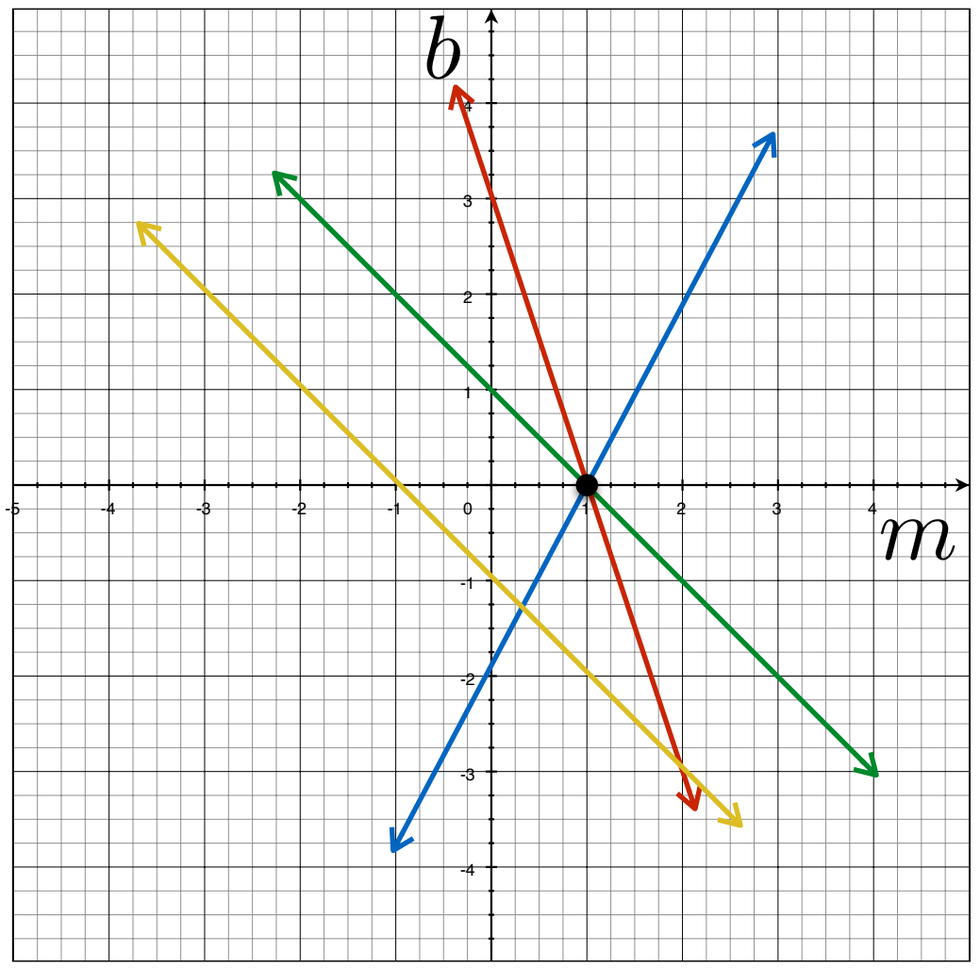


Image space

Parameter space

How would you find the best fitting line?

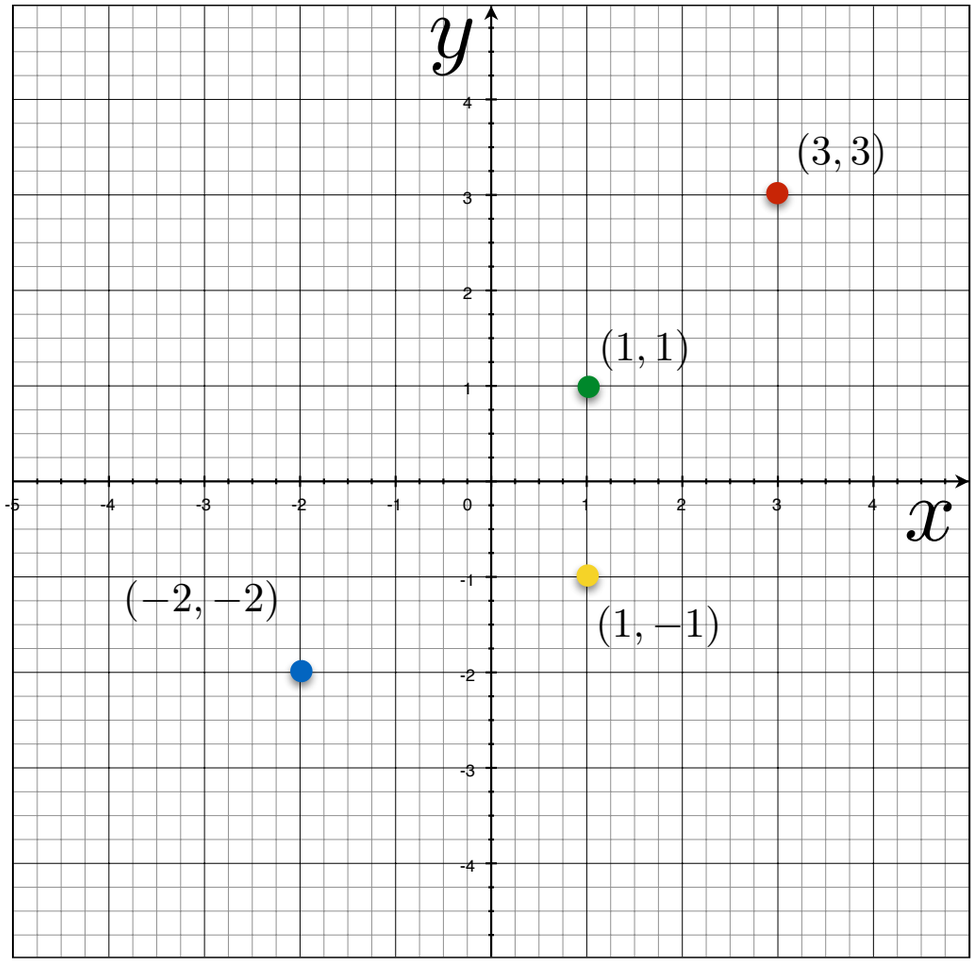
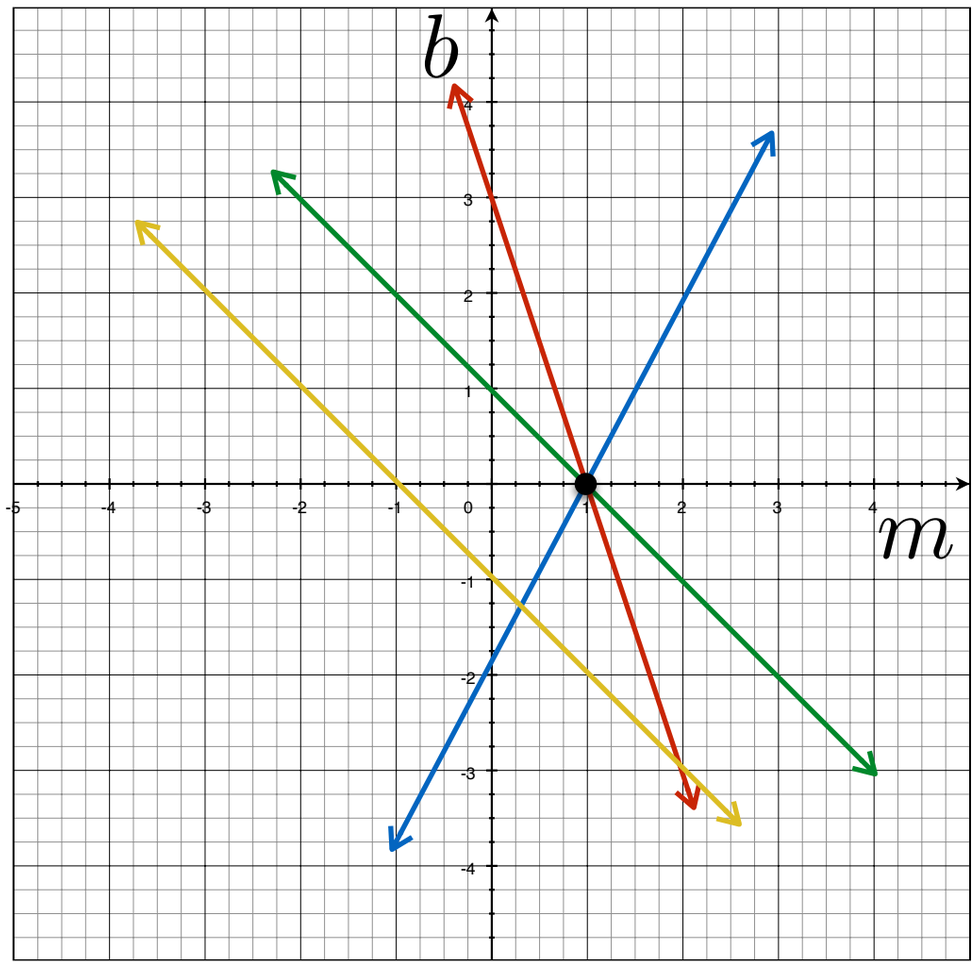


Image space



Parameter space

Is your method robust to measurement noise?

Is your method robust to outliers?

Line Detection by Hough Transform

Algorithm:

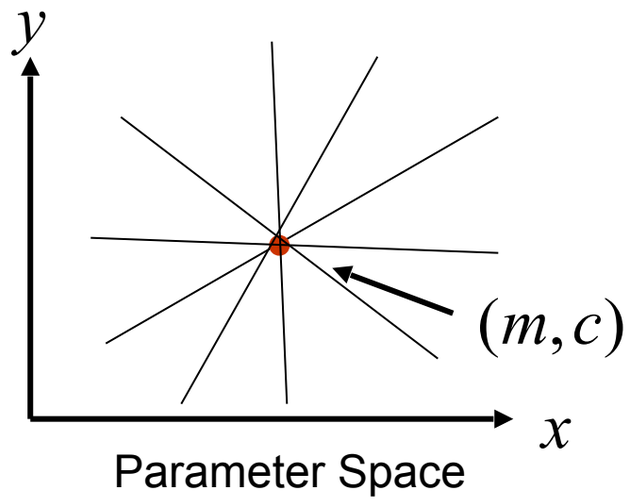
1. Quantize Parameter Space (m,c)

2. Create Accumulator Array $A(m,c)$

3. Set $A(m,c) = 0 \quad \forall m,c$

4. For each image edge (x_i, y_i)
   For each element in $A(m,c)$
   If (m,c) lies on the line: $c = -x_i m + y_i$
   Increment $A(m,c) = A(m,c) + 1$

5. Find local maxima in $A(m,c)$



$A(m,c)$

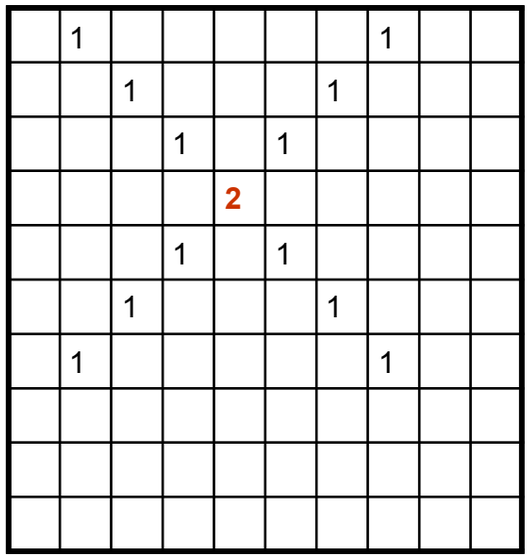
	1						1		
		1				1			
			1		1				
				2					
			1		1				
		1				1			
	1						1		

Problems with parameterization

What's wrong with the parameterization (m, c) *?*

How big does the accumulator need to be?

$A(m, c)$

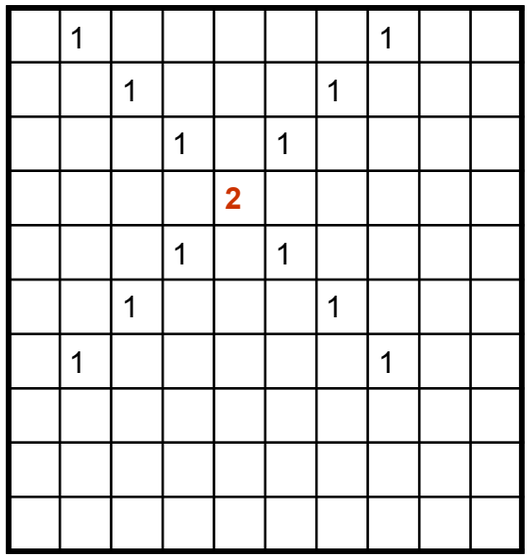


Problems with parameterization

What's wrong with the parameterization (m, c) *?*

How big does the accumulator need to be?

$A(m, c)$



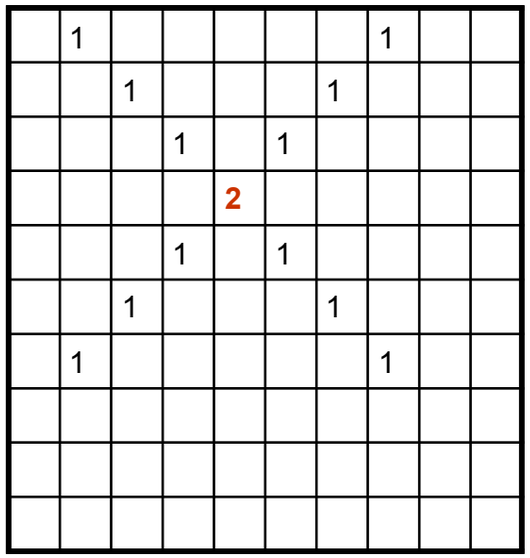
The space of m is huge! The space of c is huge!

Problems with parameterization

What's wrong with the parameterization (m, c) *?*

How big does the accumulator need to be?

$A(m, c)$



The space of m is huge! The space of c is huge!

$$-\infty \leq m \leq \infty$$

Better Parameterization

Use normal form:

$$x\cos\theta + y\sin\theta = \rho$$

Given points (x_i, y_i) find (ρ,θ)

Hough Space Sinusoid

$$0 \le \theta \le 2\pi$$

$$0 \le \rho \le \rho_{max}$$

(Finite Accumulator Array Size)

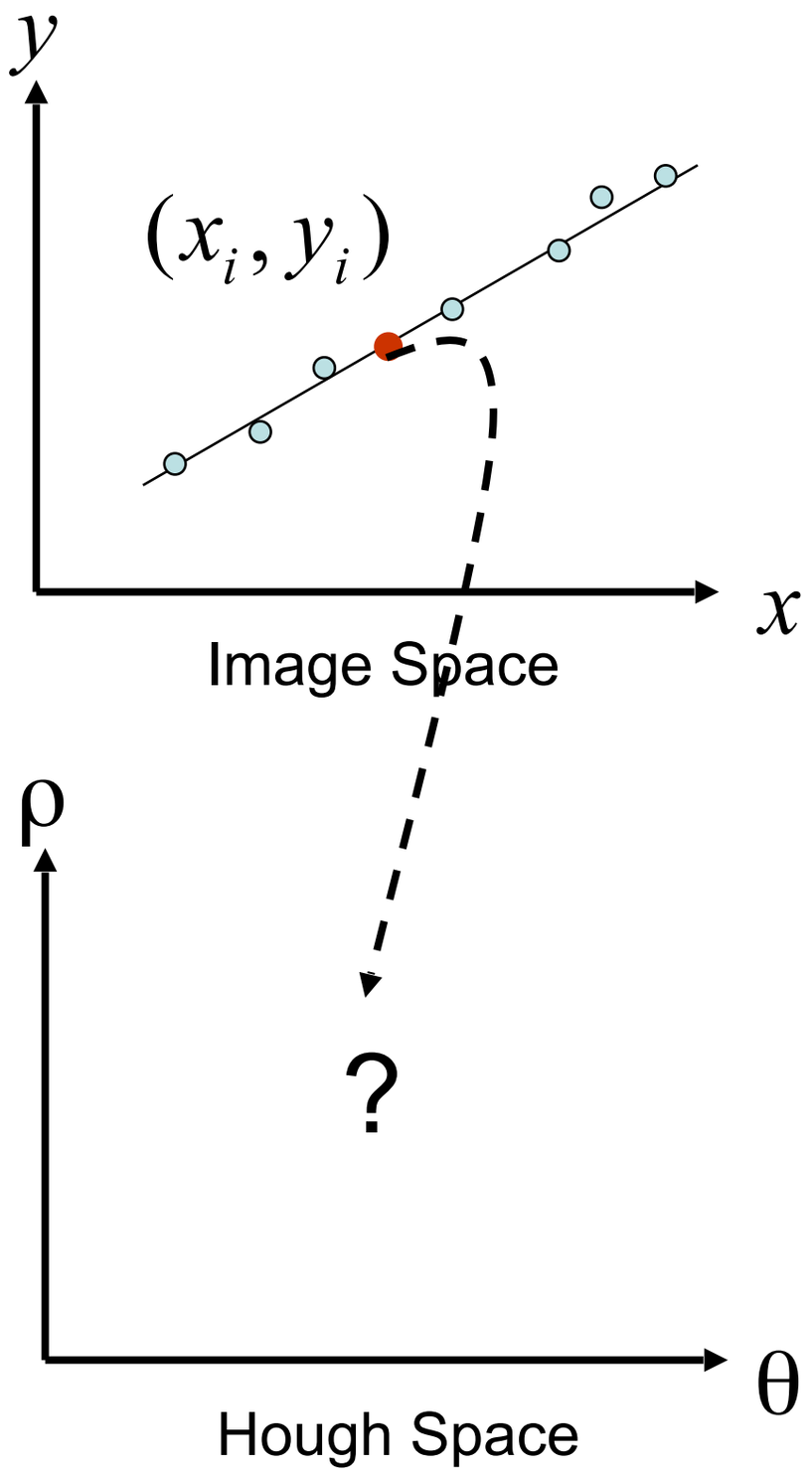


Image and parameter space

variables

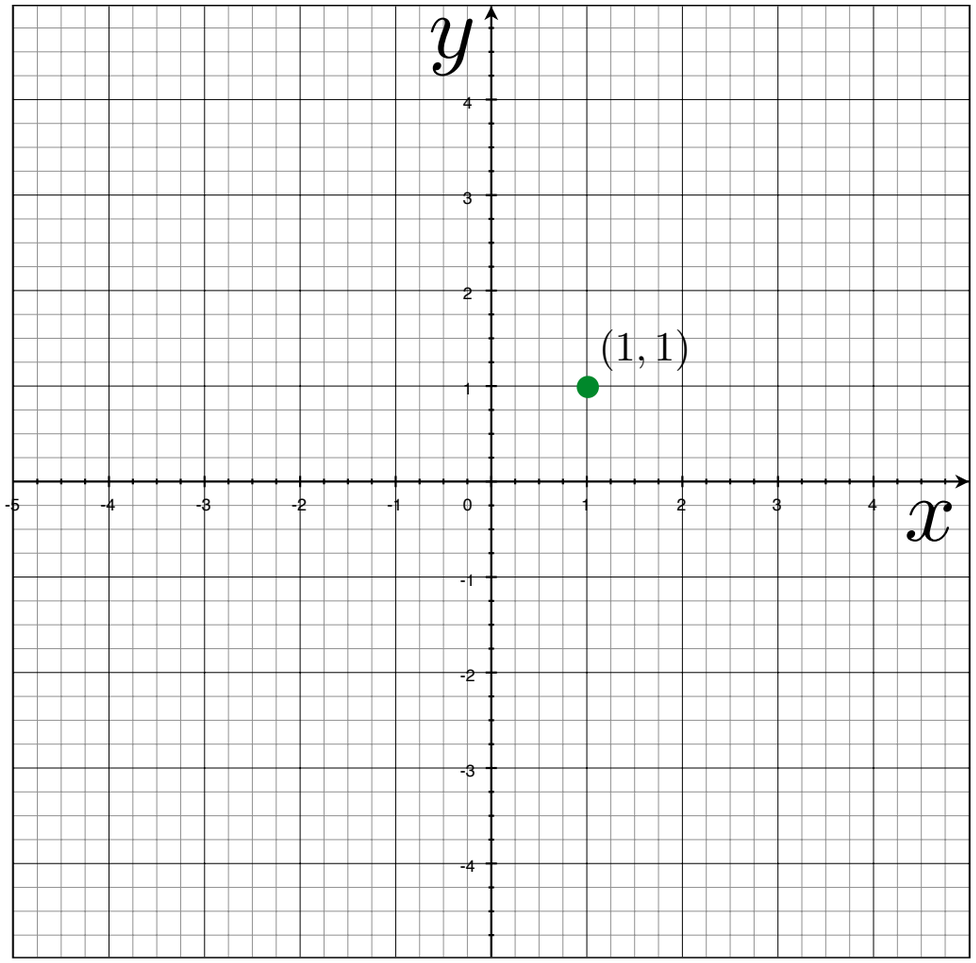
$$y = mx + b$$

parameters

parameters

$$x\cos\theta + y\sin\theta = \rho$$

variables



a point becomes a wave

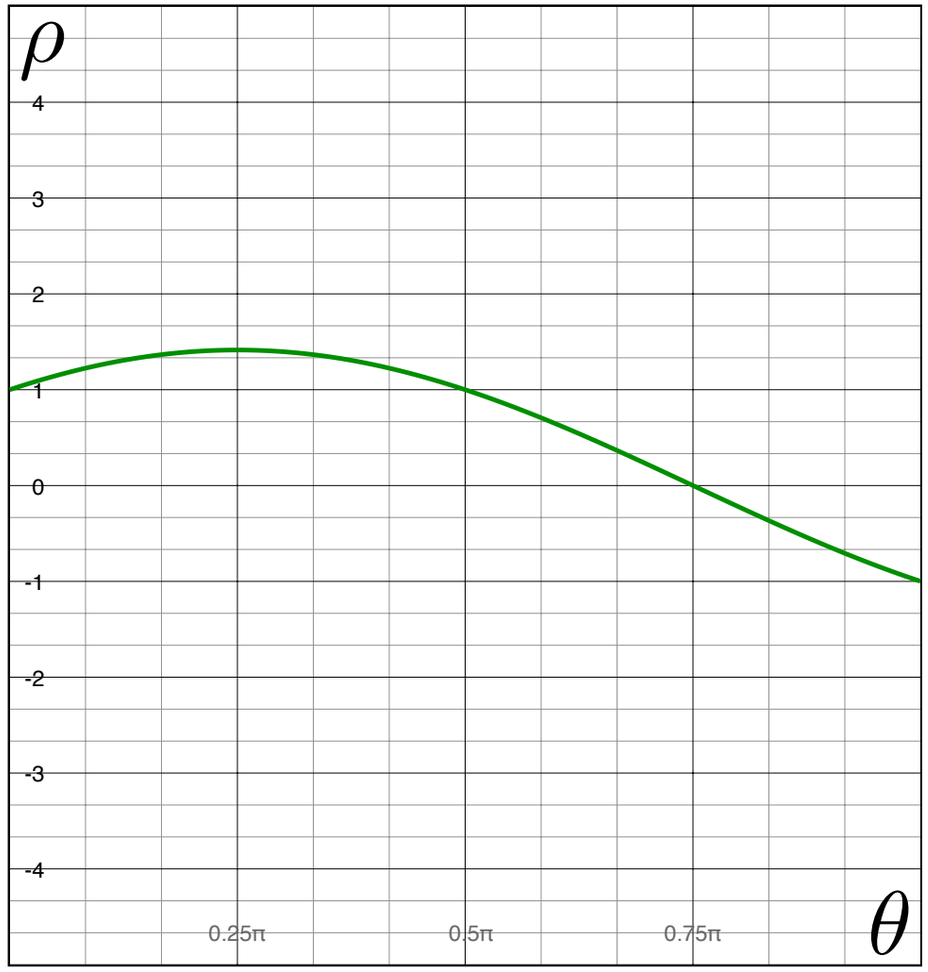


Image space

Parameter space

Image and parameter space

variables

$$y = mx + b$$

parameters

$$x\cos\theta + y\sin\theta = \rho$$

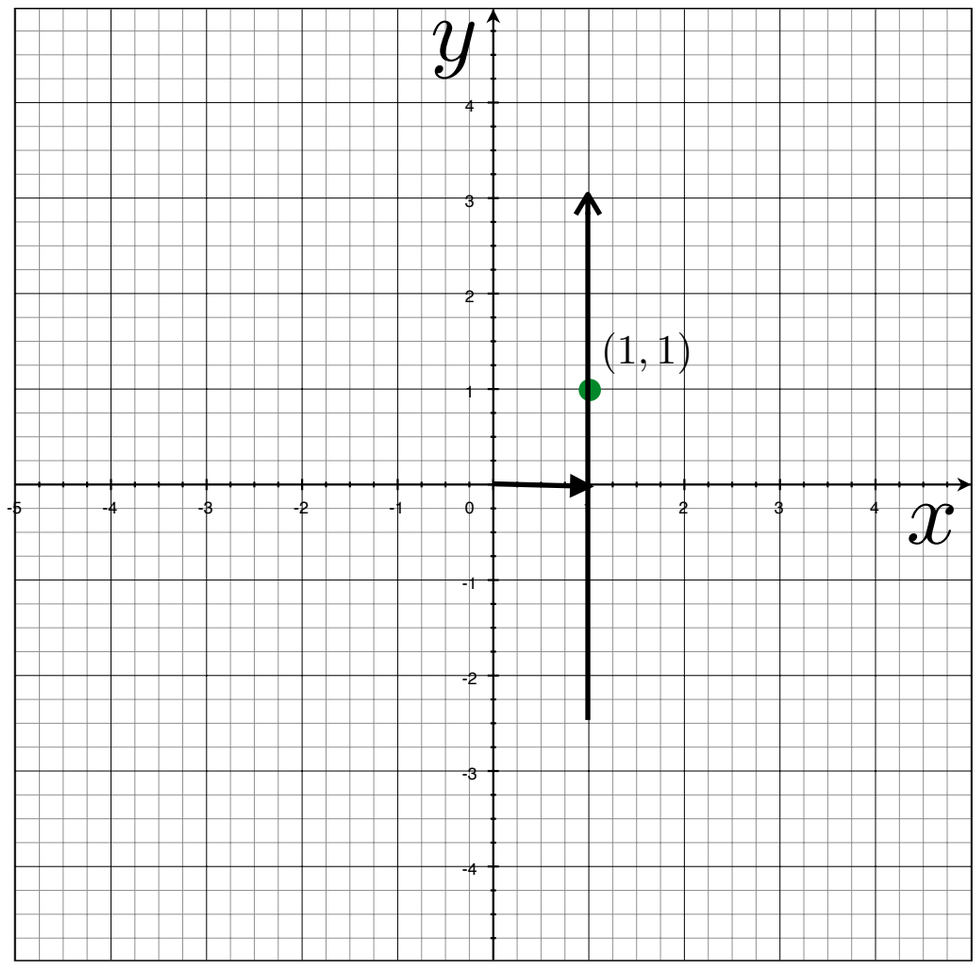
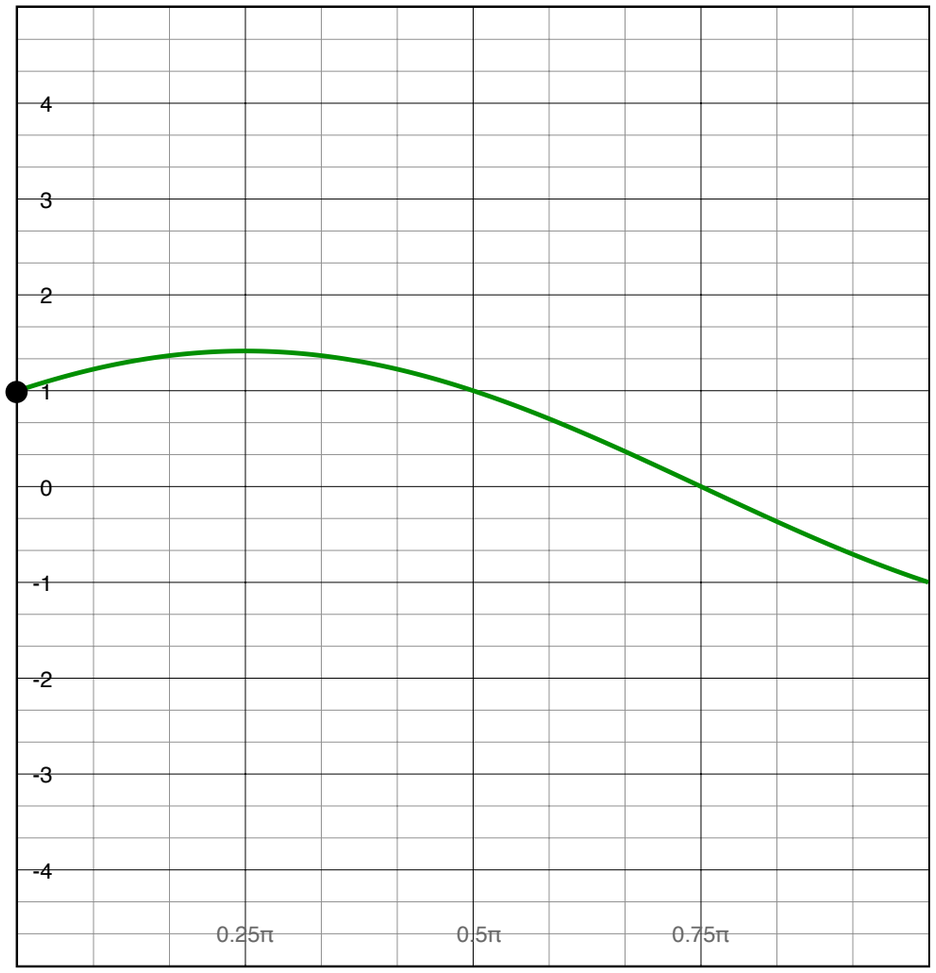


Image space

a line becomes a point



Parameter space

Image and parameter space

variables

$$y = mx + b$$

parameters

$$x\cos\theta + y\sin\theta = \rho$$

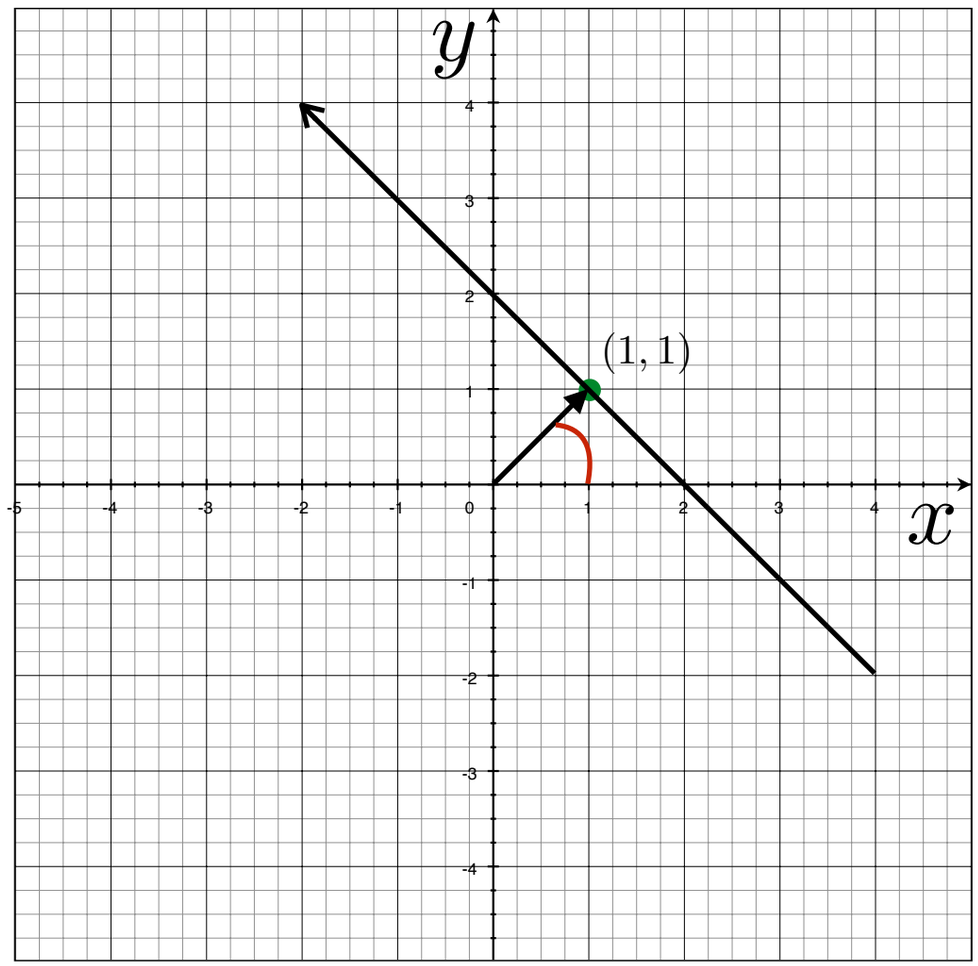
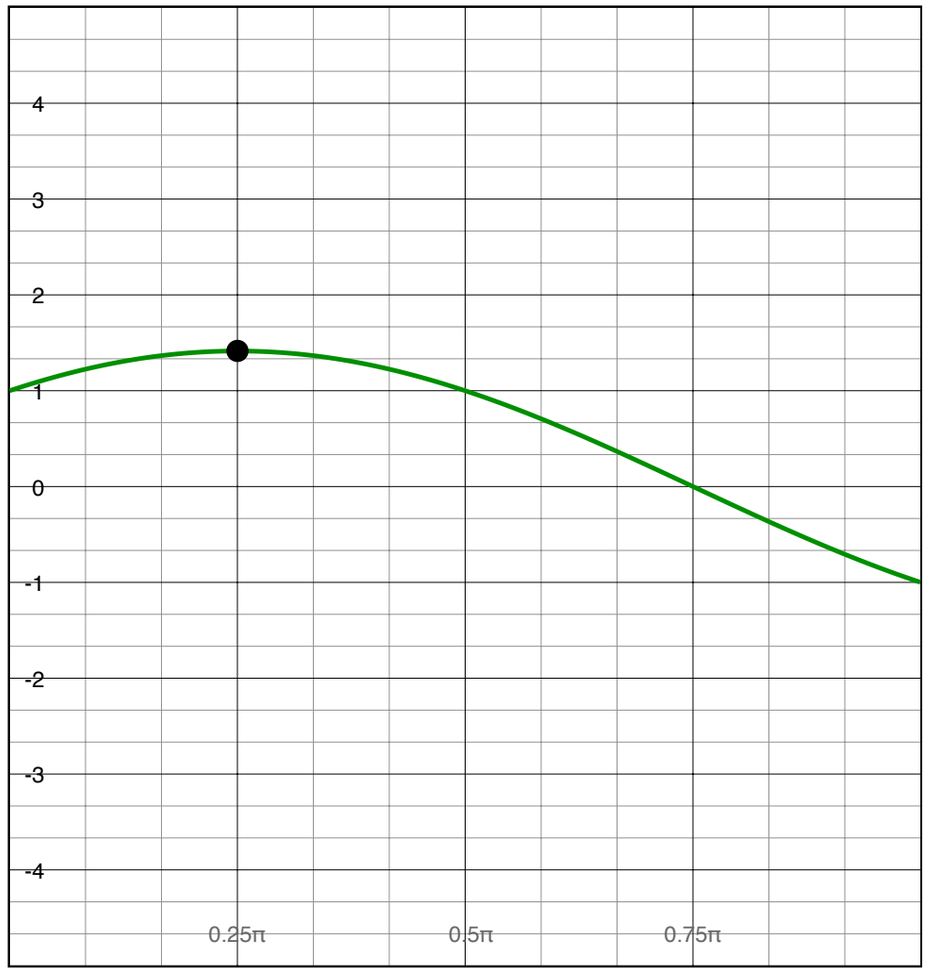


Image space

a line becomes a point



Parameter space

Image and parameter space

variables

$$y = mx + b$$

parameters

$$x \cos\theta + y \sin\theta = \rho$$

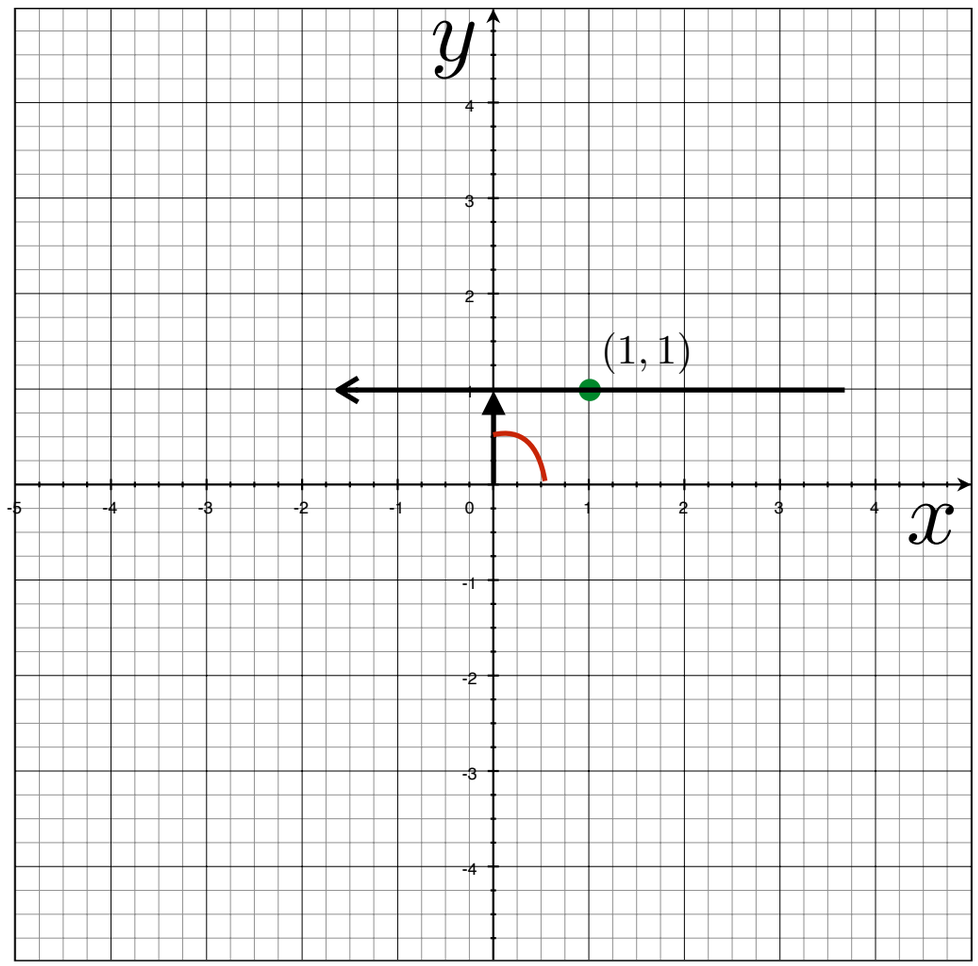
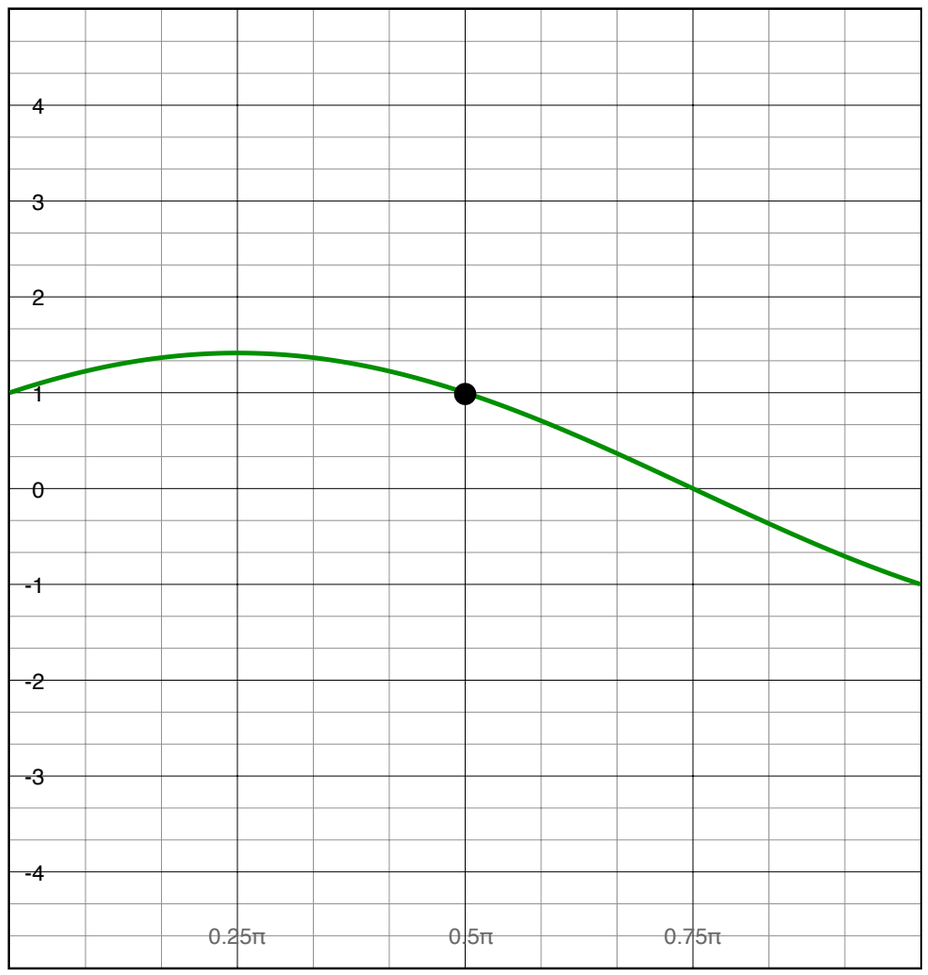


Image space

a line becomes a point



Parameter space

Image and parameter space

variables

$$y = mx + b$$

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$$x\cos\theta + y\sin\theta = \rho$$

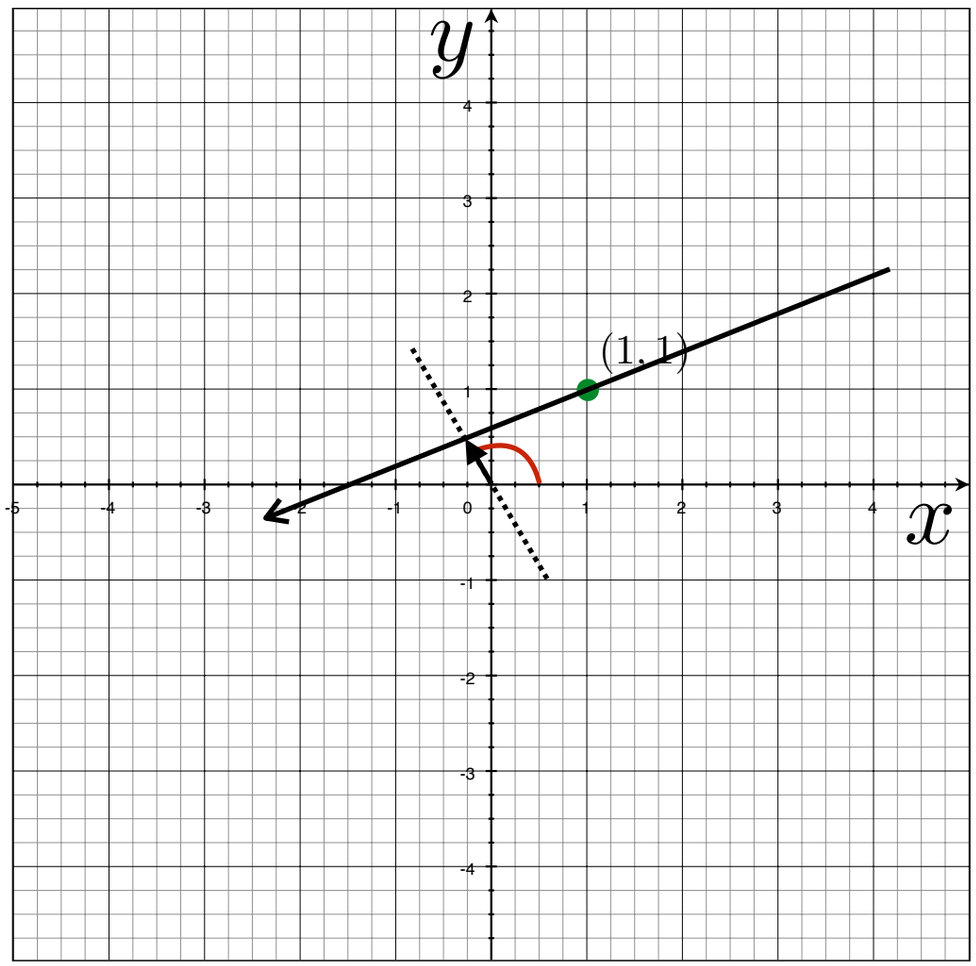
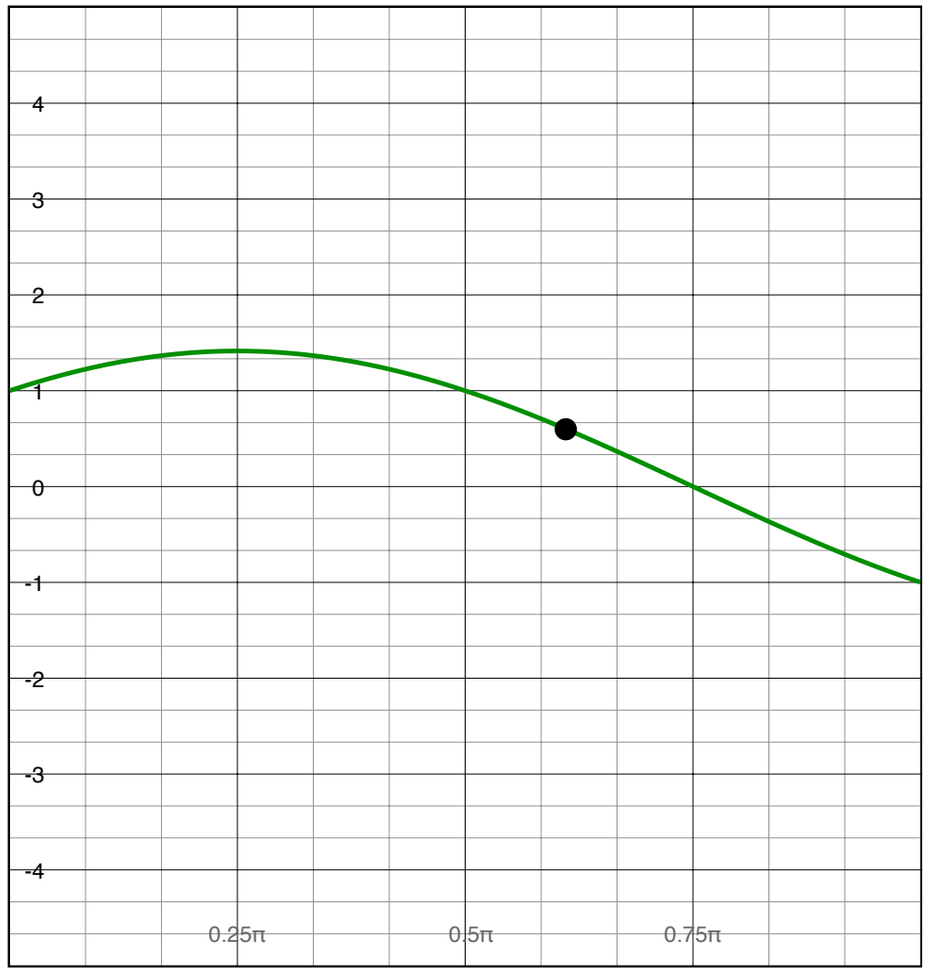


Image space

a line becomes a point



Parameter space

Image and parameter space

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$$y = mx + b$$

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$$x\cos\theta + y\sin\theta = \rho$$

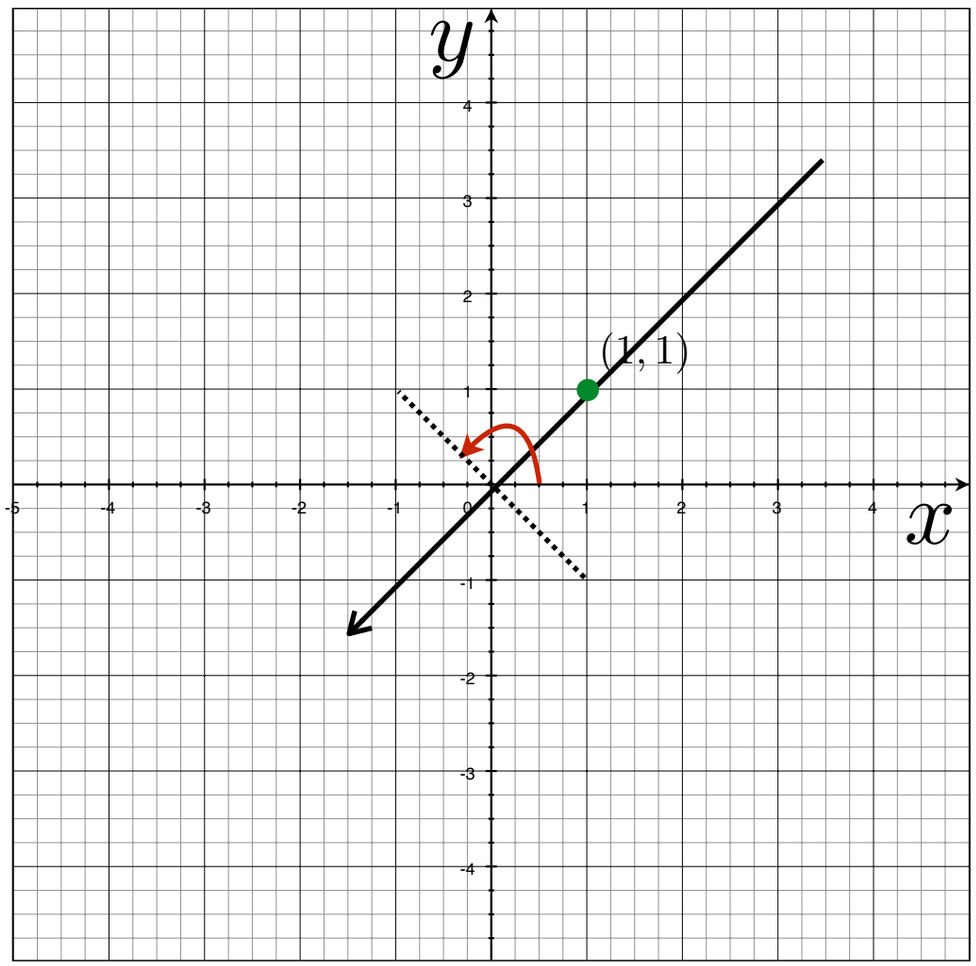
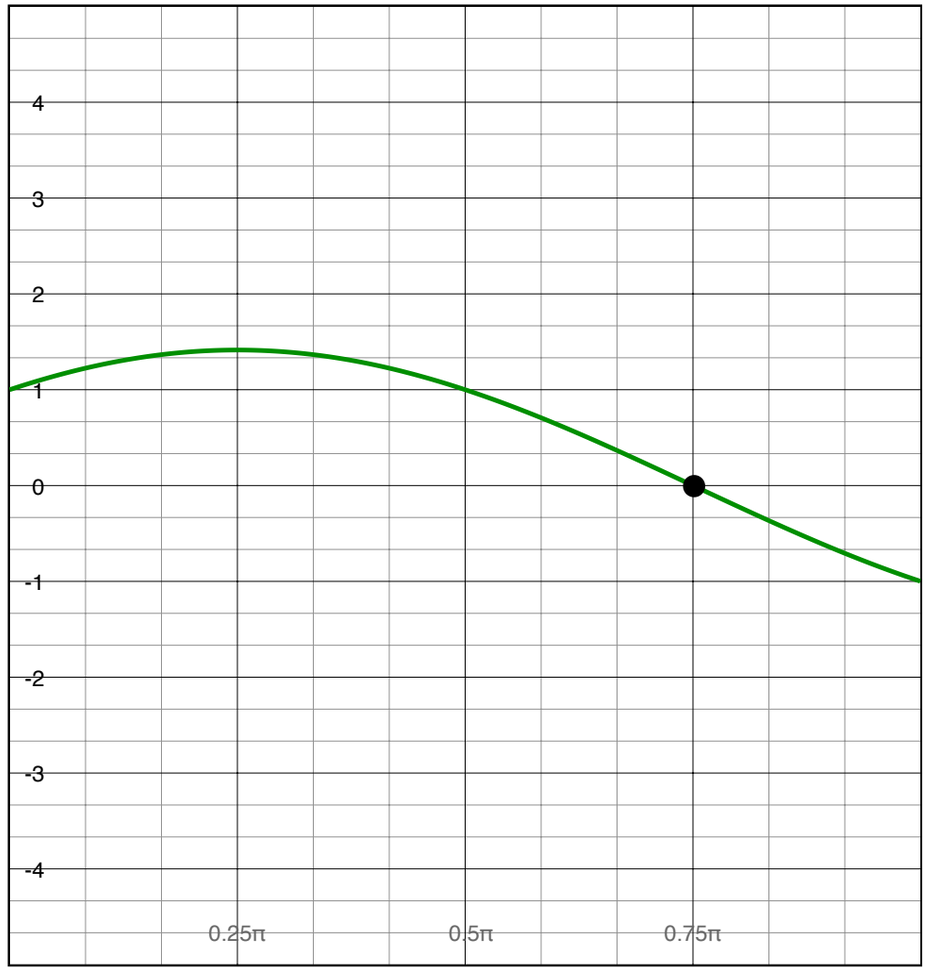


Image space

a line becomes a point



Parameter space

Image and parameter space

variables

$$y = mx + b$$

parameters

$$x \cos\theta + y \sin\theta = \rho$$

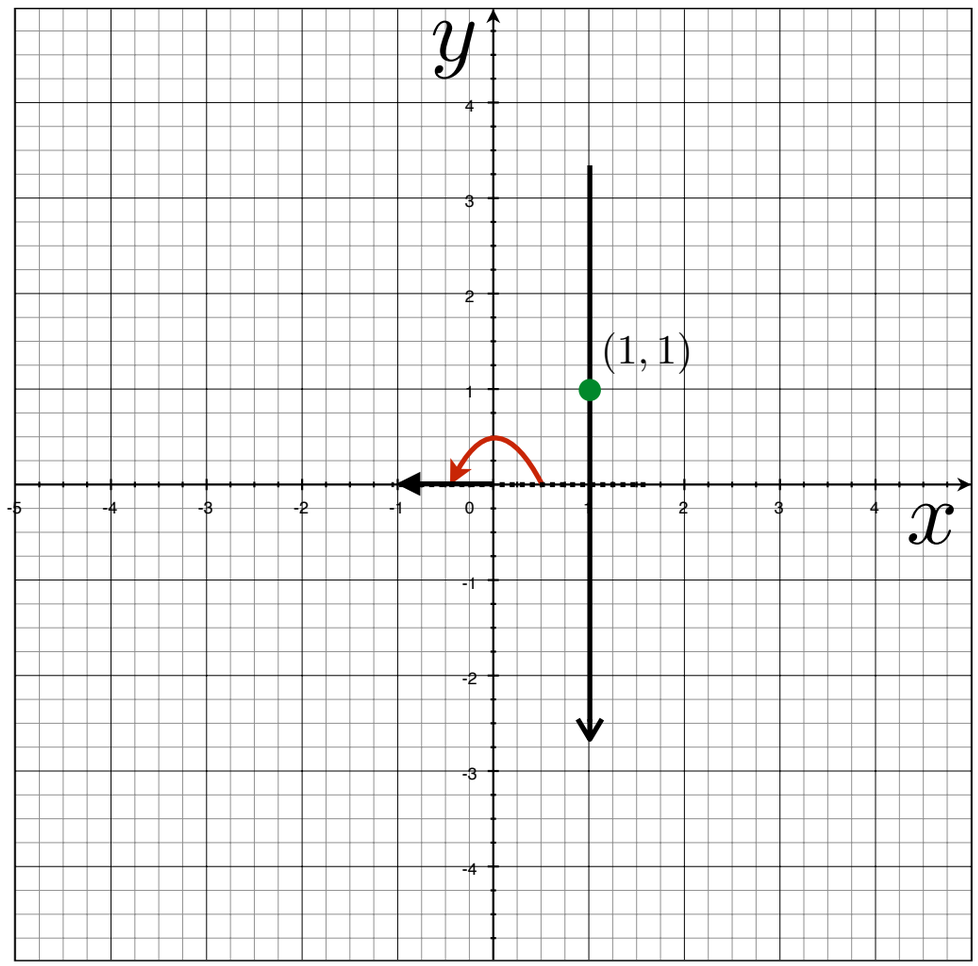
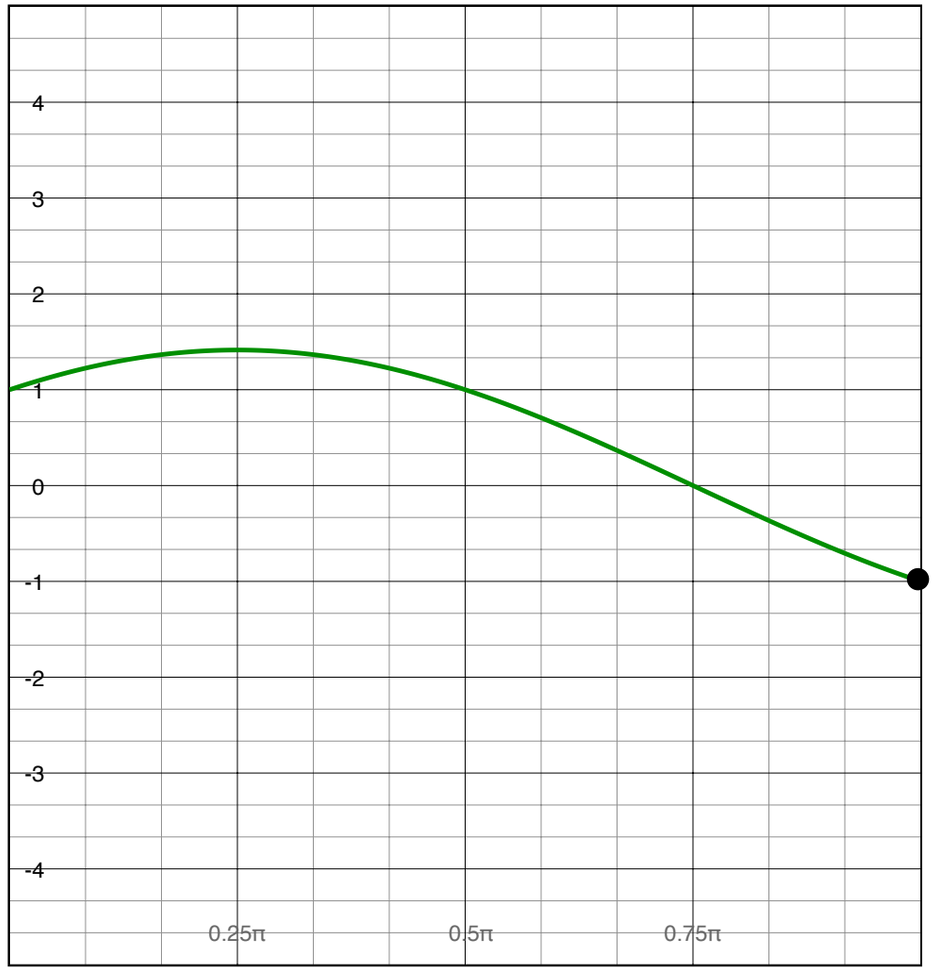


Image space

a line becomes a point



Parameter space

Image and parameter space

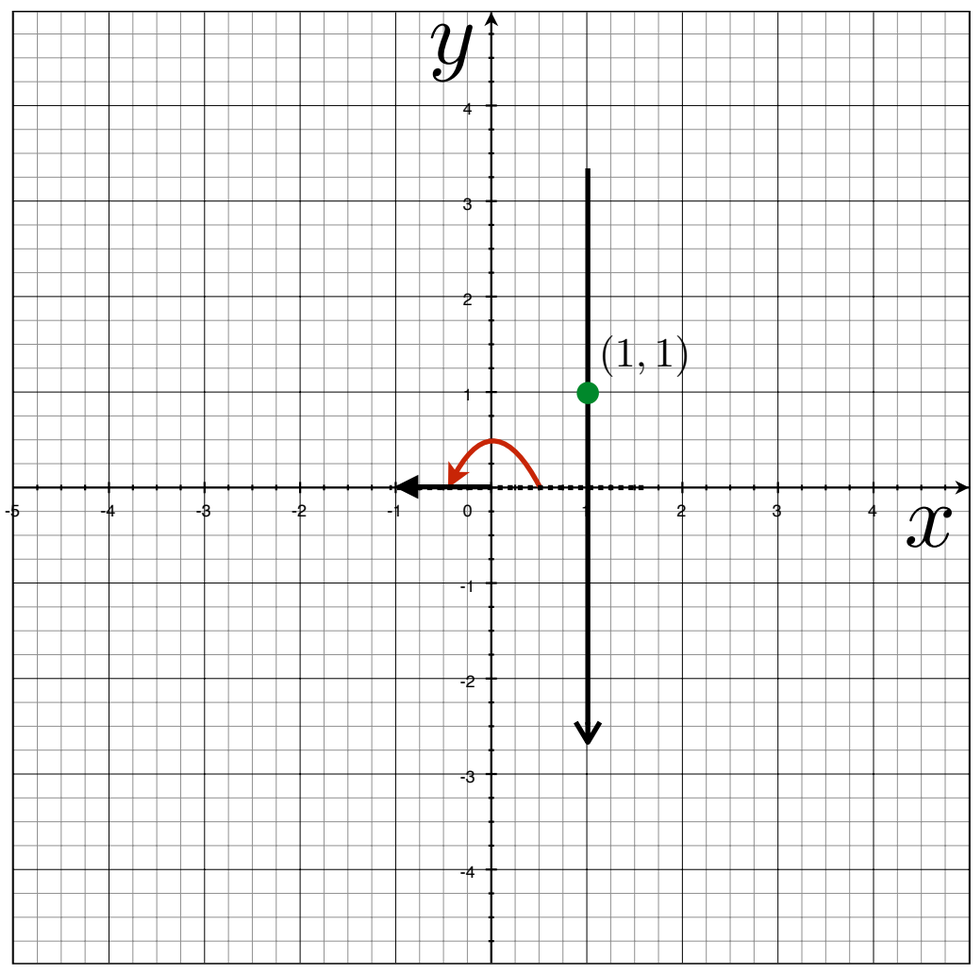
variables

$$y = mx + b$$

parameters

$$x\cos\theta + y\sin\theta = \rho$$

Wait …why is rho negative?



a line becomes a point

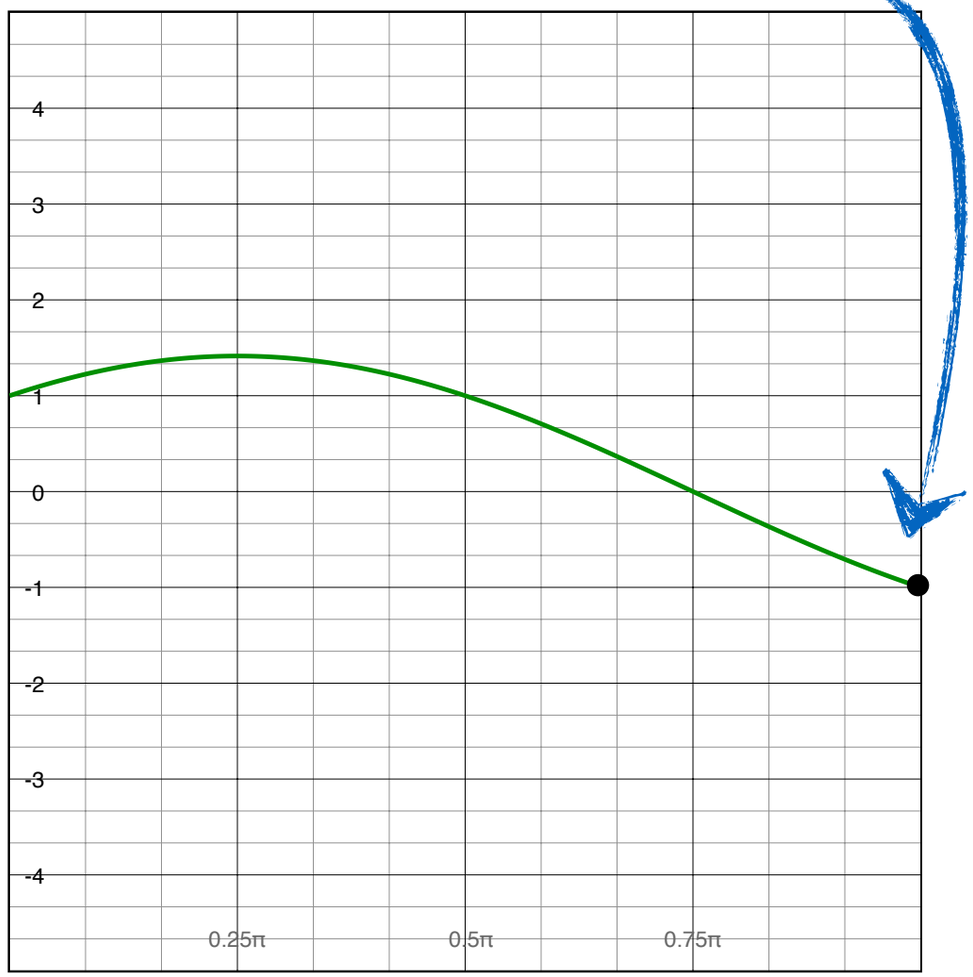


Image space

Parameter space

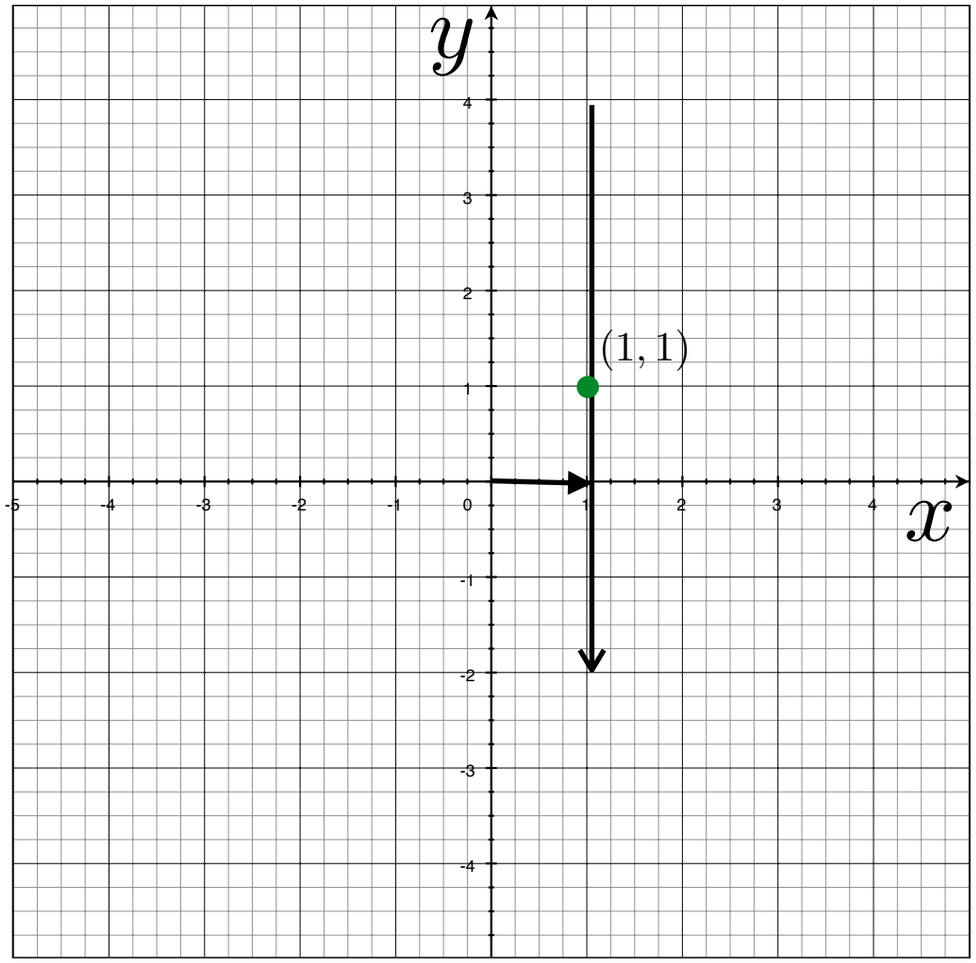
Image and parameter space

variables

$$y = mx + b$$

parameters

$$x\cos\theta + y\sin\theta = \rho$$



a line becomes a point

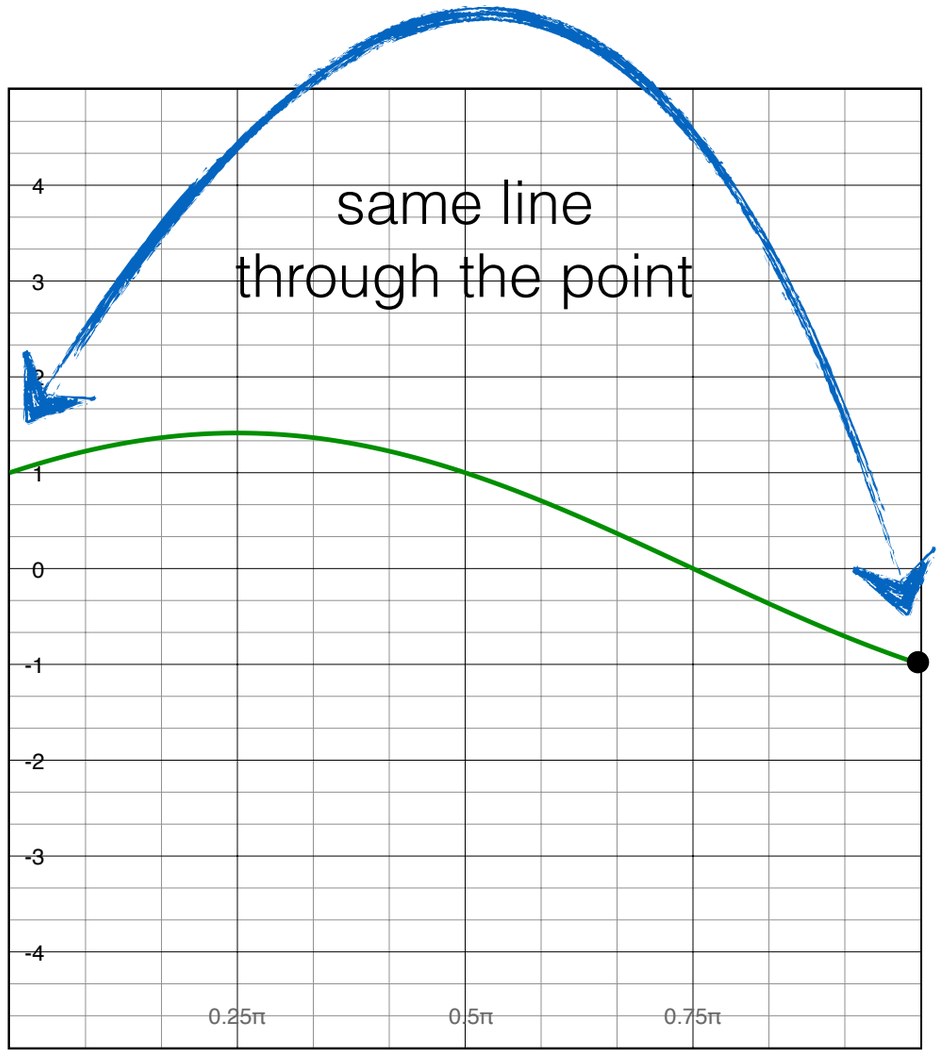


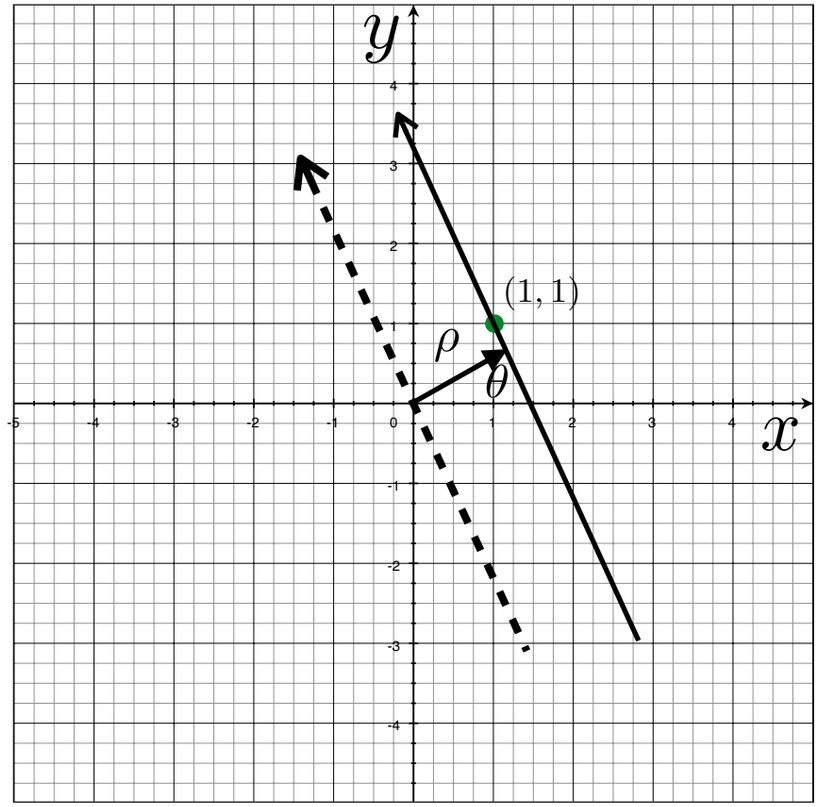
Image space

Parameter space

There are two ways to write the same line:

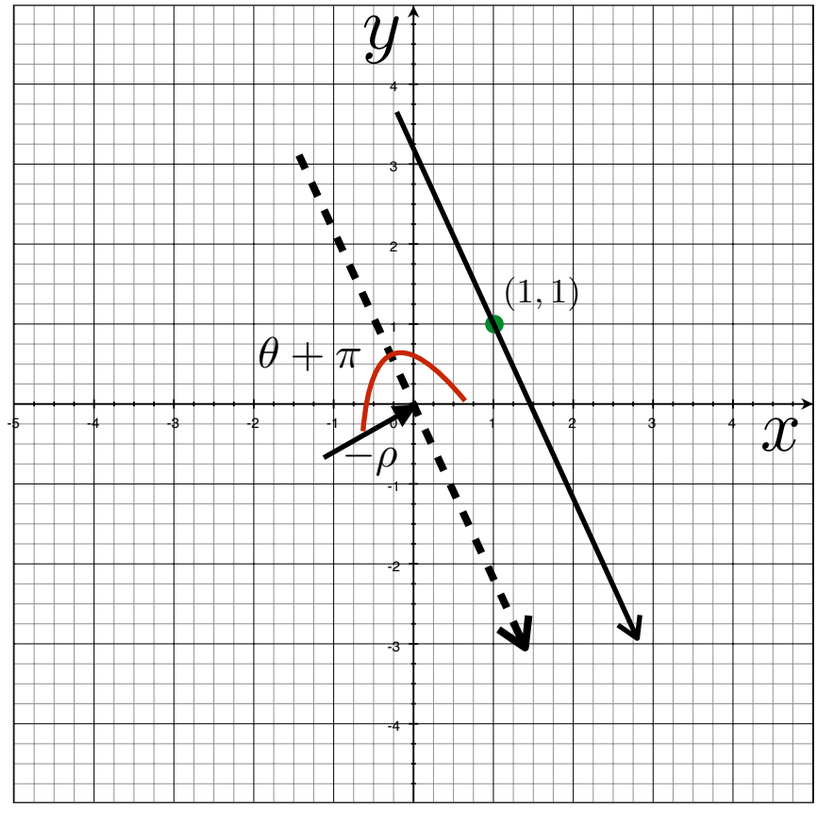
Positive rho version:

$$x\cos\theta + y\sin\theta = \rho$$



Negative rho version:

$$x\cos(\theta+\pi) + y\sin(\theta+\pi) = -\rho$$



Recall:

$$\sin(\theta) = -\sin(\theta+\pi)$$

$$\cos(\theta) = -\cos(\theta+\pi)$$

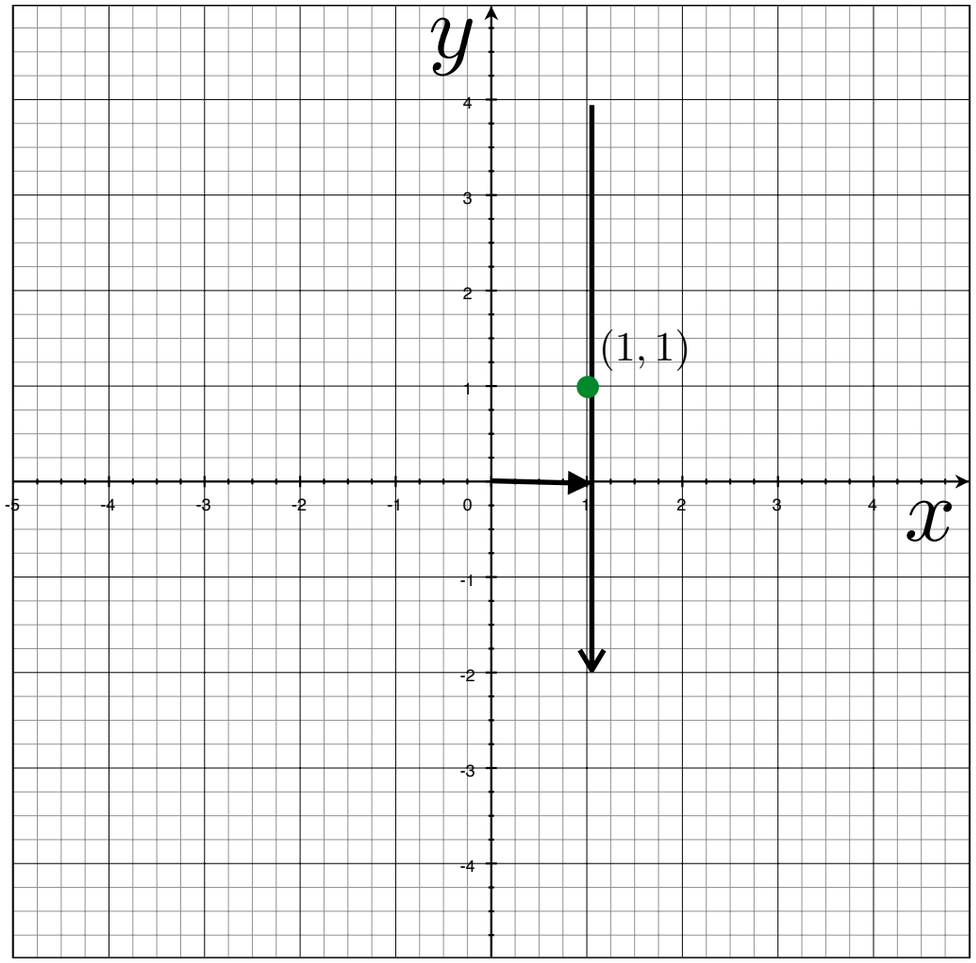
Image and parameter space

variables

$$y = mx + b$$

parameters

$$x\cos\theta + y\sin\theta = \rho$$



a line becomes a point

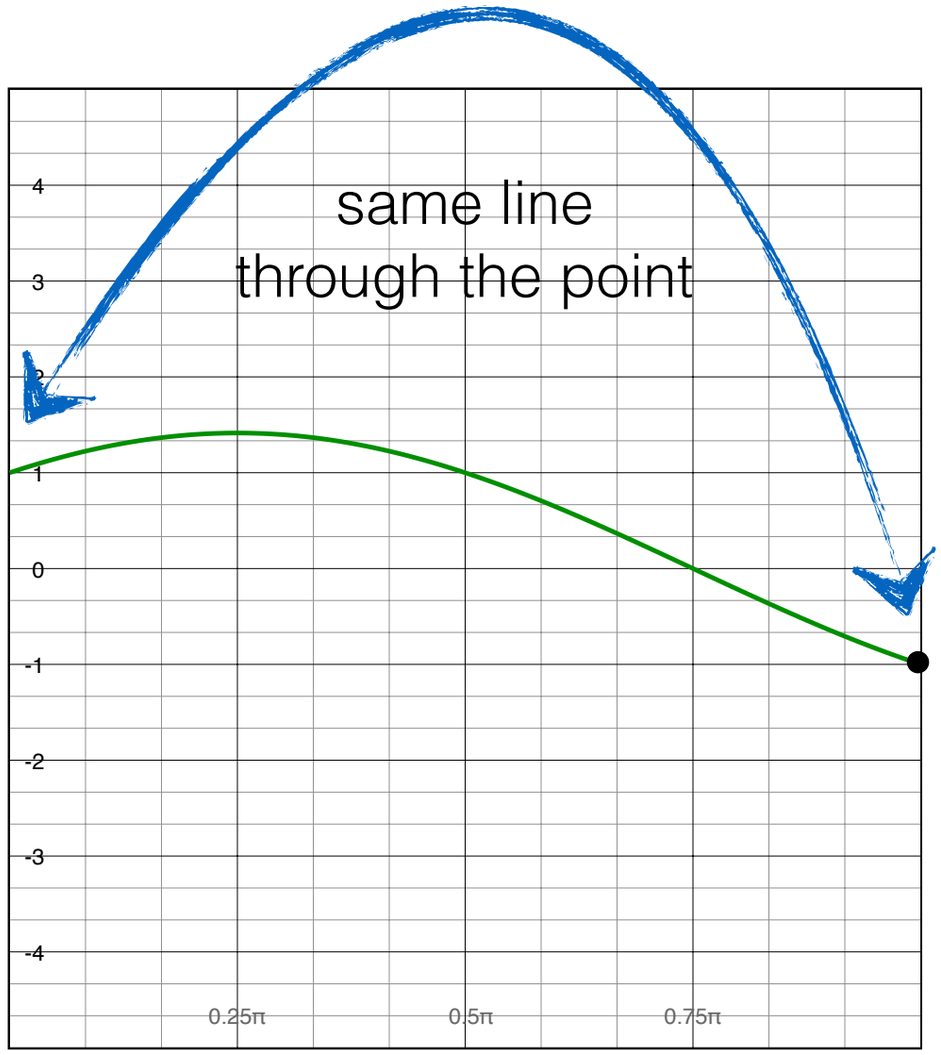


Image space

Parameter space

Image and parameter space

variables

$$y = mx + b$$

parameters

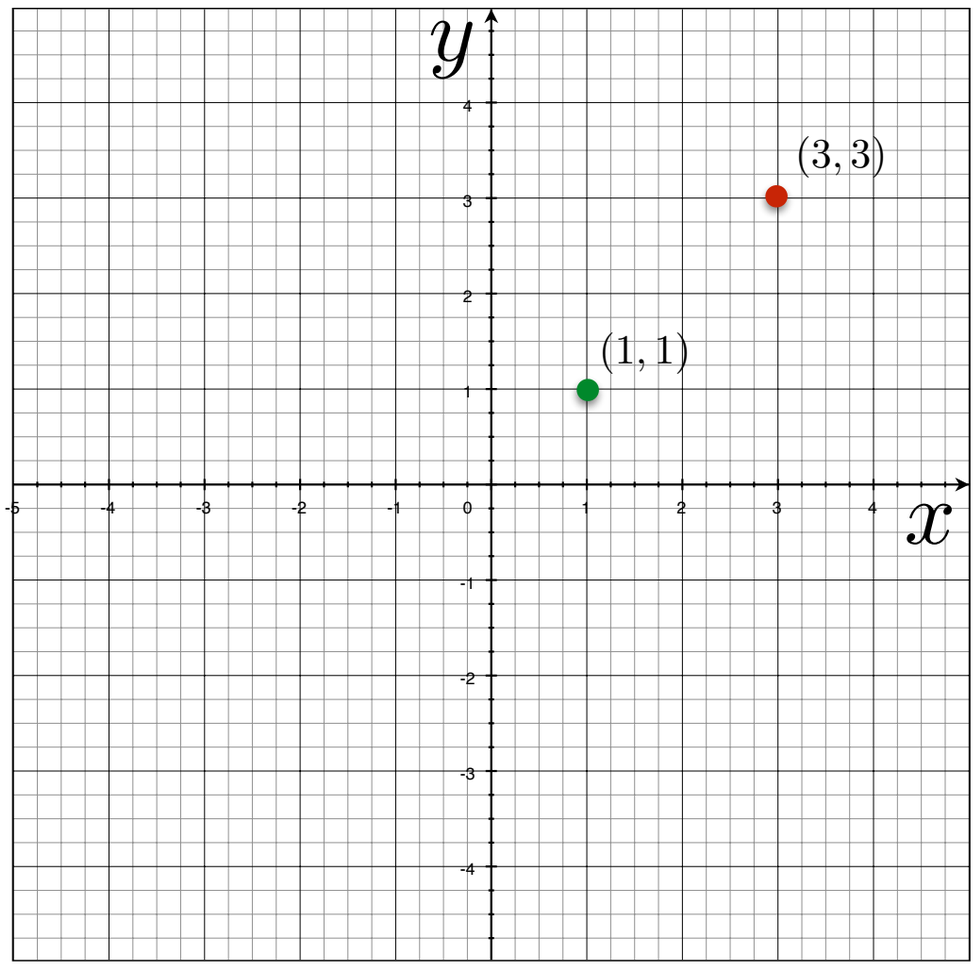
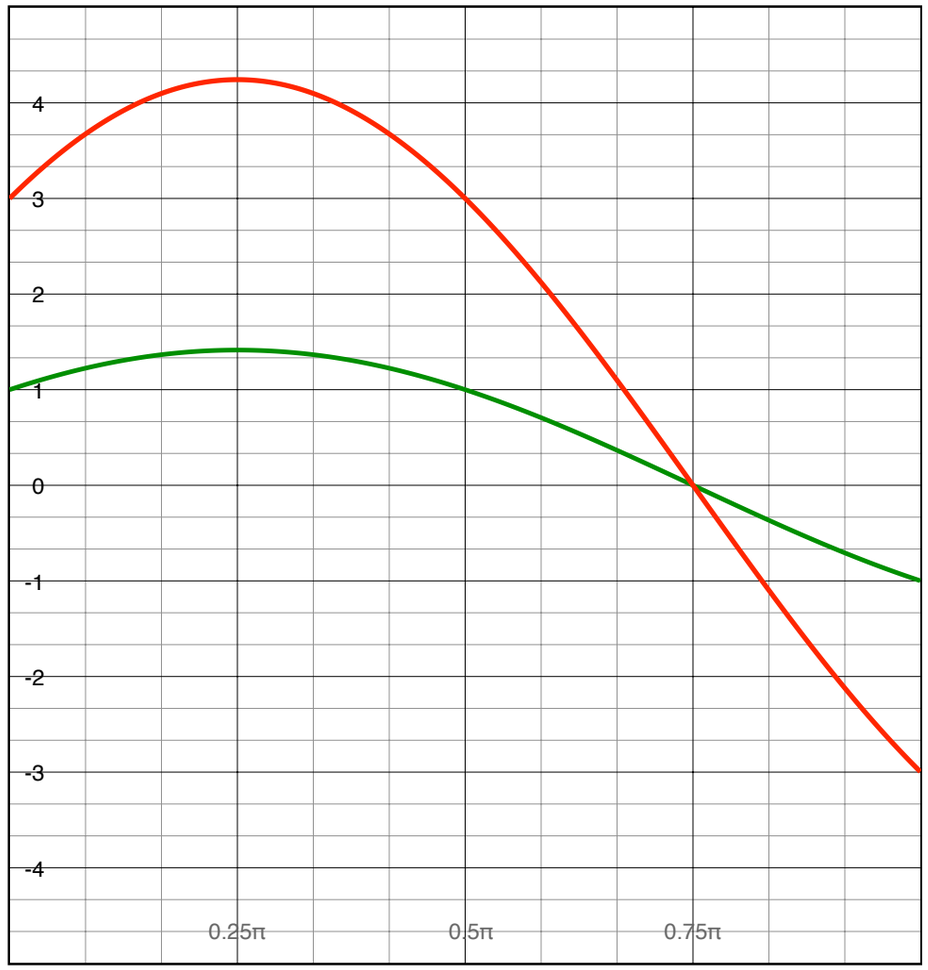


Image space

two points become ?



Parameter space

Image and parameter space

variables

$$y = mx + b$$

parameters

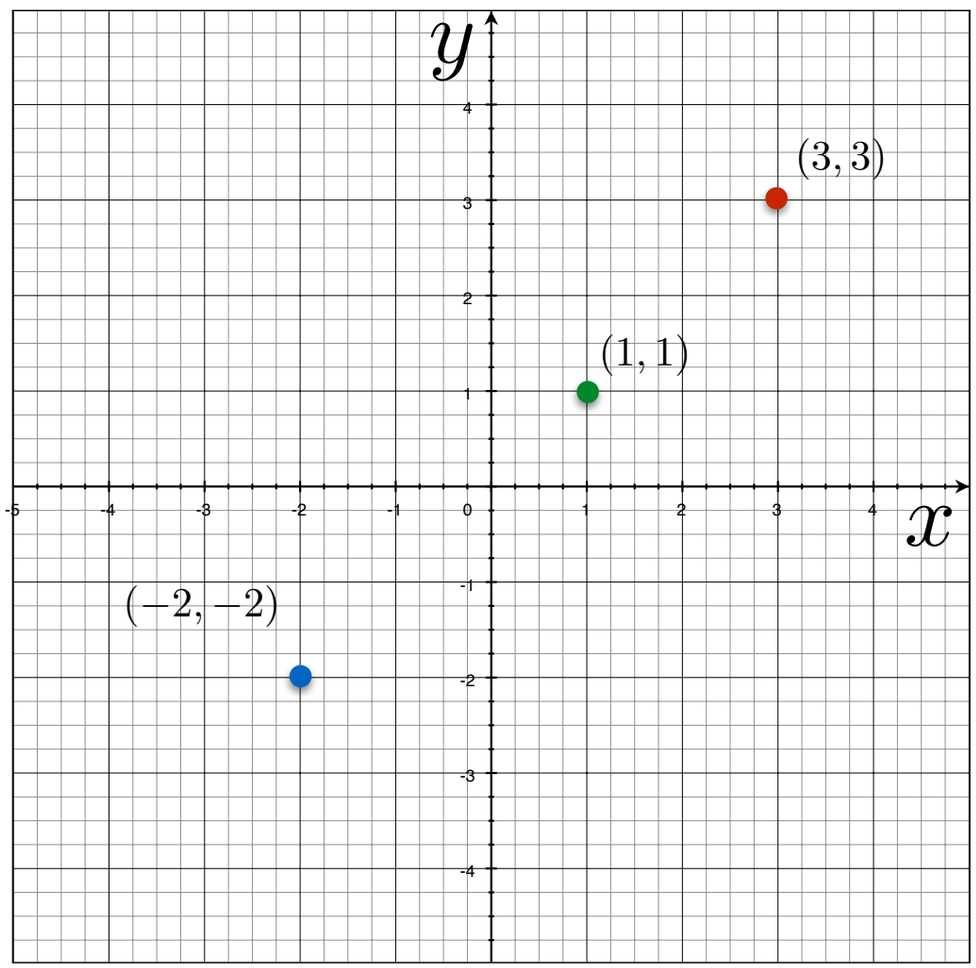
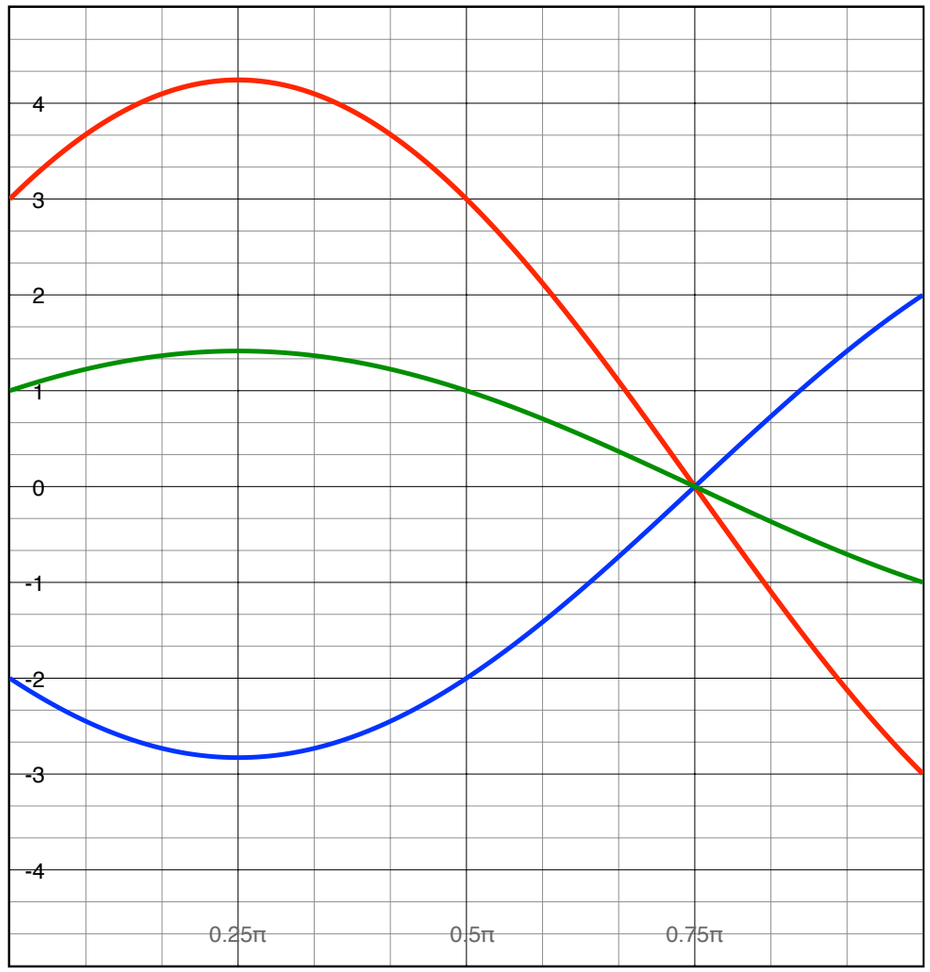


Image space

three points become ?



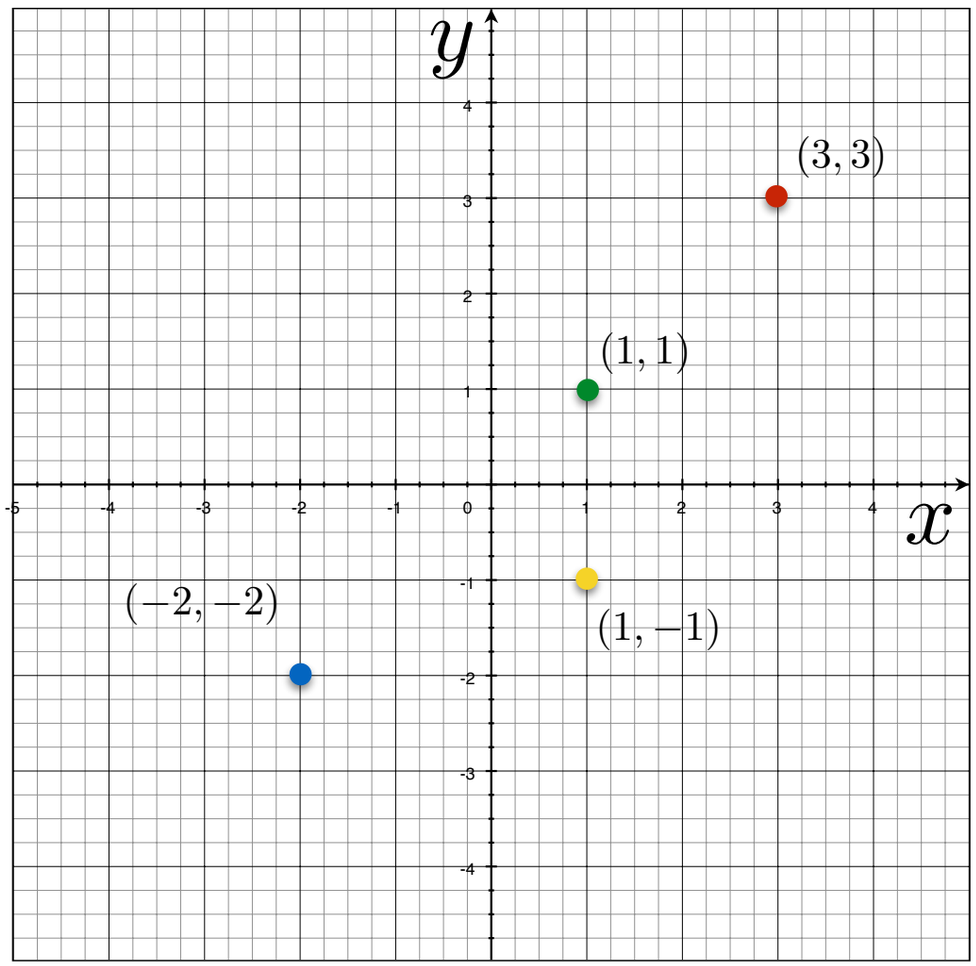
Parameter space

Image and parameter space

variables

$$y = mx + b$$

parameters



four points become ?

Image space

Parameter space

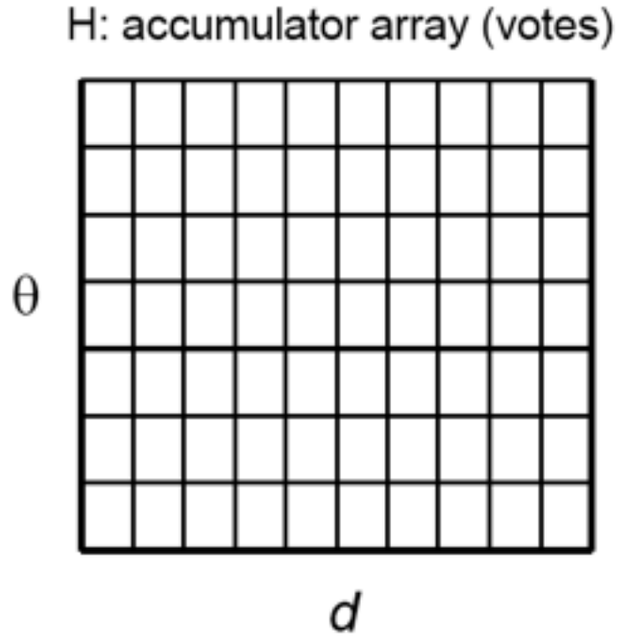
implementation

1. Initialize accumulator H to all zeros

2. For each edge point (x,y) in the image
   For θ = 0 to 180
      ρ = x cos θ + y sin θ
      H(θ, ρ) = H(θ, ρ) + 1
   end
end

3. Find the value(s) of (θ, ρ) where H(θ, ρ) is a local maximum

4. The detected line in the image is given by
   ρ = x cos θ + y sin θ



NOTE: Watch your coordinates. Image origin is top left!

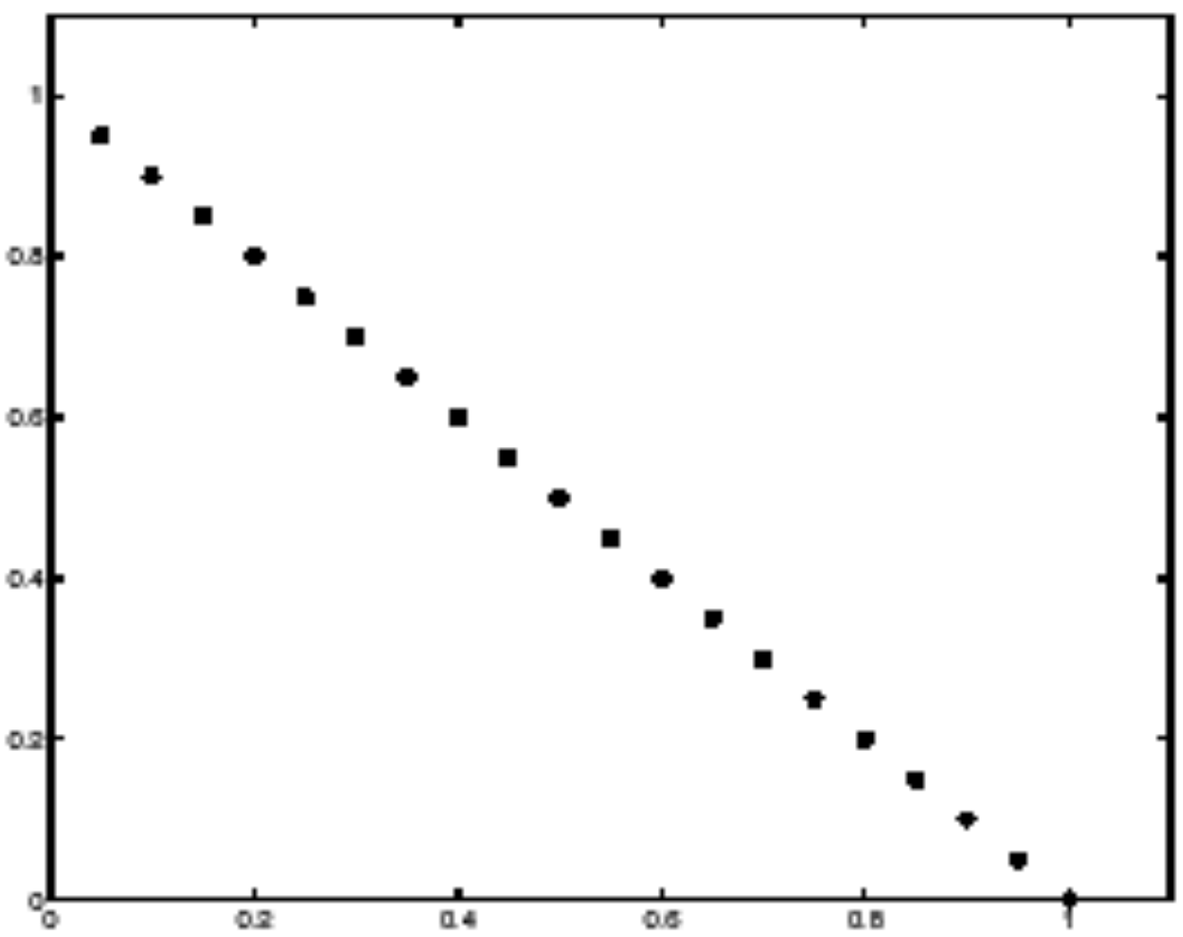
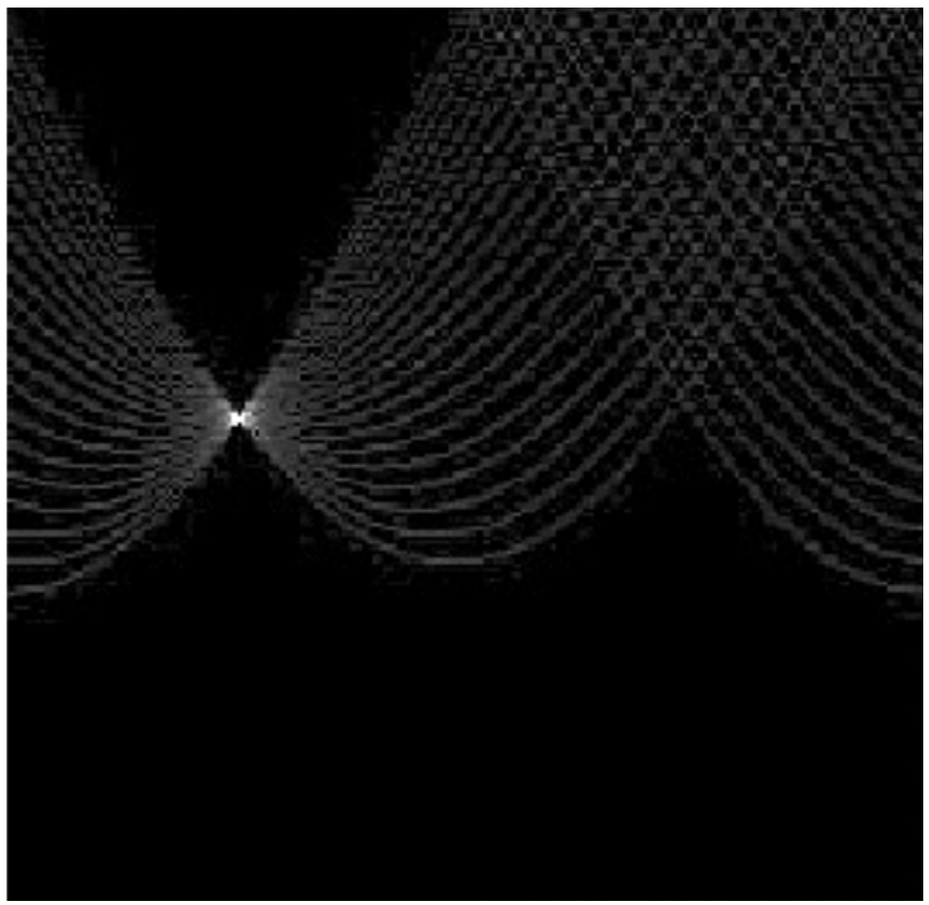


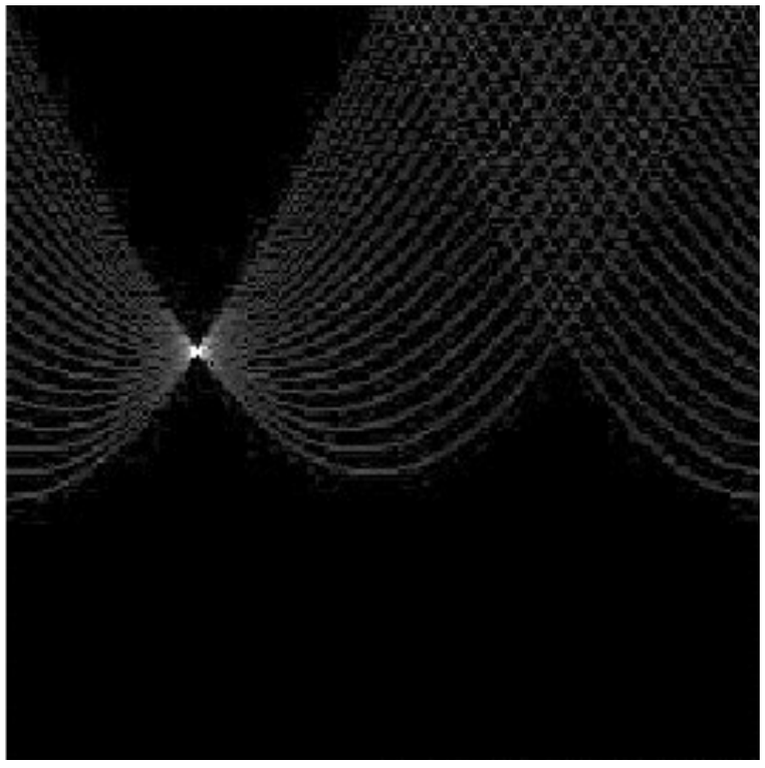
Image space

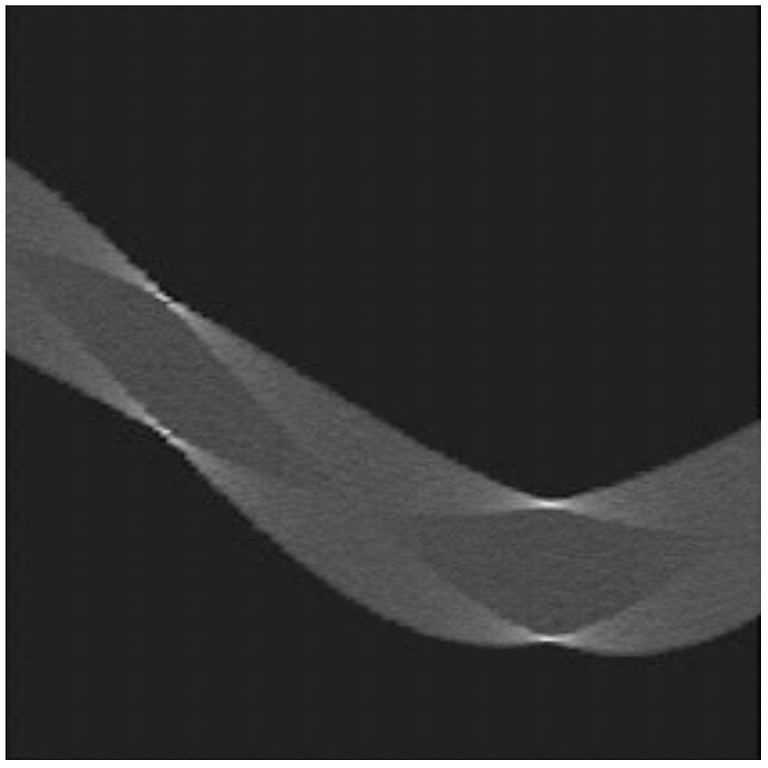


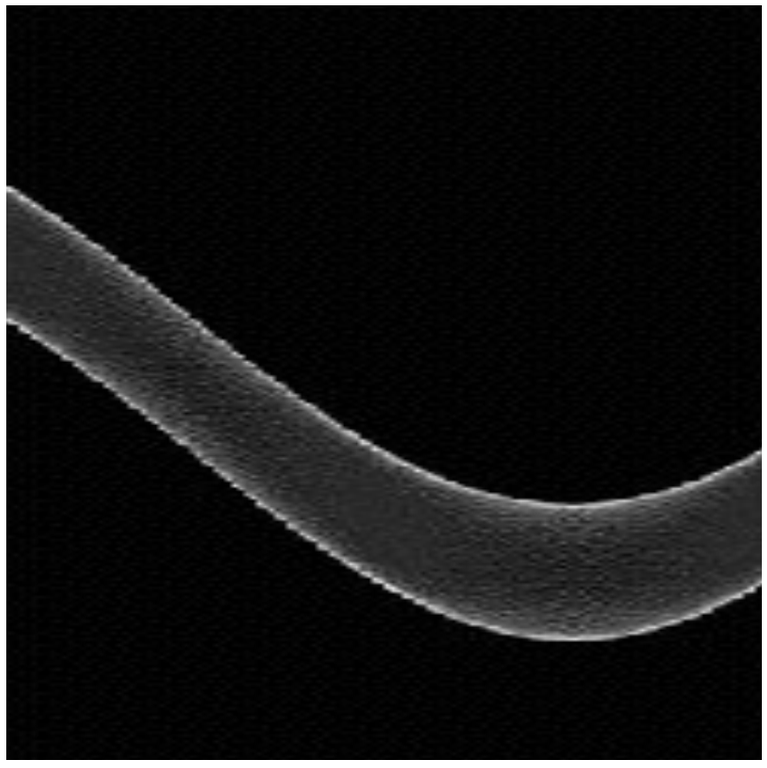
Votes

Basic shapes

(in polar parameter space)



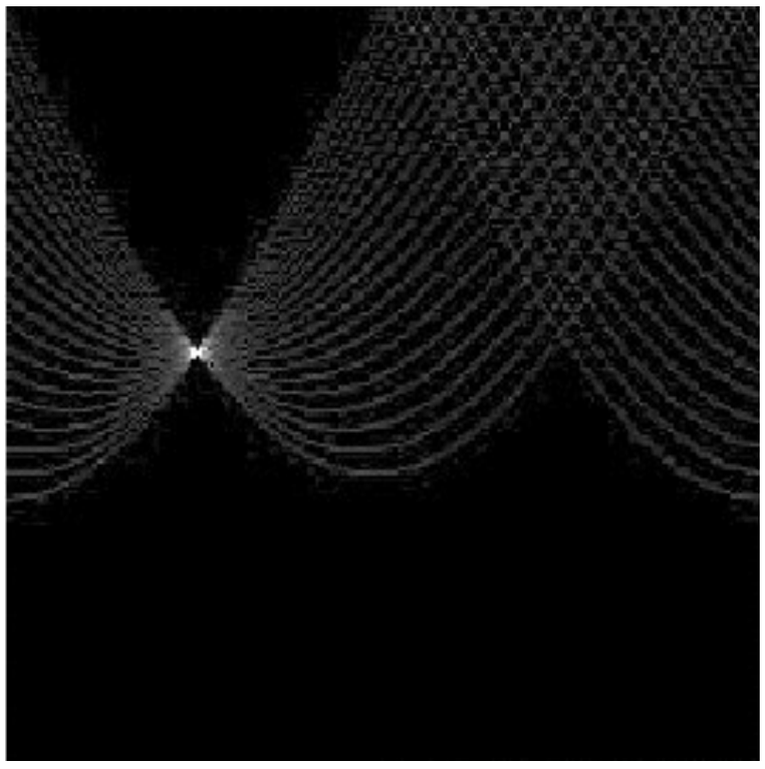




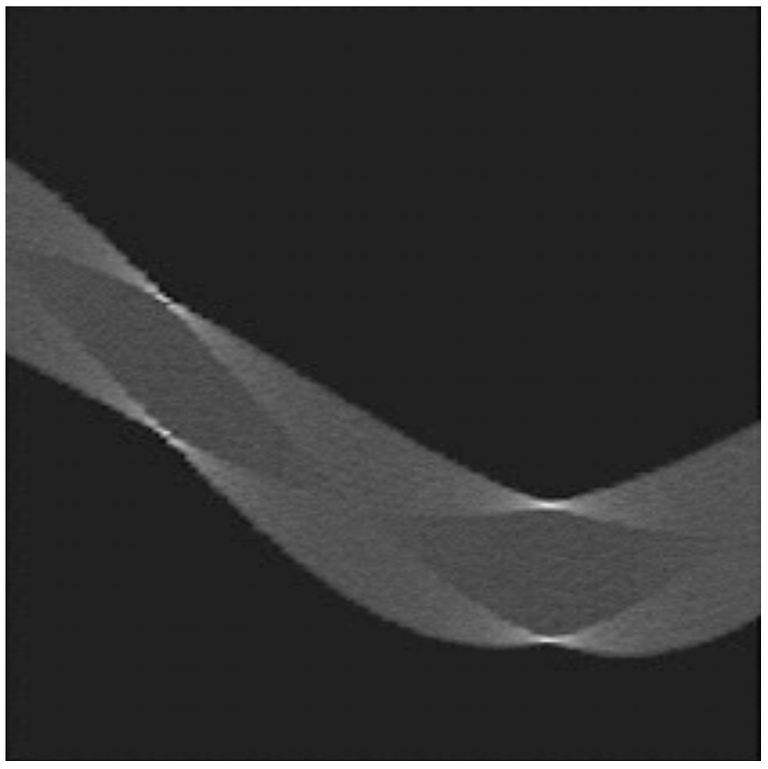
can you guess the shape?

Basic shapes

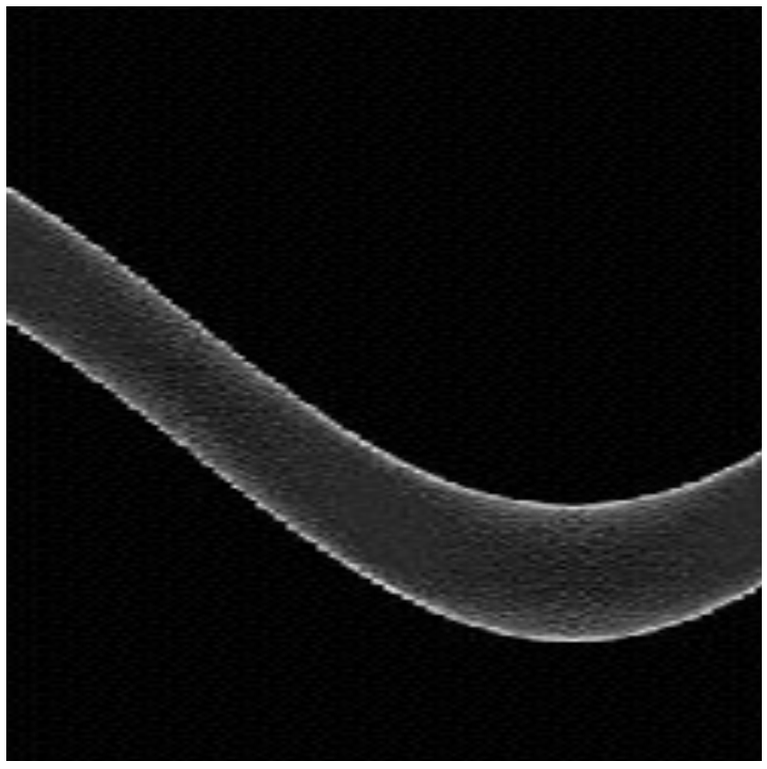
(in polar parameter space)



line

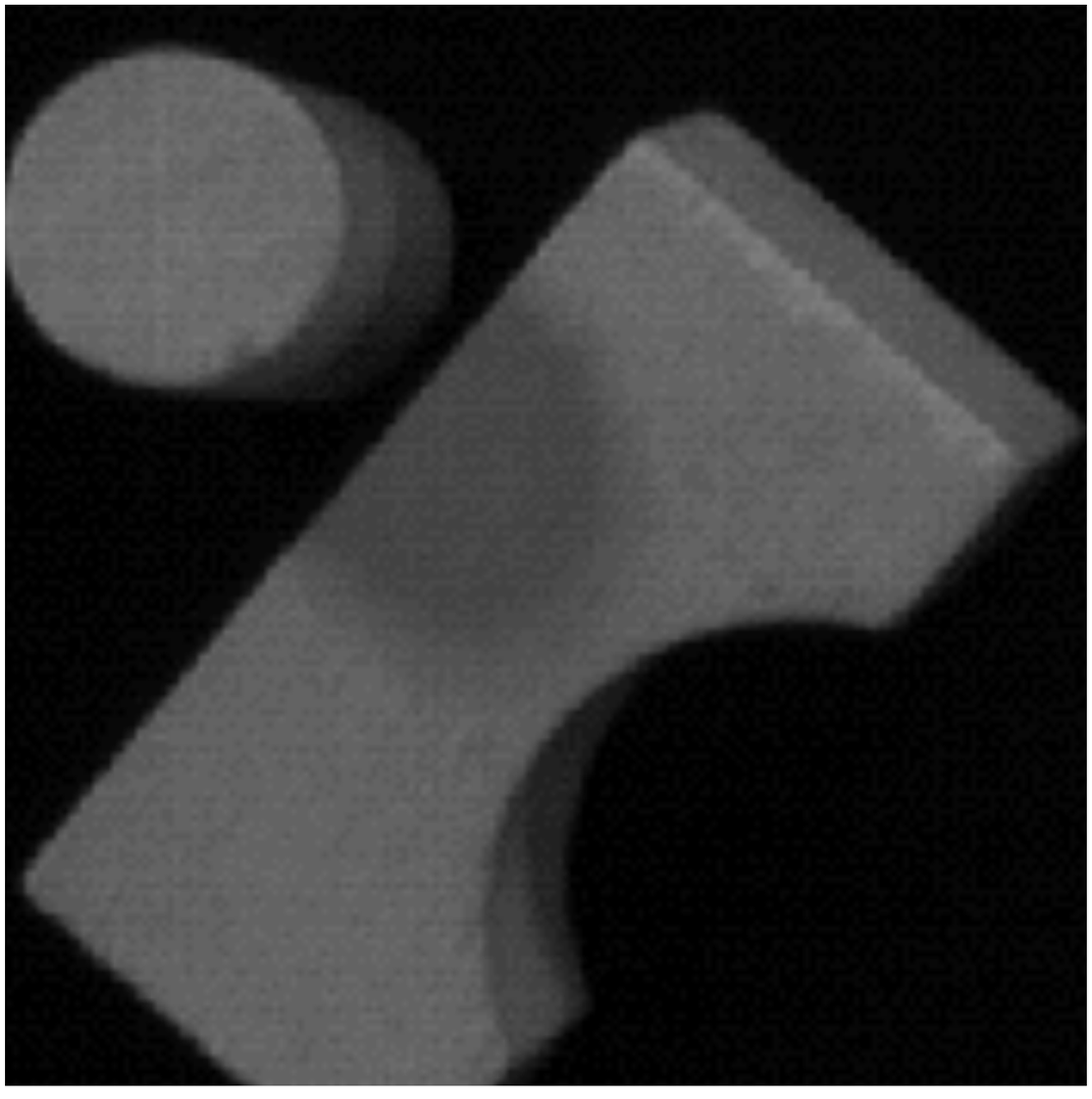


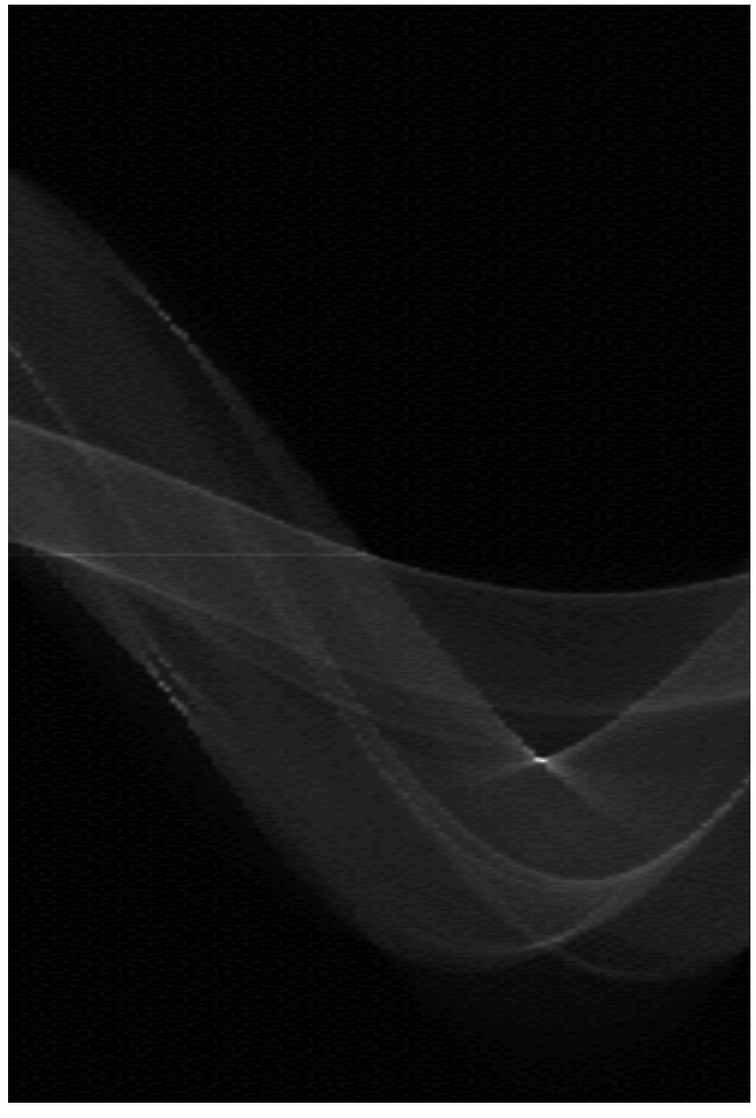
rectangle



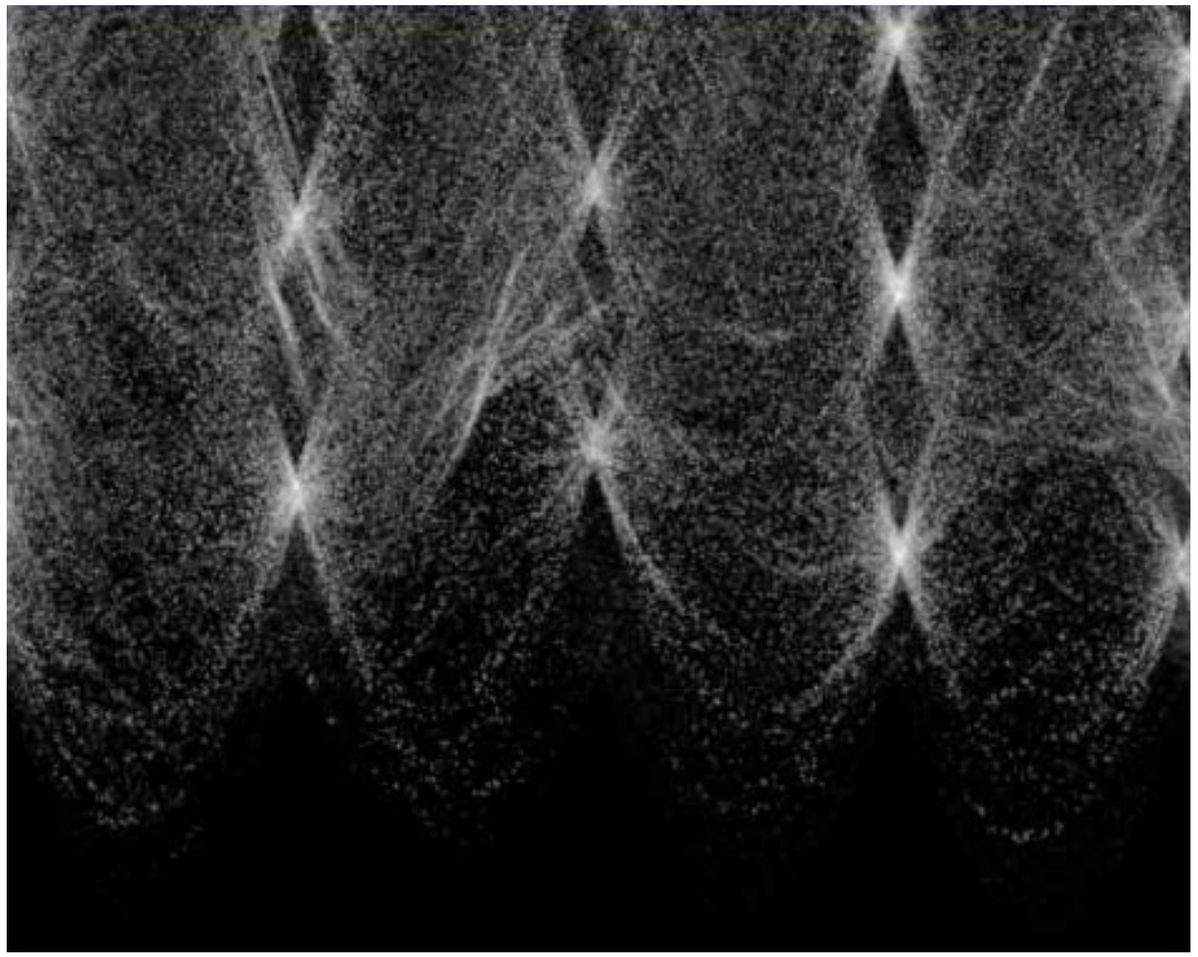
circle

Basic Shapes





More complex image



In practice, measurements are noisy…

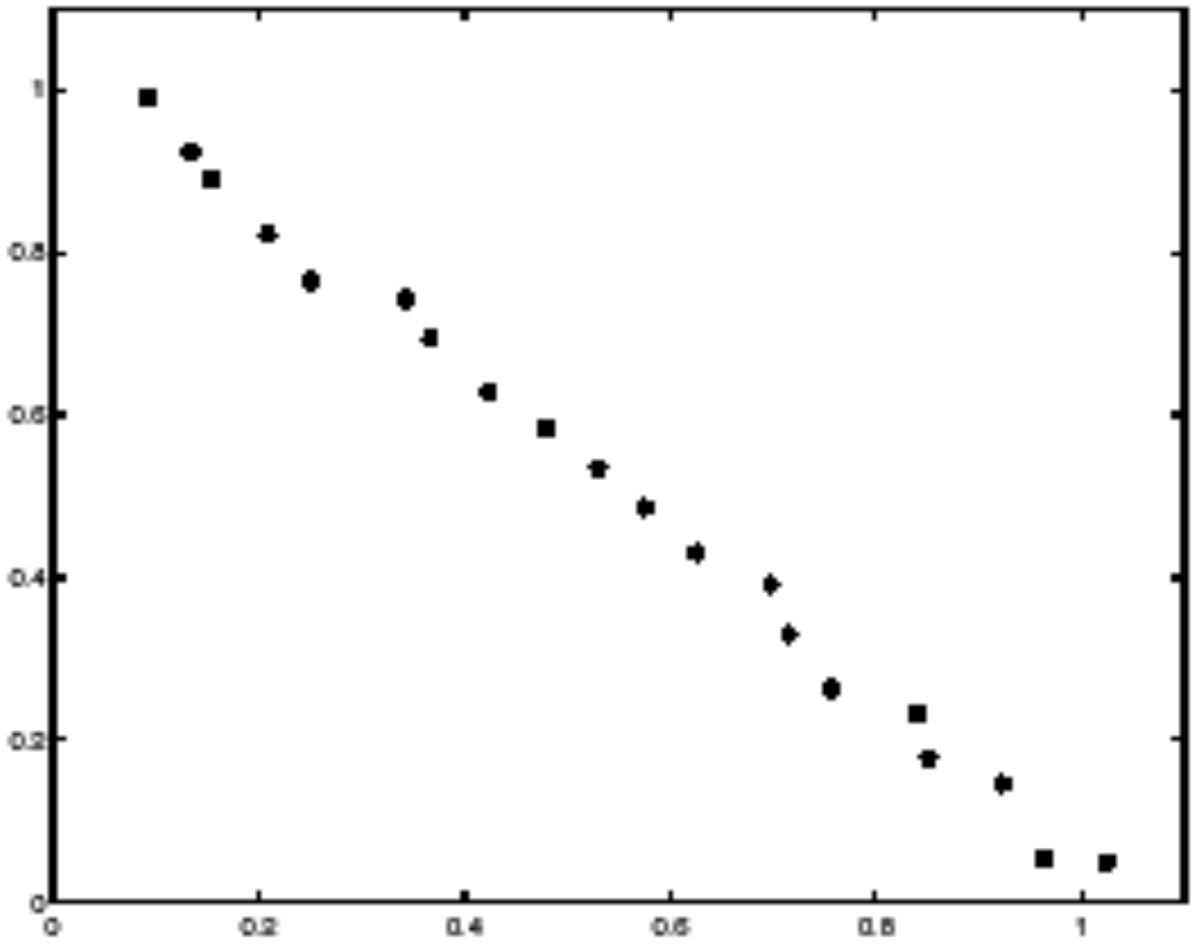
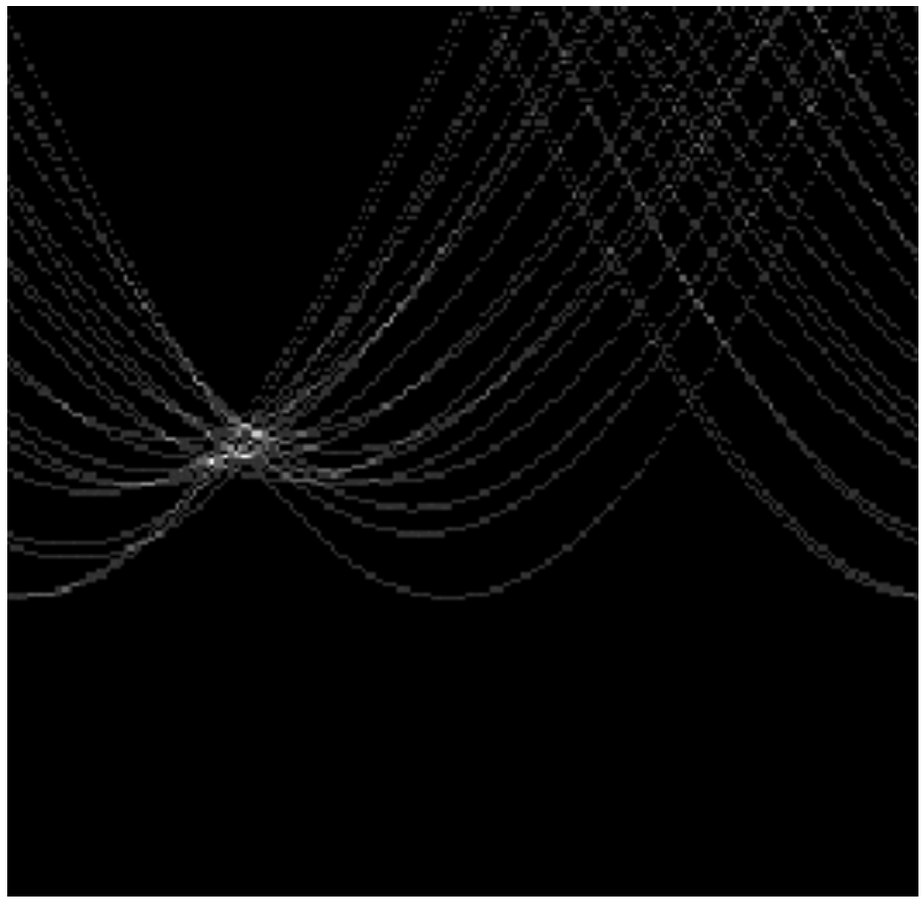


Image space



Votes

Too much noise …

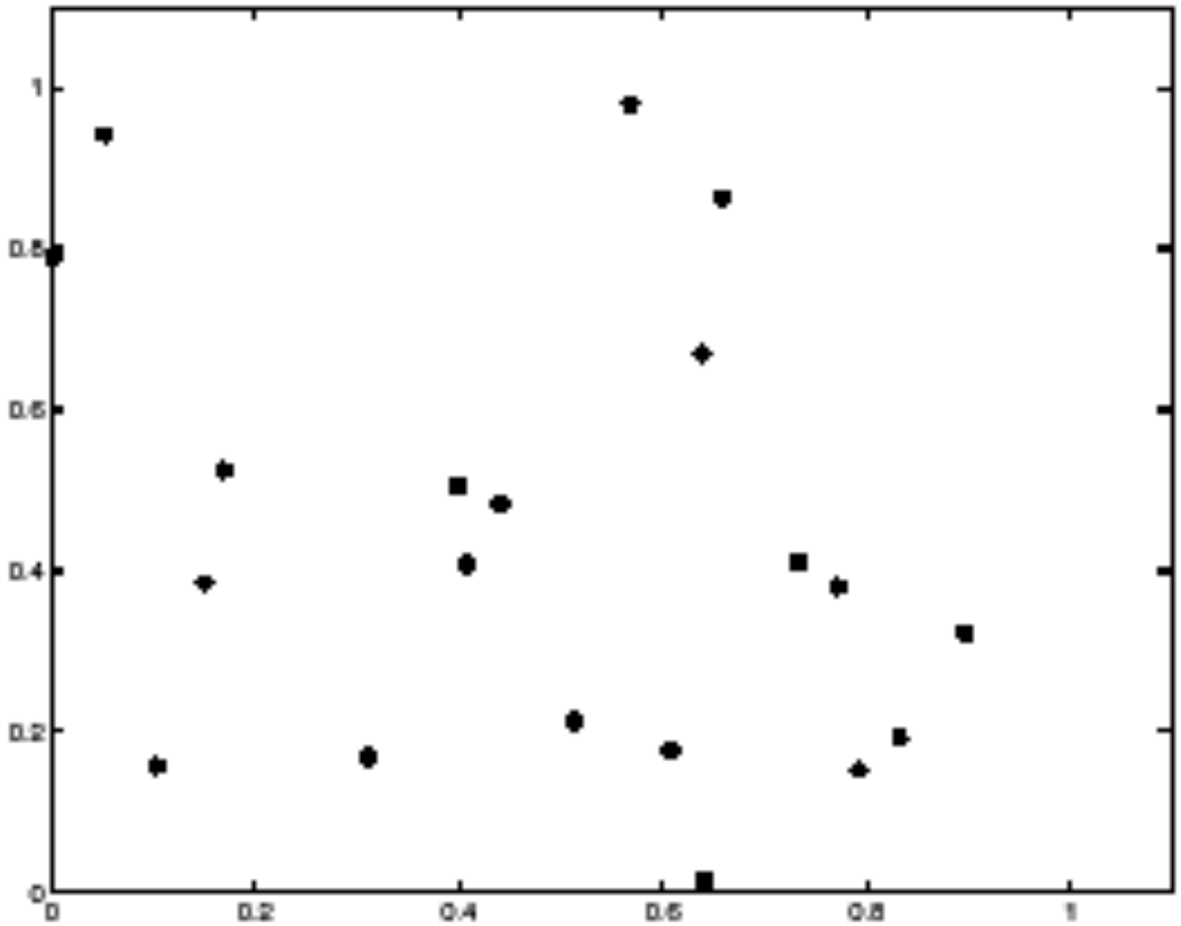
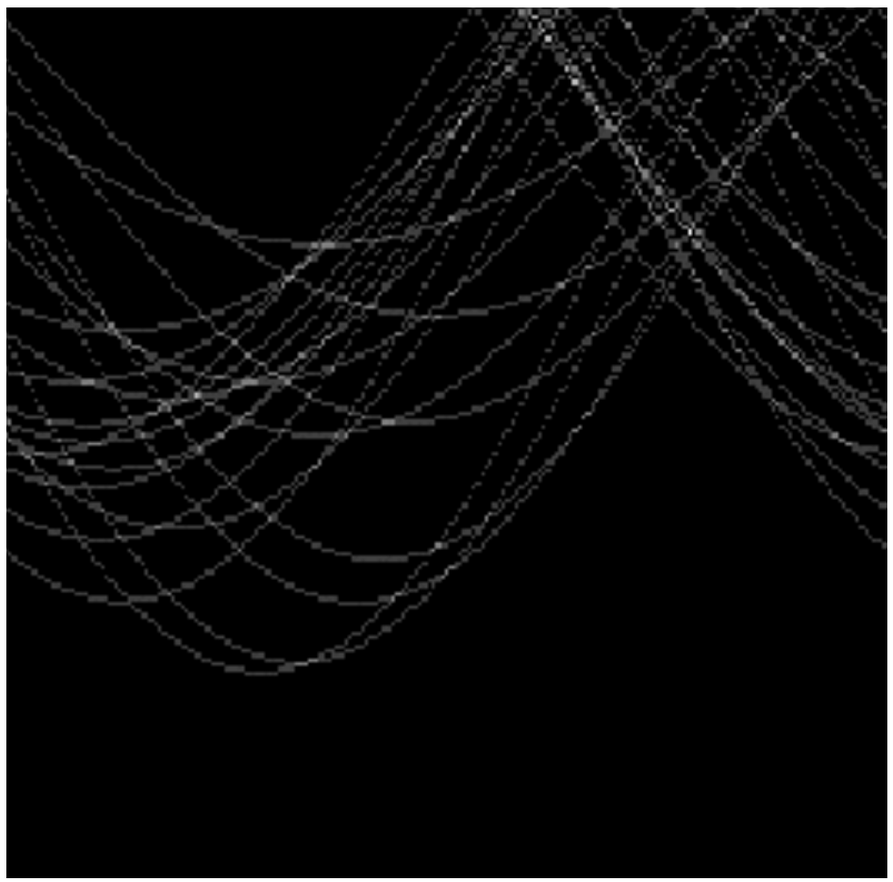
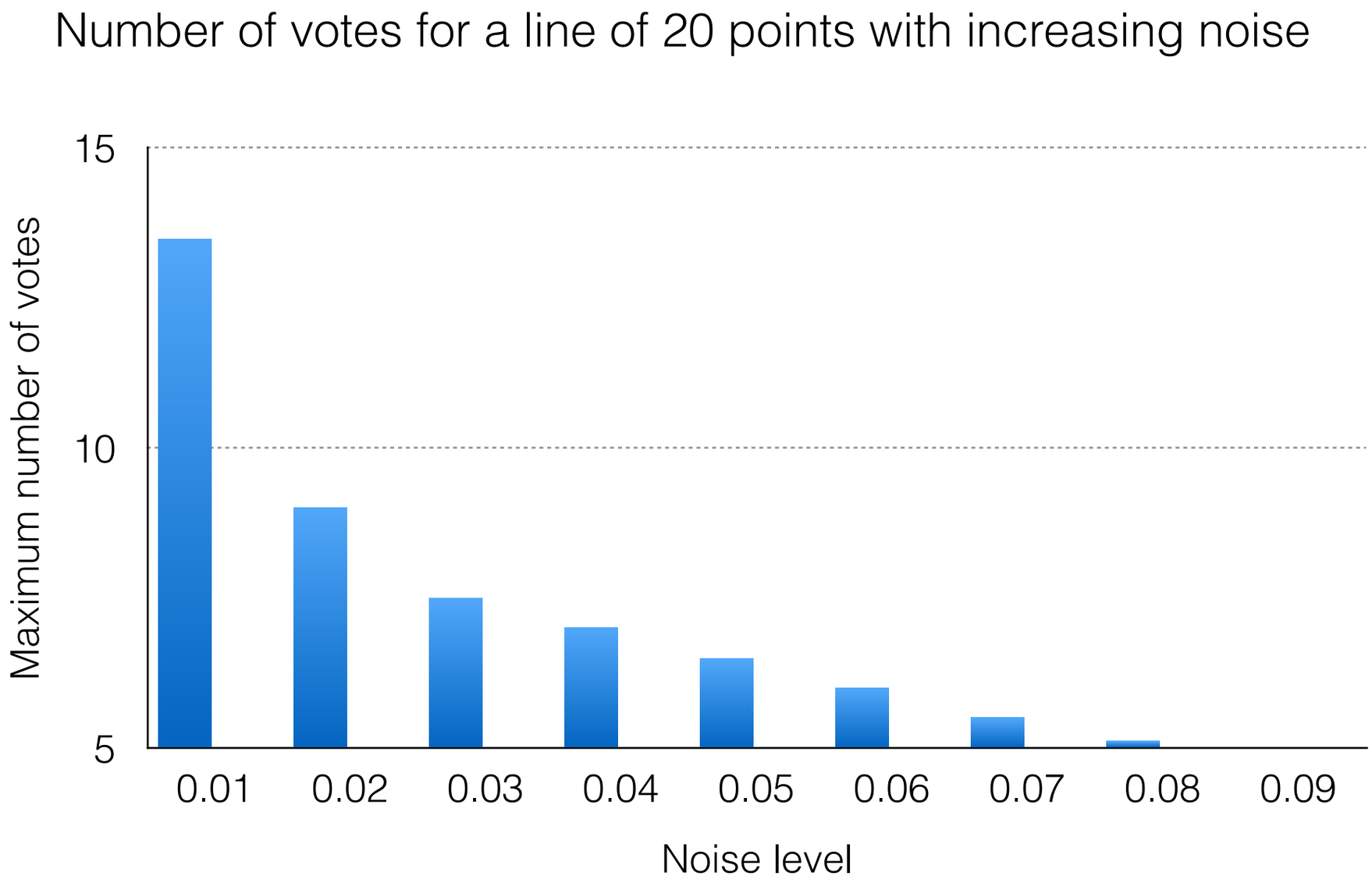


Image space



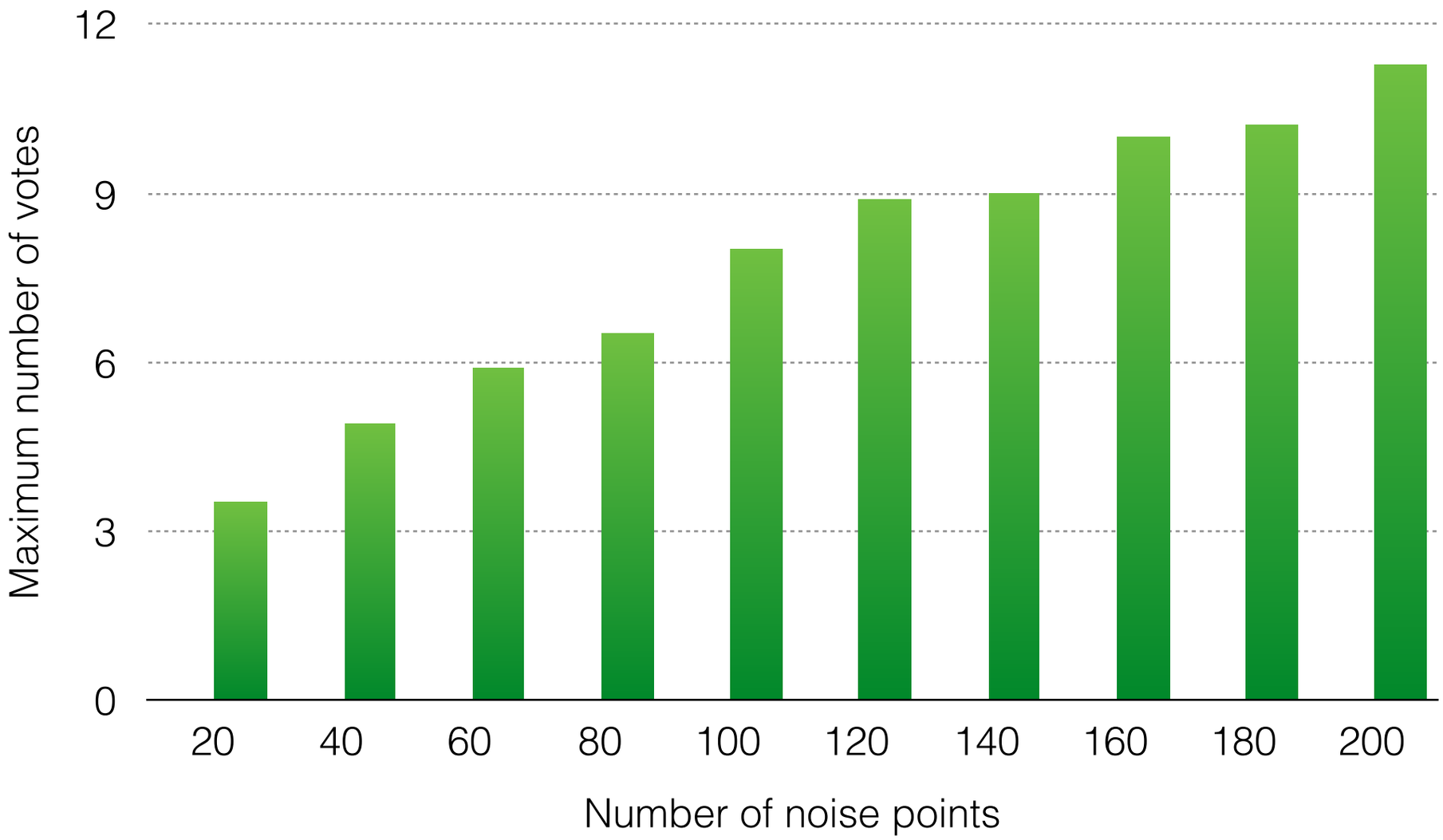
Votes

Effects of noise level



More noise, less votes (in the right bin)

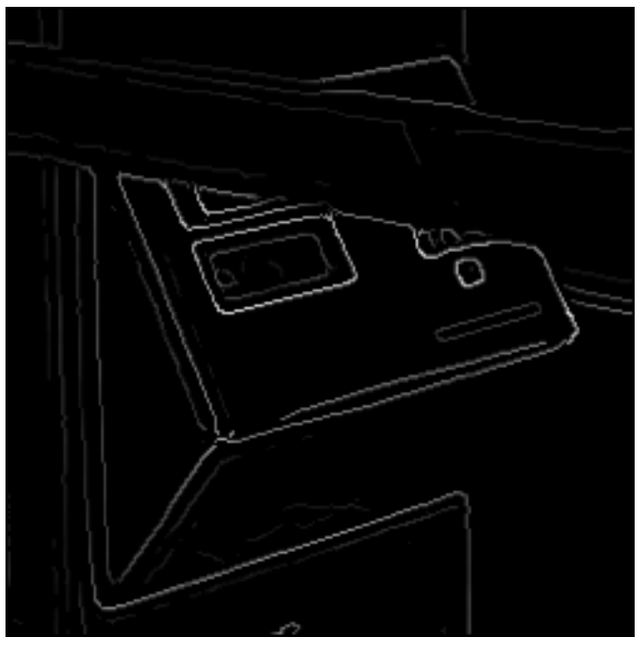
Effect of noise points



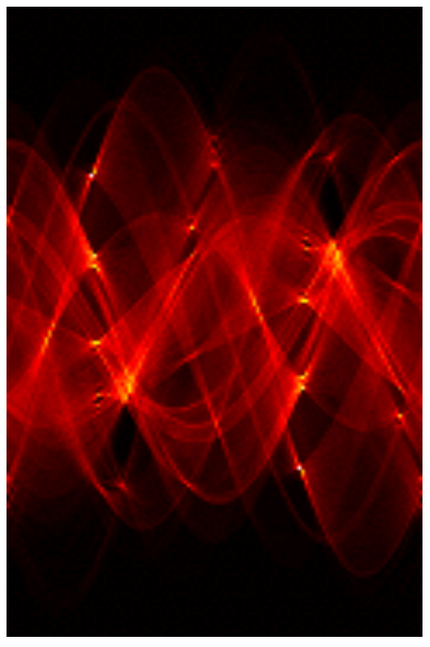
More noise, more votes (in the wrong bin)

Real-world example

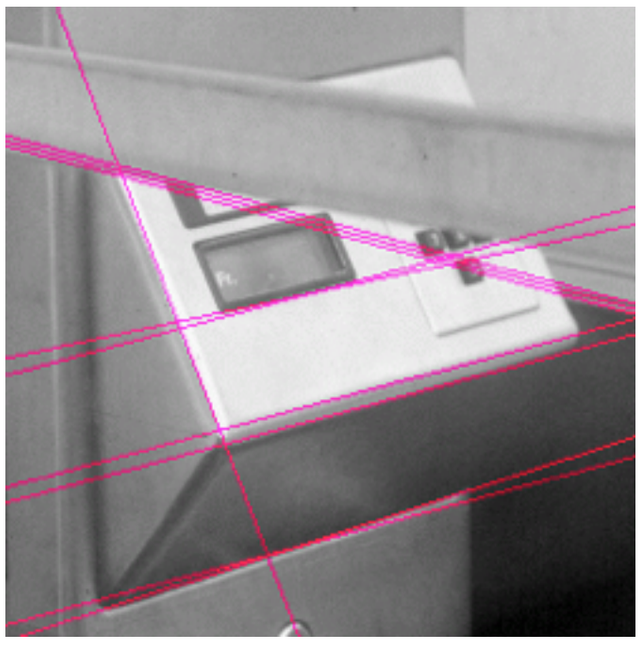
Original



Edges



parameter space



Hough Lines